

Inflation in Supergravity and cosmological attractors

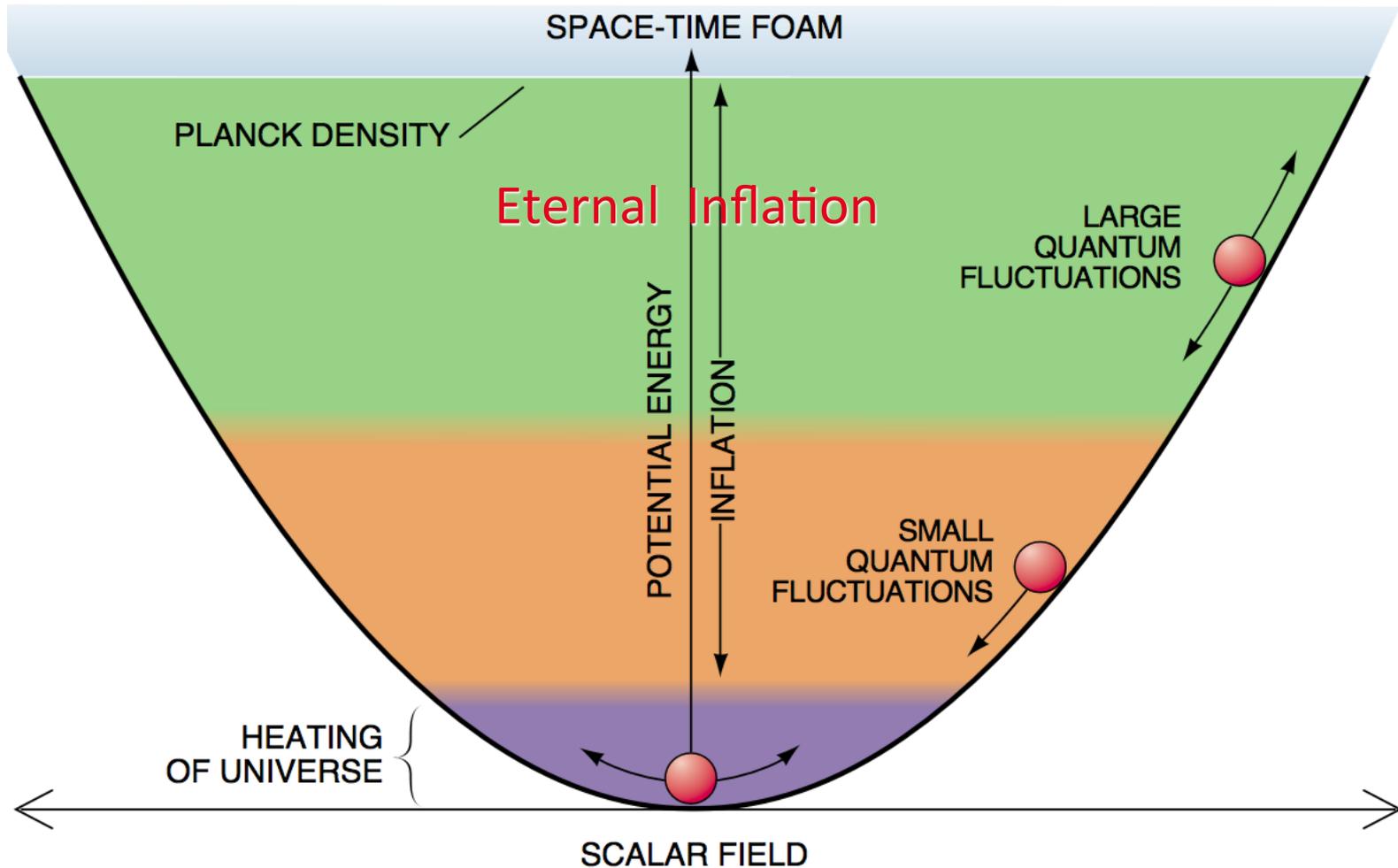
Andrei Linde

with Kallosh, Ferrara, Porrati, Van Proeyen, Roest, Verhocke, Westphal, Wrase

Our goal is to find inflationary models which are flexible enough to fit the data (either Planck 2013 or BICEP2), which can be implemented in string theory or supergravity, and which may tell us something interesting and instructive.

The simplest chaotic inflation model

$$V(\phi) = \frac{m^2}{2}\phi^2$$

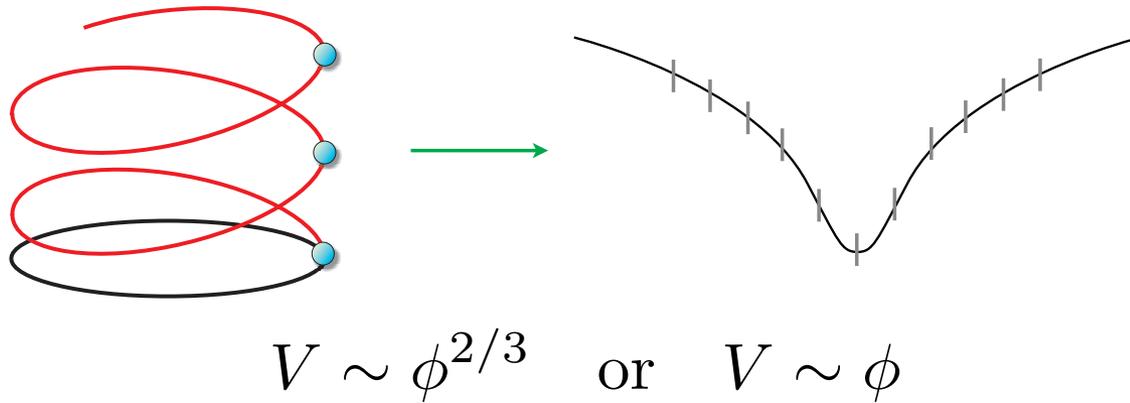


Chaotic Inflation in String Theory

An elegant example: Axion monodromy

Silverstein, Westphal and McAllister, Silverstein, Westphal, 2008

- unwind a periodic field direction into a *monodromy*
→ e.g. by employing a wrapped brane



New results

[McAllister, Silverstein, Westphal, Wrase, 1405.3652](#)

Monomial potentials $\mu^{4-p} \phi^p$

$p = 2/3, 4/3, 2, \text{ and } 3.$

$r \approx 0.04, 0.09, 0.13, \text{ and } 0.2, \text{ respectively}$

Chaotic inflation in supergravity

Main problem:

$$V(\phi) = e^K \left(K_{\Phi\bar{\Phi}}^{-1} |D_{\Phi}W|^2 - 3|W|^2 \right)$$

Canonical Kahler potential is $K = \Phi\bar{\Phi}$

Therefore the potential blows up at large $|\phi|$, and slow-roll inflation is impossible:

$$V \sim e|\Phi|^2$$

Too steep, no inflation...

A solution: shift symmetry

Kawasaki, Yamaguchi, Yanagida 2000

Kahler potential $\mathcal{K} = S\bar{S} - \frac{1}{2}(\Phi - \bar{\Phi})^2$

and superpotential $W = mS\Phi$

The potential is very curved with respect to S and $\text{Im } \Phi$, so these fields vanish. But Kahler potential does not depend on

$$\phi = \sqrt{2} \text{Re } \Phi = (\Phi + \bar{\Phi})/\sqrt{2}$$

The potential of this field has the simplest form, as in chaotic inflation, without any exponential terms:

$$V = \frac{m^2}{2} \phi^2$$

Quantum corrections do not change this result

More general models

Kallos, AL 1008.3375, Kallos, AL, Rube,1011.5945

$$W = S f(\Phi)$$

Superpotential must be a REAL holomorphic function. (We must be sure that the potential is symmetric with respect to $\text{Im}\Phi$, so that $\text{Im}\Phi = 0$ is an extremum (then we will check that it is a minimum). The Kahler potential is any function of the type

$$\mathcal{K}((\Phi - \bar{\Phi})^2, S\bar{S})$$

The potential as a function of the real part of Φ at $S = 0$ is

$$V = |f(\Phi)|^2$$

FUNCTIONAL FREEDOM in choosing inflationary potential

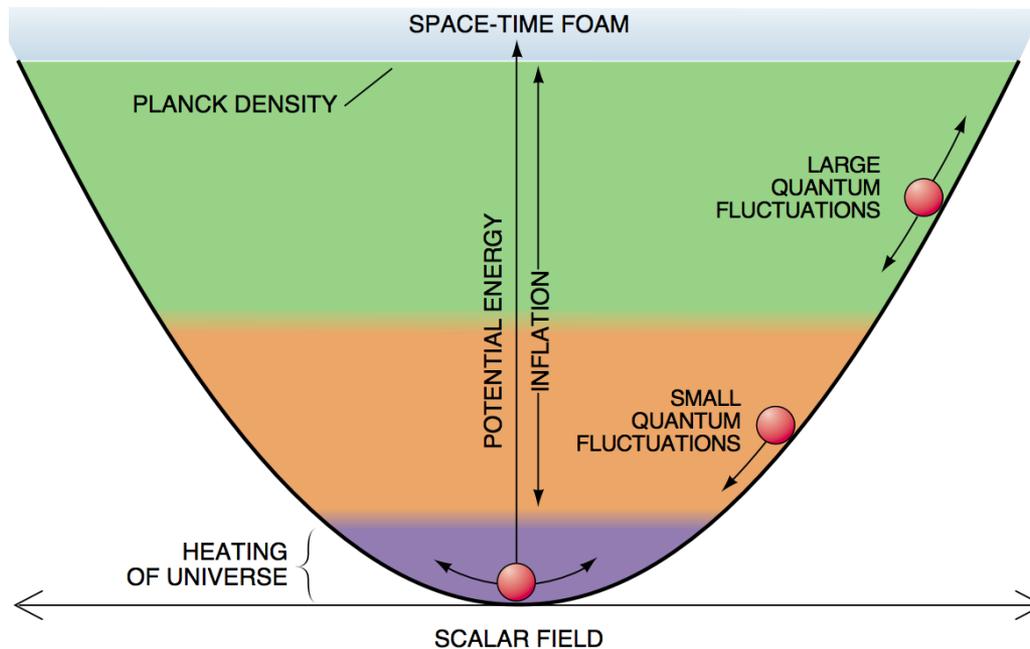
Here S is a goldstino multiplet: supersymmetry is broken only in the goldstino direction

FUNCTIONAL FREEDOM in choosing inflationary potential **in supergravity** allows us to fit **any set** of n_s and r .

Example: $W = mS\Phi$

During inflation $S = 0$, $\text{Im } \Phi = 0$, $\text{Re } \Phi = \sqrt{2} \phi$

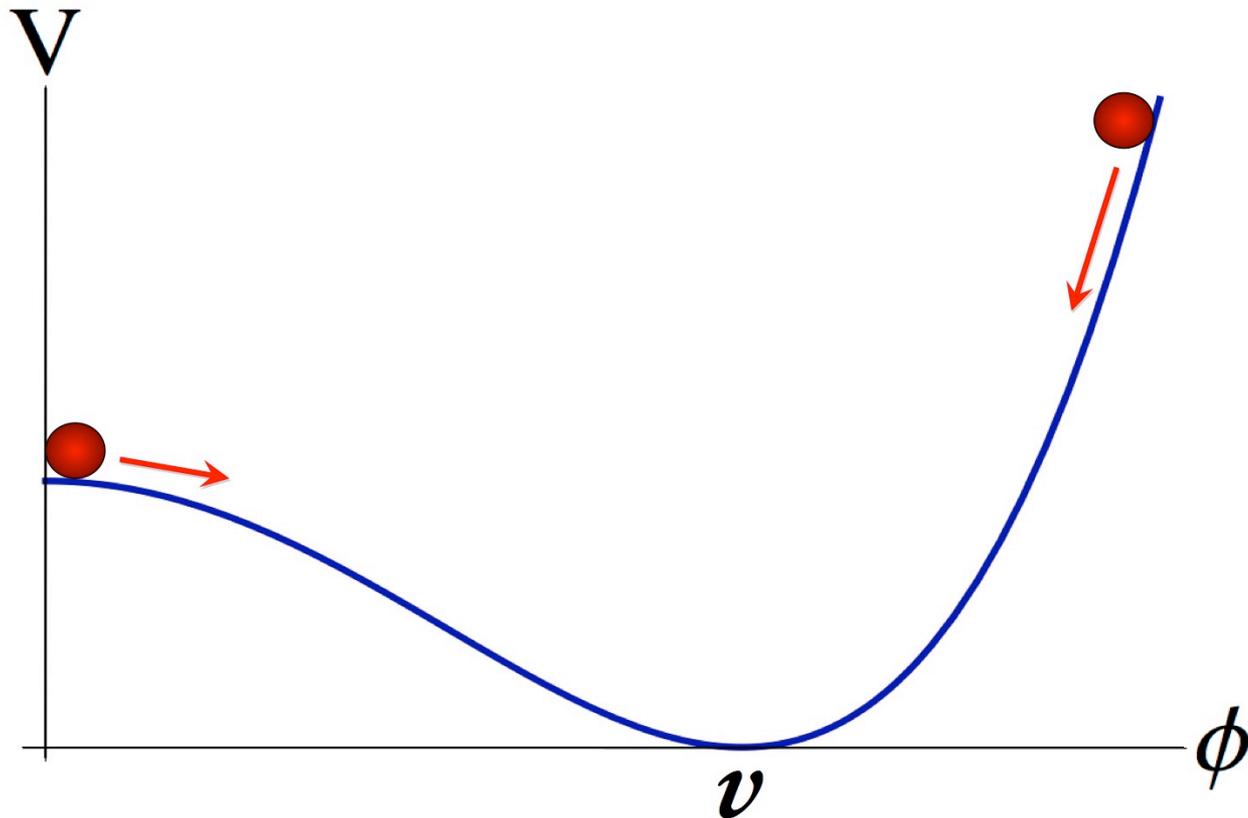
$$V = \frac{m^2}{2} \phi^2$$



Example: $W = mS\Phi(1 - \alpha\Phi)$

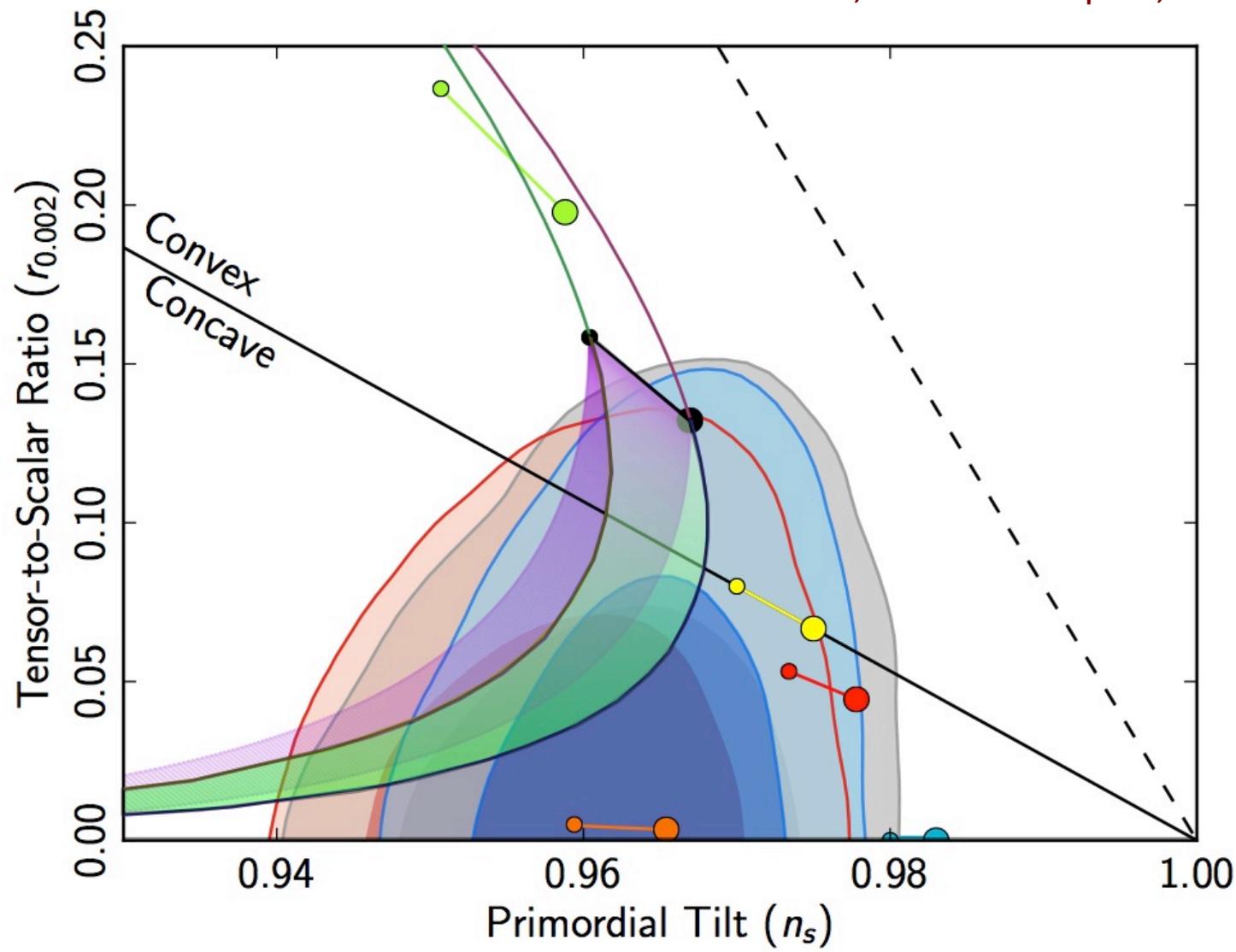
During inflation $S = 0$, $\text{Im } \Phi = 0$, $\text{Re } \Phi = \sqrt{2} \phi$

$$V(\phi) = \frac{\lambda^2}{4} (\phi^2 - v^2)^2 \quad \text{where} \quad v \sim \frac{1}{\alpha} \gg 1$$



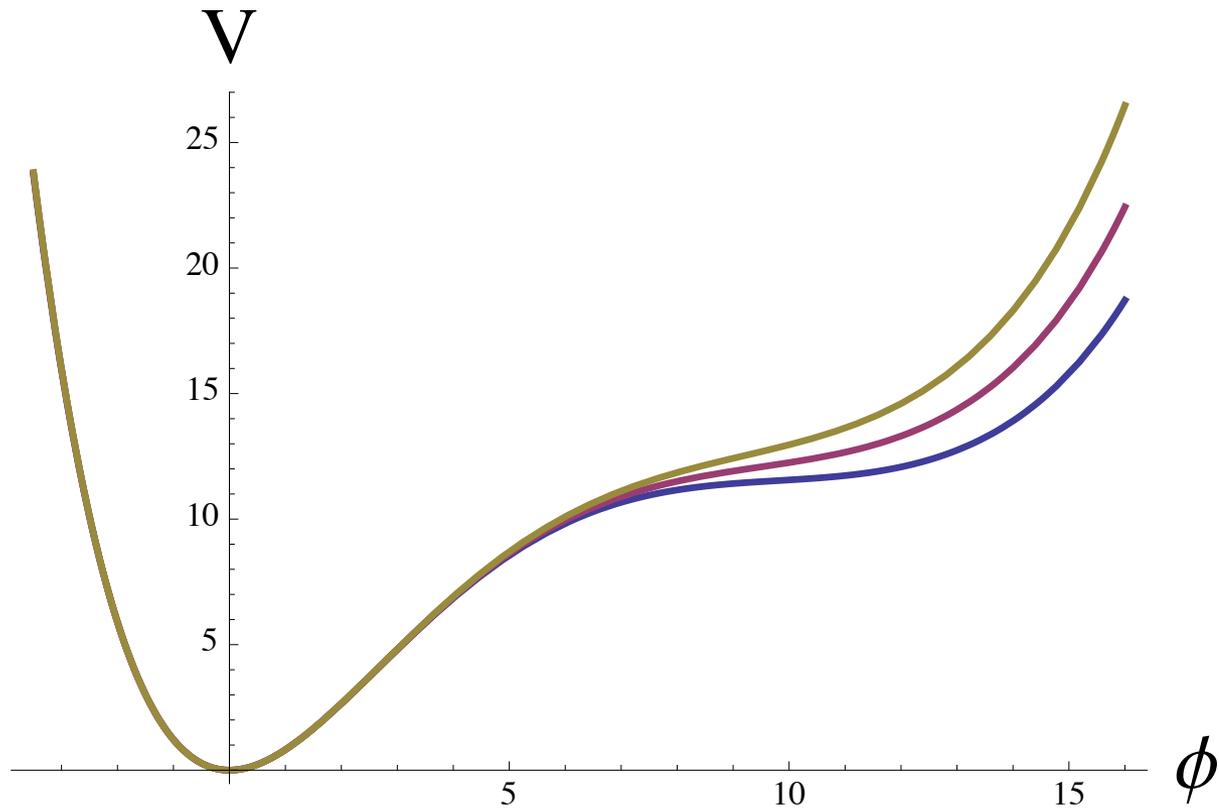
$$V(\phi) = \frac{\lambda}{4} (\phi^2 - v^2)^2$$

Kallosch, AL and Westphal, 1405.0270



$$W = mS\Phi(1 - \alpha\Phi + \beta\Phi^2)$$

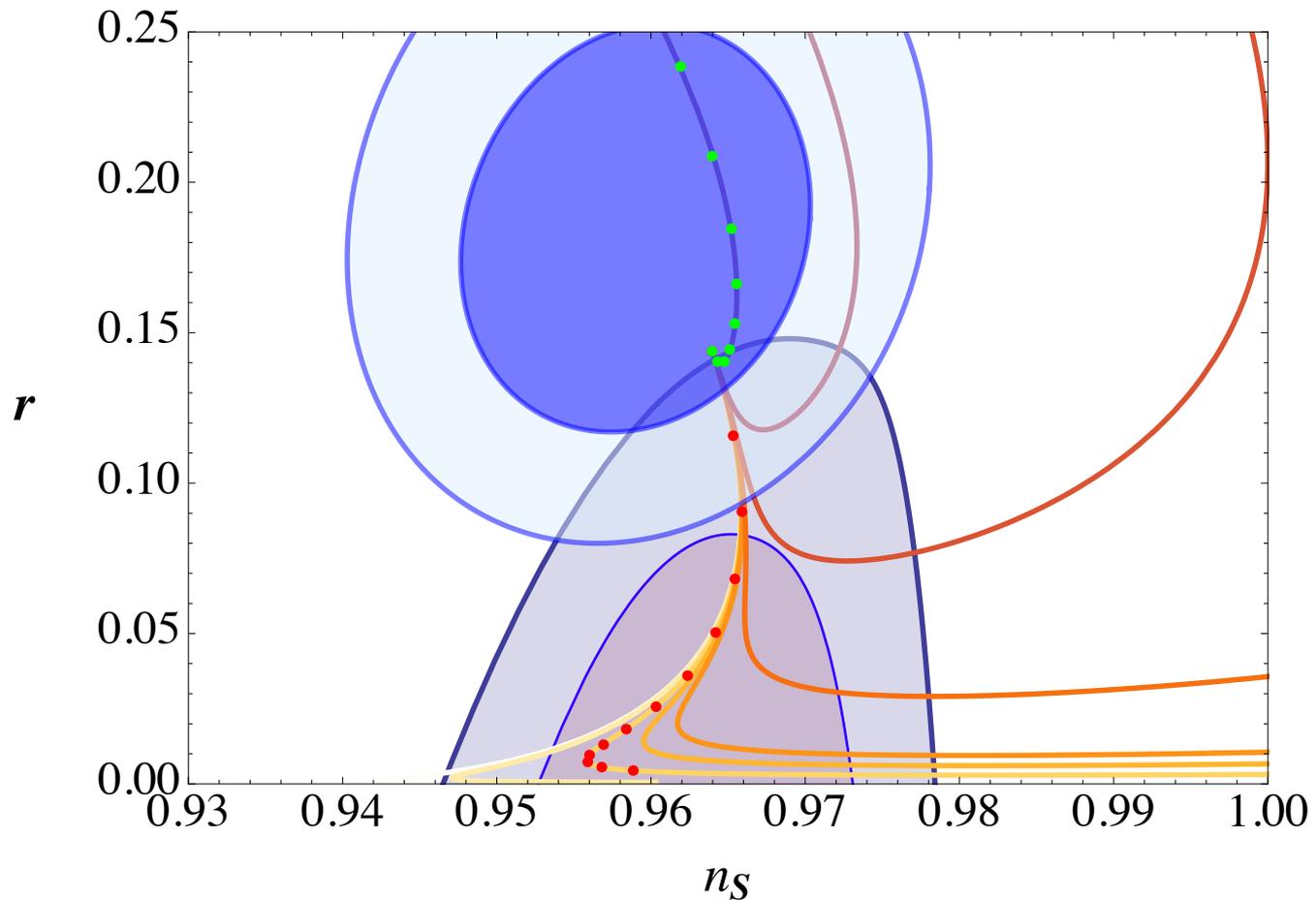
$$V(\phi) = \frac{m^2\phi^2}{2} (1 - a\phi + a^2b\phi^2)^2$$



$$V(\phi) = \frac{m^2 \phi^2}{2} (1 - a\phi + a^2 b \phi^2)^2$$

Kalosh, AL and Westphal, 1405.0270

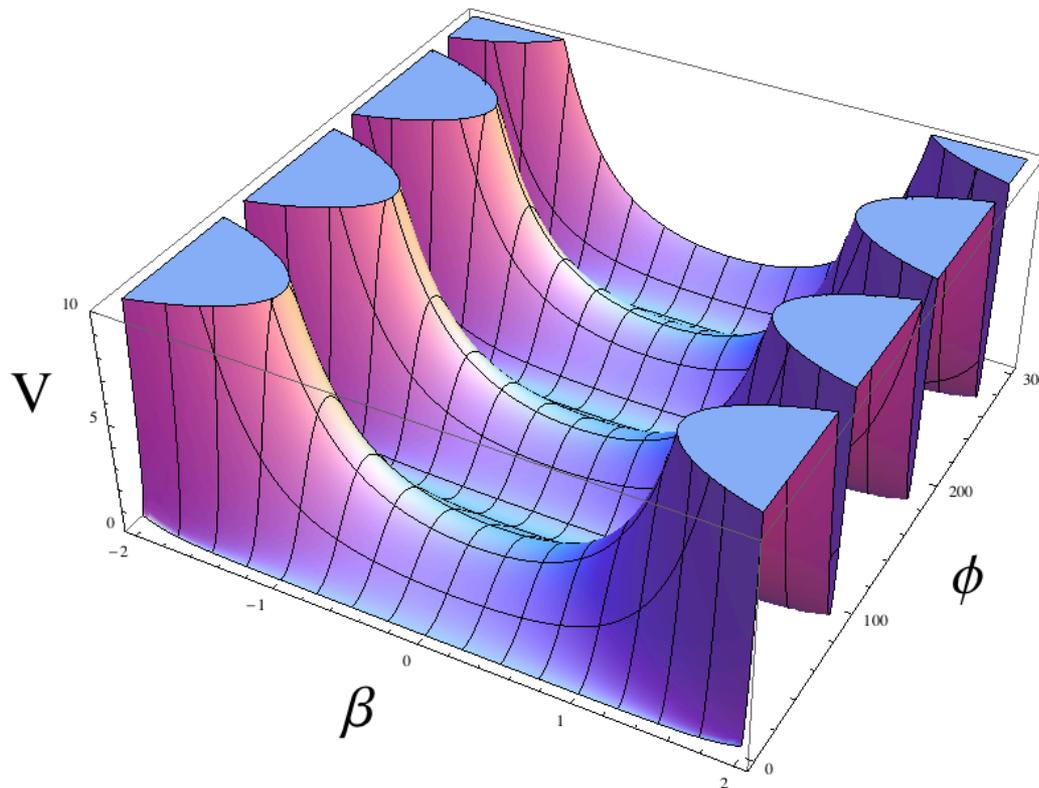
Using **3 parameters** (m , a , b) one can fit any set of **3 observational results**:
The amplitude of perturbations, n_s , and r .



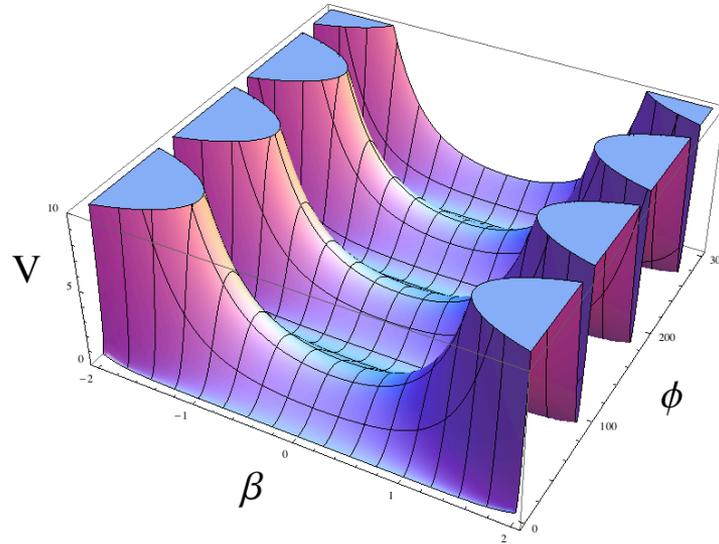
Natural Inflation in Supergravity

Natural inflation in theories with axion potentials is known for nearly 25 years (Freese et al 1990), but **until now it did not have any stable supergravity generalization**. Invariably, there was an instability with respect to some moduli, or we needed some assumptions about string theory uplifting. The problem was solved only recently:

Kallosh, AL, Vercnocke, 1404.6204



Natural Inflation in Supergravity



$$W = \Lambda^2 S(1 - e^{-aT}), \quad K = \frac{1}{2}(T + \bar{T})^2 + S\bar{S} - g(S\bar{S})^2$$

$$V|_{S=0, T+\bar{T}=0} = 2\Lambda^4 \left(1 - \cos \frac{a\phi}{\sqrt{2}}\right)$$

All non-inflaton moduli stabilized

Natural Inflation in Supergravity

$$V|_{S=0, T+\bar{T}=0} = 2\Lambda^4 \left(1 - \cos \frac{a\phi}{\sqrt{2}}\right)$$

Requires $a \ll 1$. Hard to achieve in string theory, but one could do it **if more than one axion field is involved**

Blanco-Pillado, Burgess, Cline, Escoda, Gomez-Reino, Kallosh, A.L., and Quevedo, hep-th/0406230,

Kim, Nilles and Peloso, hep-ph/0409138

However, in the first of these two papers the potential was not equivalent to the potential of natural inflation, whereas in the second one there was no supersymmetry.

One axion models in string theory inspired supergravity models with (non-supersymmetric) uplifting were also proposed in Kallosh hep-th/0702059, Kallosh, Sivanandam, Soroush, 0710.3429

The improved version of this model has been suggested recently, using the goldstino superfield S , where $S=0$ at the minimum

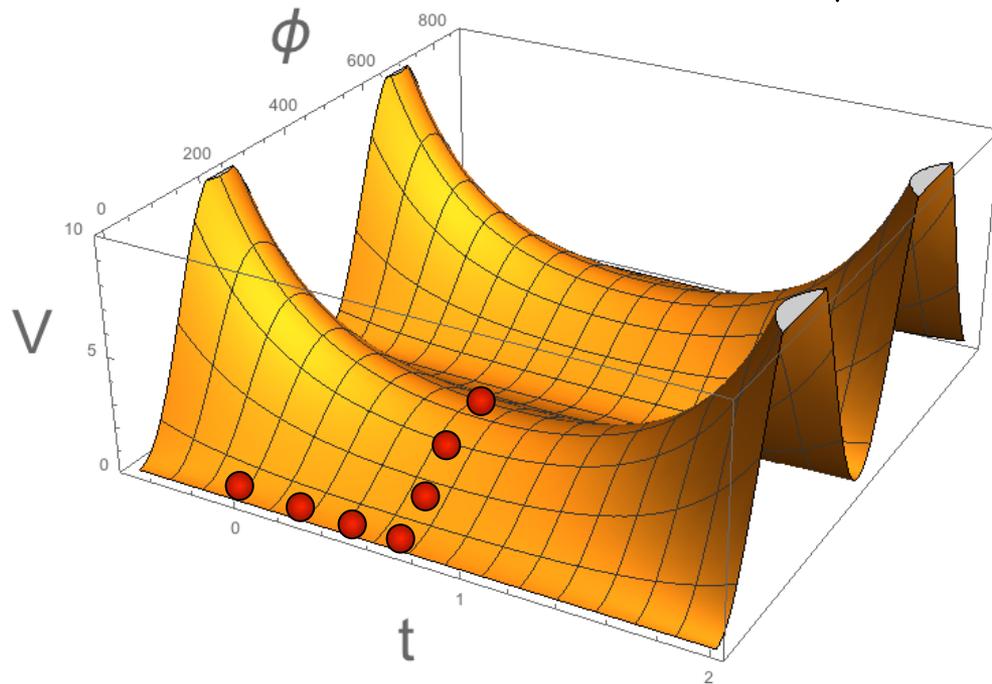
Large field natural inflation with a single axion

Kallosh, AL, Verhocke, 1404.6204

$$W = \Lambda^2 S(e^{-aT} - e^{-bT})$$

Natural inflation occurs even in theories with $a, b > 1$, as suggested by string theory. For $|a - b| \ll 1$, one can have natural inflation even in the theory with a single axion field.

$$V = 2\Lambda^4 e^{-a^2/2} \left(1 - \cos \frac{(a-b)\phi}{\sqrt{2}} \right)$$

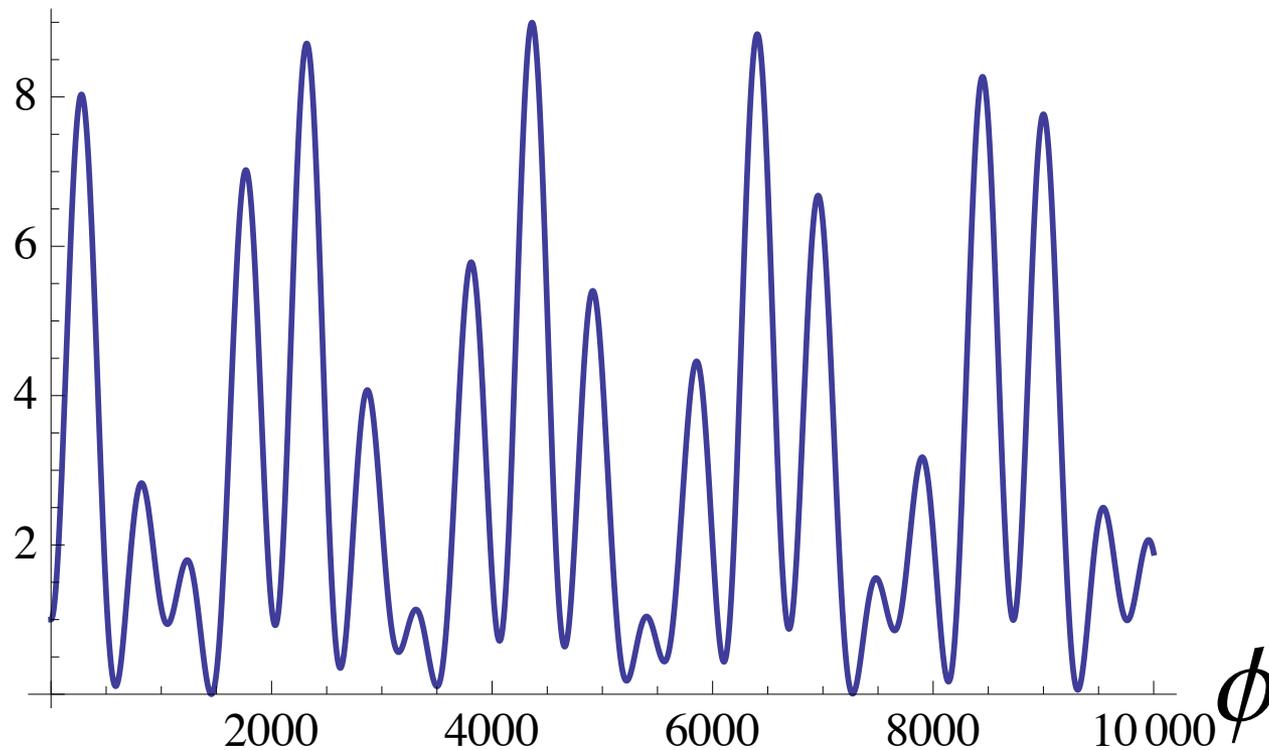


Irrationally natural inflation 😊

$$W = \Lambda^2 S(1 - Ae^{-aT} - Be^{-bT})$$

V

If a/b is irrational – an infinite landscape of different dS minima and inflationary regimes

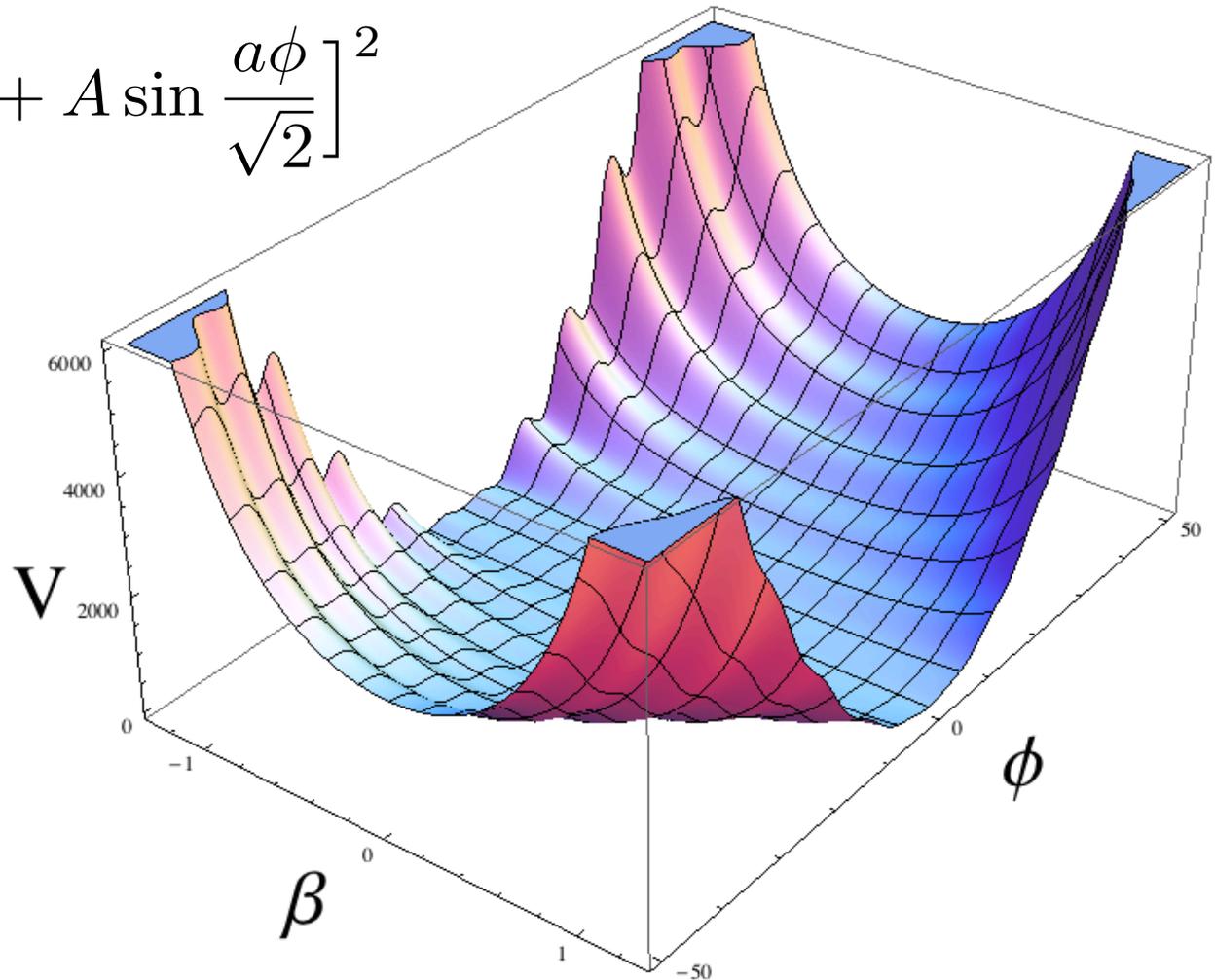


$$V = \Lambda^4 \left(1 + A^2 + B^2 - 2A \cos \frac{a\phi}{\sqrt{2}} + 2AB \cos \frac{(a-b)\phi}{\sqrt{2}} - 2B \cos \frac{b\phi}{\sqrt{2}} \right)$$

Modulated chaotic inflation potentials in supergravity

$$W = S \left[f(T) + A \sin(aT) \right] \quad K = \frac{1}{2} (T + \bar{T})^2 + S \bar{S}$$

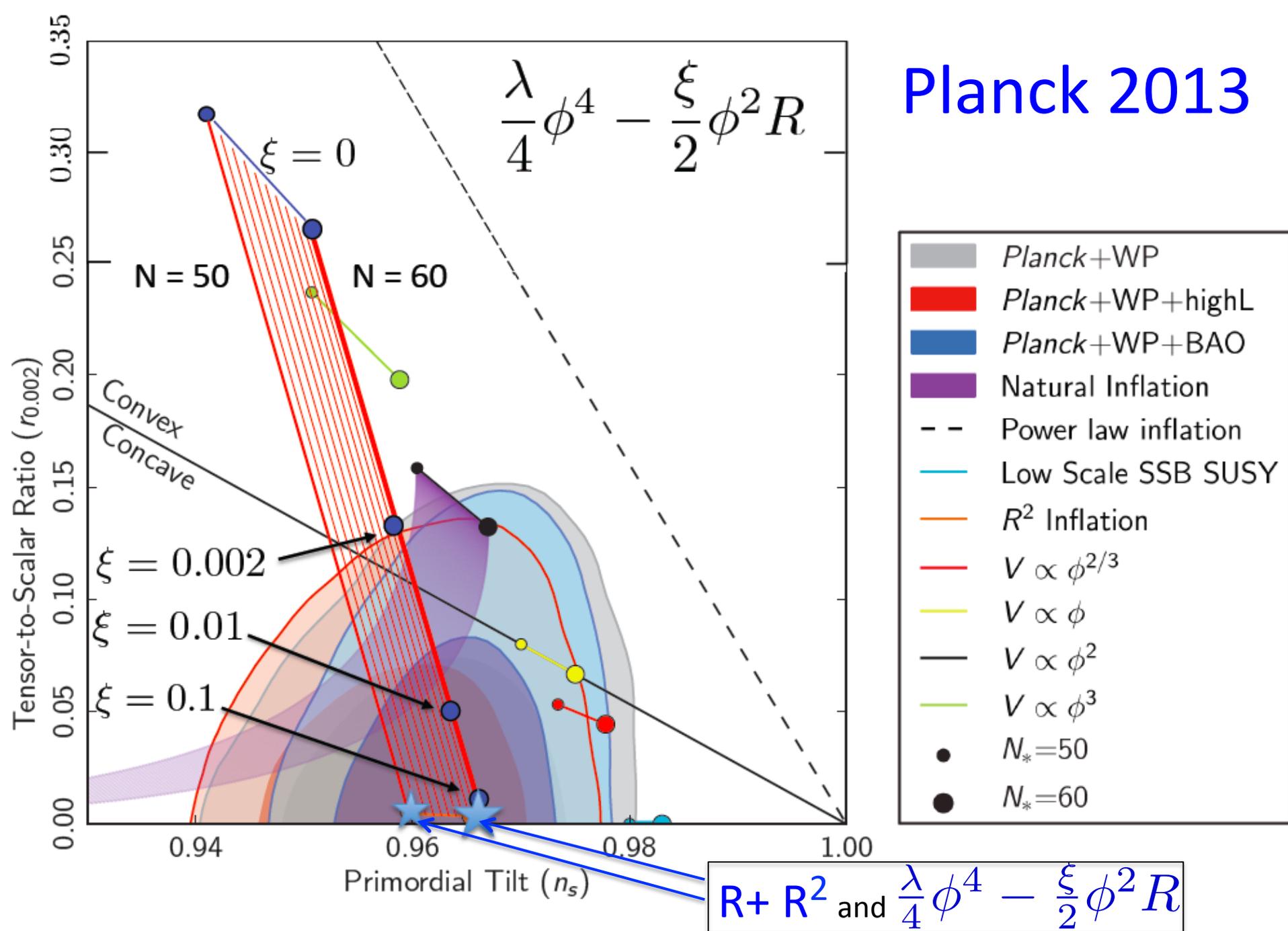
$$V = \left[f\left(\frac{\phi}{\sqrt{2}}\right) + A \sin\frac{a\phi}{\sqrt{2}} \right]^2$$



Models described above can easily explain large r . Good for BICEP2. They can also describe $r \ll 1$, but not without tuning. Can we do it naturally?

Let us return to Planck and some mysteries related to its results.

Planck 2013



Miracles to be explained:

MANY apparently unrelated theories make same prediction

$$1 - n_s = \frac{2}{N}, \quad r = \frac{12}{N^2}$$

This point is at the sweet spot of the Planck allowed region.

Here $N = O(60)$ is the required number of e-foldings of inflation corresponding to perturbations on the scale of the observable part of the universe.

What is going on? Why predictions of different theories converge at the same point? Why convergence is so fast? What is the relation to non-minimal coupling to gravity? Anything related to broken conformal invariance? Any way to explain it or at least account for it in supergravity?

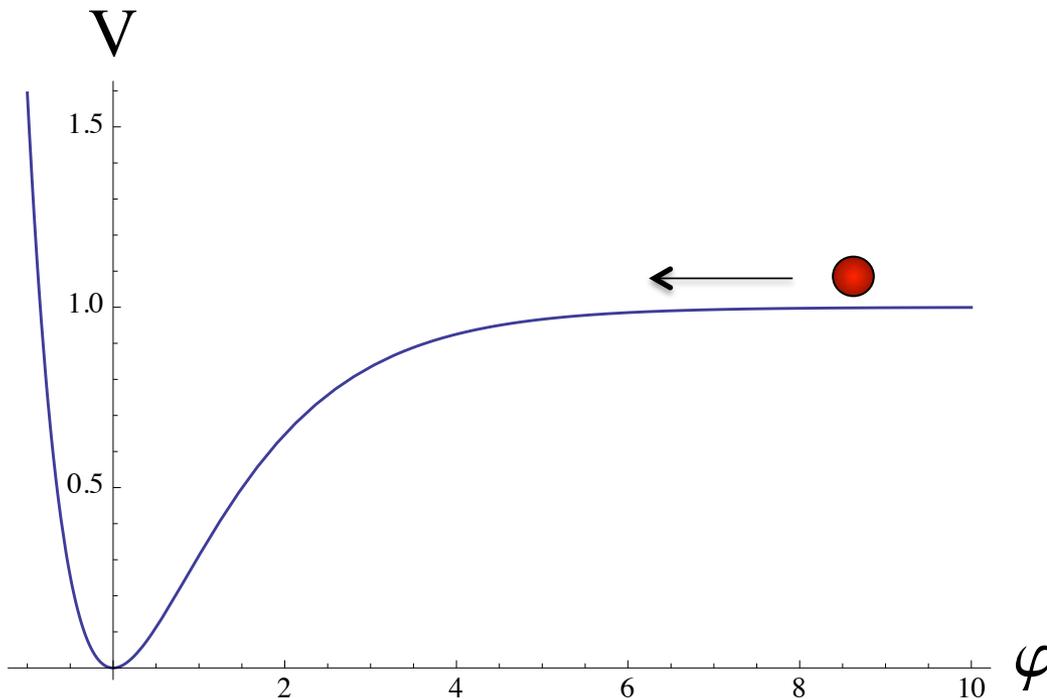
Starobinsky model

$$L = \sqrt{-g} \left(\frac{1}{2} R + \frac{R^2}{12M^2} \right)$$

$$\tilde{g}_{\mu\nu} = \left(1 + \frac{\phi}{3M^2} \right) g_{\mu\nu}$$

$$\varphi = \sqrt{\frac{3}{2}} \ln \left(1 + \frac{\phi}{3M^2} \right)$$

$$L = \sqrt{-\tilde{g}} \left[\frac{1}{2} \tilde{R} - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{3}{4} M^2 \left(1 - e^{-\sqrt{2/3} \varphi} \right)^2 \right]$$



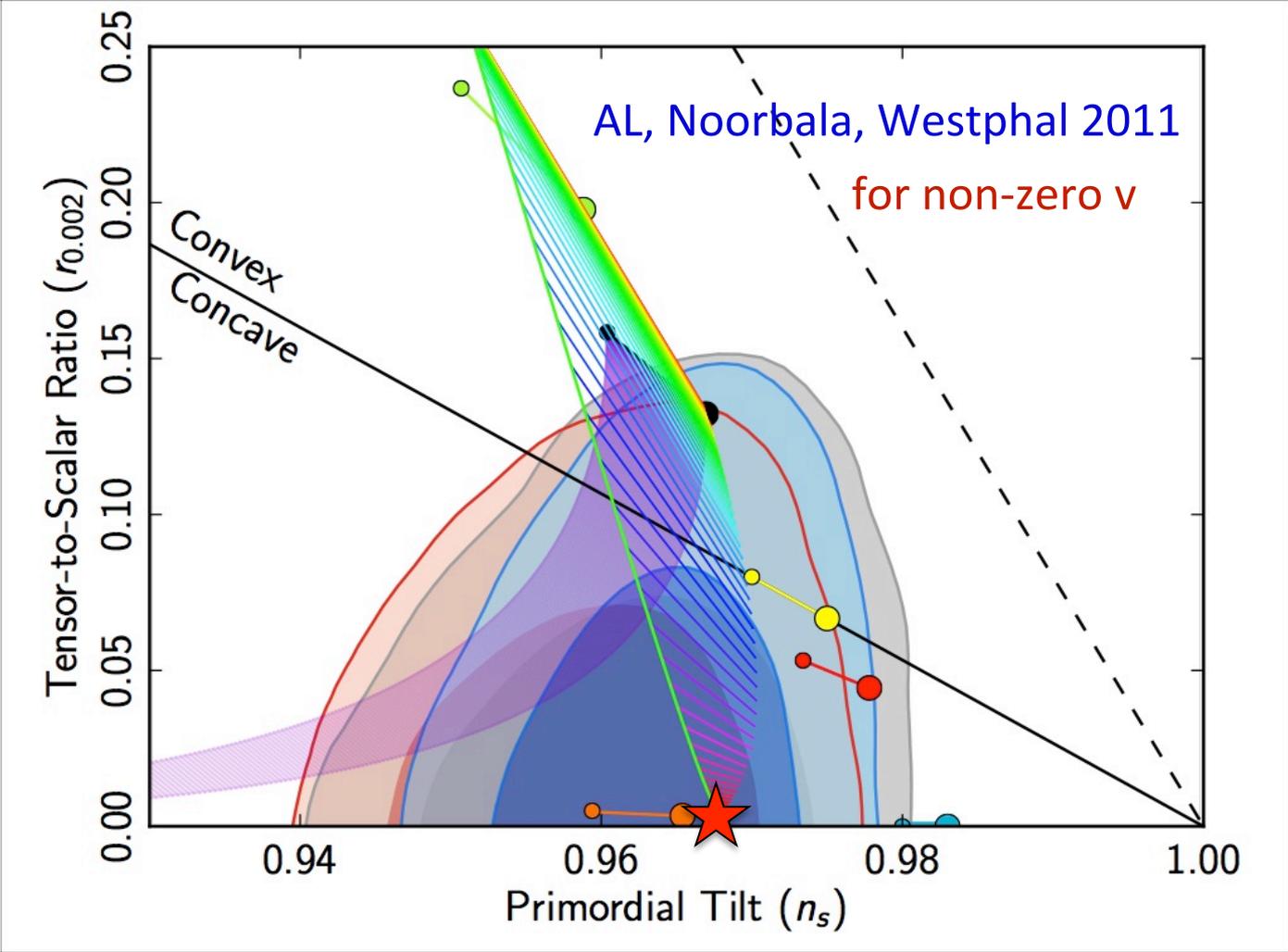
Pay attention to this potential, you will see its cousins many times in this talk

“Higgs Inflation”

Salopek, Bond and Bardeen, 1989
Bezrukov, Shaposhnikov 2008
Ferrara, Kallosh, A.L., Marrani, Van Proeyen 2011

$$-\frac{\xi}{2}\phi^2 R + \frac{\lambda}{4}(\phi^2 - v^2)^2$$

$$\xi > 0$$



Jordan frame

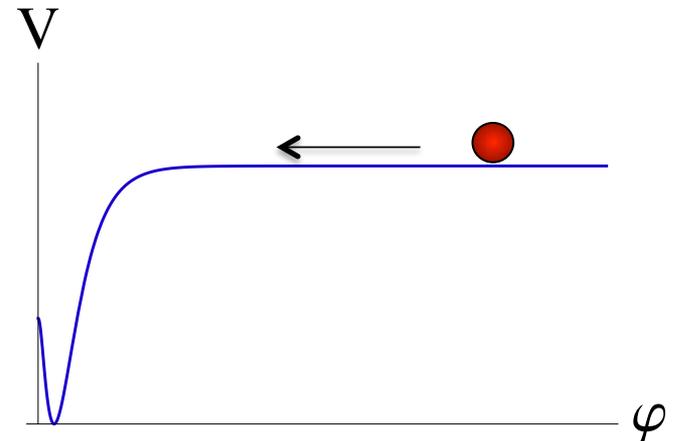
$$L = \frac{1}{2} \sqrt{-g_J} \left[(1 + \xi \phi^2) R_J - g_J^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V_J(\phi) \right]$$

Einstein frame

$$L = \frac{1}{2} \sqrt{-g_E} \left[R_E - g_E^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 2V_E(\phi(\varphi)) \right]$$

$$V_E(\phi) = \frac{V_J(\phi)}{(1 + \xi \phi^2)^2} \quad \left(\frac{d\varphi}{d\phi} \right)^2 = \frac{1 + \xi \phi^2 + 6\xi^2 \phi^2}{(1 + \xi \phi^2)^2}$$

$$-\frac{\xi}{2} \phi^2 R + \frac{\lambda}{4} (\phi^2 - v^2)^2 \quad \longrightarrow$$



GENERALIZATION:

Kallosh, AL, Roest 1310.3950

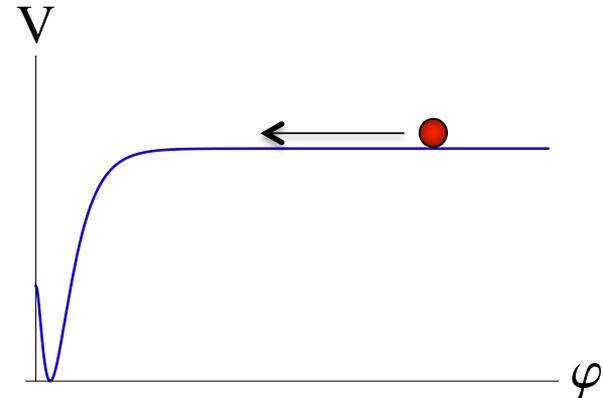
Jordan frame, arbitrary $V(\phi)$

$$\mathcal{L}_J = \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} \zeta R \sqrt{V(\phi)} - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

Einstein frame, large ζ

$$\mathcal{L}_E = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\partial\varphi)^2 - \zeta^{-2} (1 - e^{-\sqrt{2/3}\varphi})^2 \right]$$

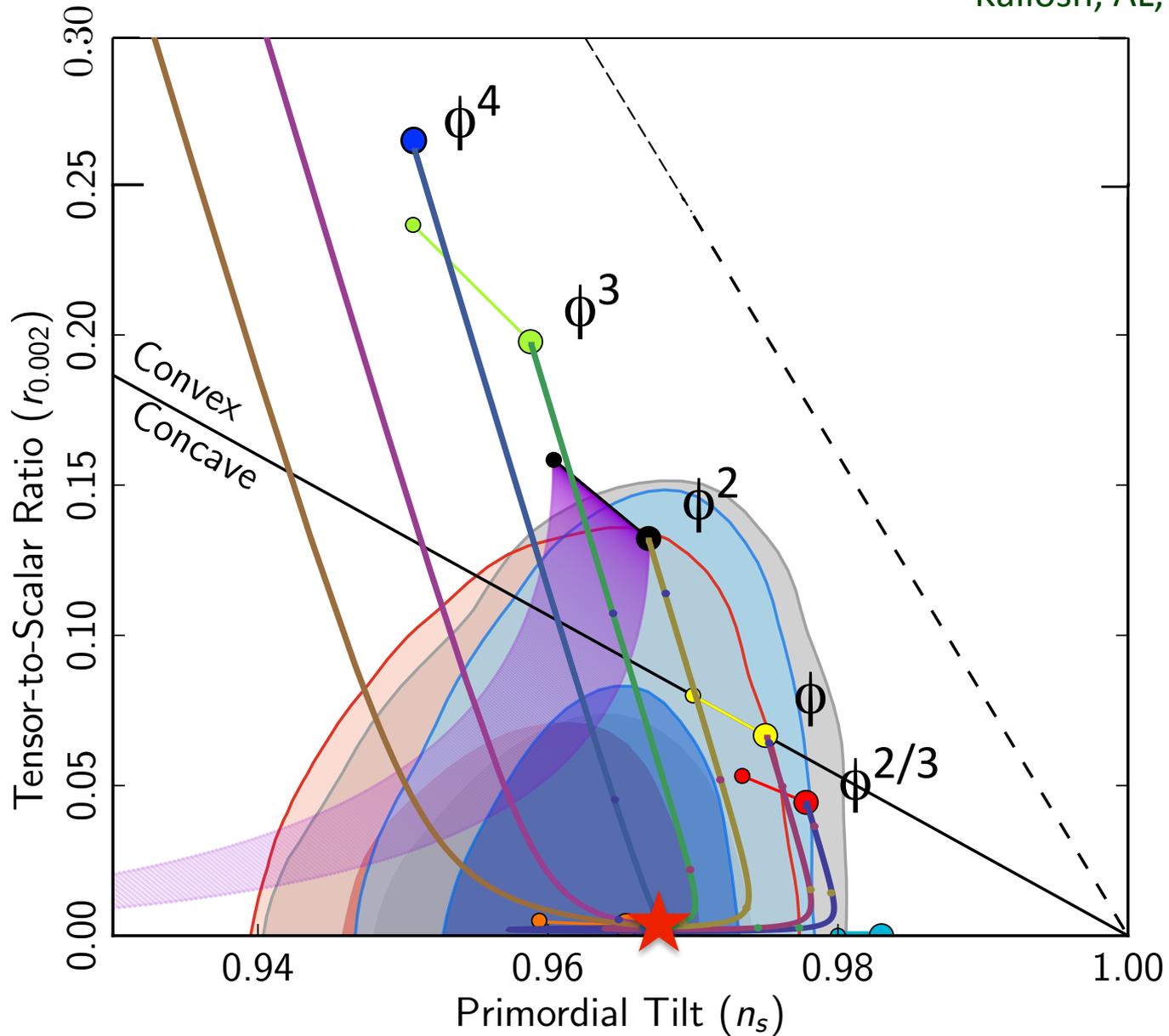
Just as before, with the same
observational consequences,
independently of $V(\phi)$



But at small ζ it reduces to the original theory $V(\phi)$, so by changing ζ we can interpolate between the original theory and the universal attractor point at large ζ .

“Combing” Chaotic Inflation

Kalosh, AL, Roest 2013



$\alpha - \beta$ model in supergravity

Ferrara, Kallosh, AL, Porrati

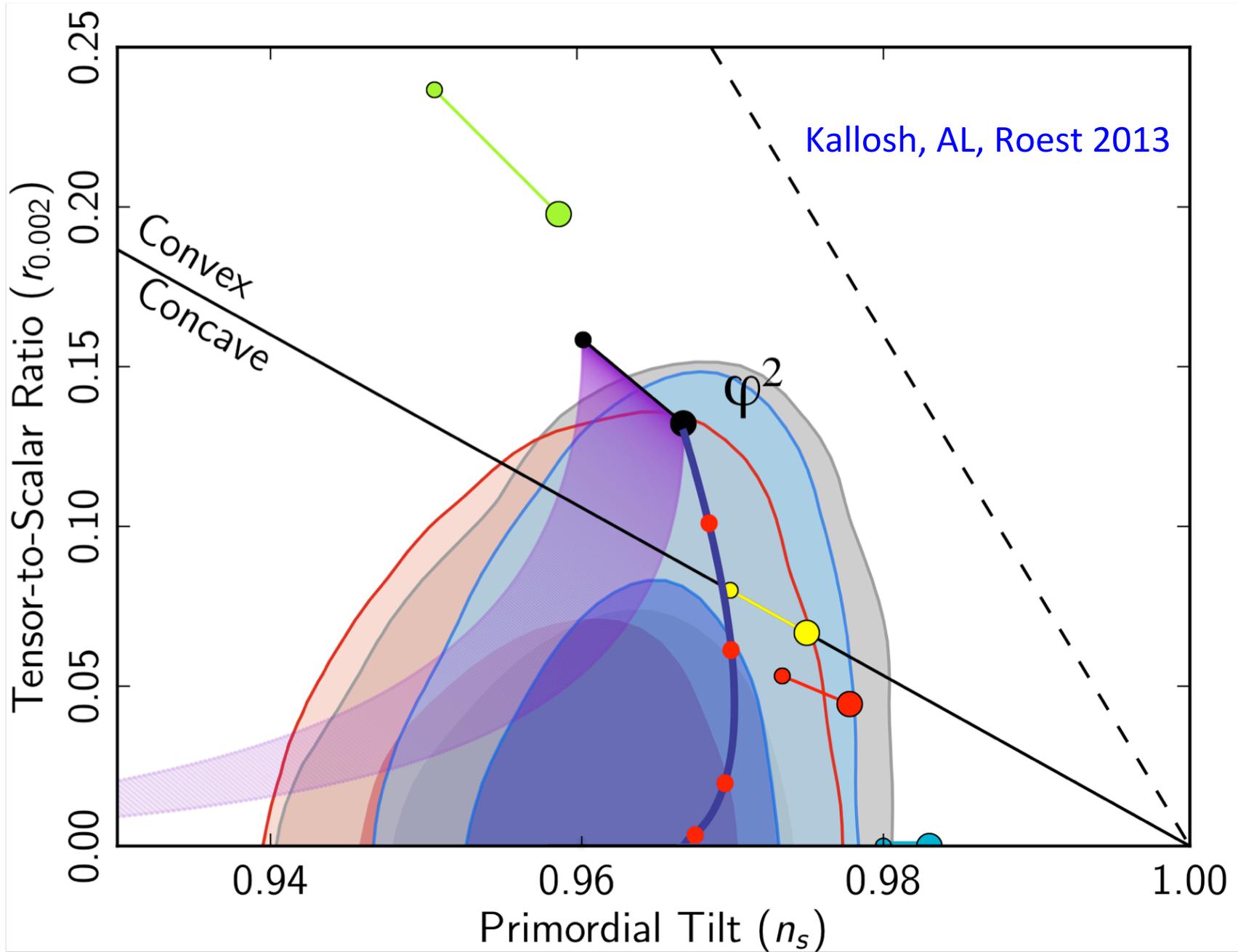
$$V \sim \left(\beta - \alpha e^{-\sqrt{\frac{2}{3\alpha}}\varphi} \right)^2$$

In this model, n_s and r do not depend on β . For $\alpha, \beta = 1$, the potential is the same as in Starobinsky model. For small α

$$n_s = 1 - \frac{2}{N}, \quad r = \alpha \frac{12}{N^2}$$

For large α the predictions are the same as in the simplest chaotic inflation with a quadratic potential:

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{8}{N}$$



$$\mathfrak{m} \geq 0$$

Cosmological attractors

Kallosh, AL 2013

We found a new class of chaotic inflation models with spontaneously broken conformal or superconformal invariance. Observational consequences of such models are stable with respect to strong deformations of the scalar potential.

In this class of models, inflation is possible even in the theories with very steep potentials because of their exponential flattening at the boundary of the moduli space.

De Sitter from spontaneously broken conformal symmetry

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2} \partial_\mu \chi \partial_\nu \chi g^{\mu\nu} + \frac{\chi^2}{12} R(g) - \frac{\lambda}{4} \chi^4 \right]$$

This theory is locally conformal invariant

$$\tilde{g}_{\mu\nu} = e^{-2\sigma(x)} g_{\mu\nu}, \quad \tilde{\chi} = e^{\sigma(x)} \chi$$

The field $\chi(x)$ is a conformal compensator, which we call '**conformon.**' It has negative sign kinetic term, but this is not a problem because it can be removed from the theory by fixing the gauge symmetry, for example

$$\chi = \sqrt{6}$$

This gauge fixing can be interpreted as a spontaneous breaking of conformal invariance due to existence of a classical field $\chi = \sqrt{6}$

The action in this gauge:
dS or AdS

$$\mathcal{L} = \sqrt{-g} \left[\frac{R(g)}{2} - 9\lambda \right]$$

The simplest conformally invariant two-field model of dS or AdS space and the SO(1,1) invariant conformal gauge

$$\mathcal{L} = \frac{\sqrt{-g}}{2} \left[(\partial_\mu \chi \partial^\mu \chi - \partial_\mu \phi \partial^\mu \phi) + \frac{\chi^2 - \phi^2}{6} R(g) - \lambda \frac{(\phi^2 - \chi^2)^2}{18} \right]$$

Local conformal symmetry

$$\tilde{g}_{\mu\nu} = e^{-2\sigma(\mathbf{x})} g_{\mu\nu}, \quad \tilde{\chi} = e^{\sigma(\mathbf{x})} \chi, \quad \tilde{\phi} = e^{\sigma(\mathbf{x})} \phi$$

The global SO(1,1) transformation is a boost between these two fields.

SO(1,1) invariant conformal gauge $\chi^2 - \phi^2 = 6$ Rapidity gauge

This gauge condition represents a hyperbola which can be parameterized by a canonically normalized field φ

$$\chi = \sqrt{6} \cosh \frac{\varphi}{\sqrt{6}}, \quad \phi = \sqrt{6} \sinh \frac{\varphi}{\sqrt{6}}$$

The action in this gauge,
dS/AdS

$$L = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - 9\lambda \right]$$

Chaotic inflation from conformal theory: **T-Model**

$$\mathcal{L} = \frac{\sqrt{-g}}{2} \left[(\partial_\mu \chi \partial^\mu \chi - \partial_\mu \phi \partial^\mu \phi) + \frac{\chi^2 - \phi^2}{6} R(g) - \frac{(\phi^2 - \chi^2)^2}{18} F(\phi/\chi) \right]$$

Here **F** is an arbitrary function of the ratio ϕ/χ . When this function is present, it breaks the SO(1,1) symmetry of the de Sitter model. This is the only possibility to keep local conformal symmetry and to deform the SO(1,1) symmetry.

In rapidity gauge it becomes

$$L = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - F\left(\tanh \frac{\varphi}{\sqrt{6}}\right) \right]$$

The attractor behavior near a critical point where SO(1,1) symmetry is restored is the following: start with generic F(tanh), almost always get

$$n_s \approx 0.967 \qquad r \approx 0.0032$$

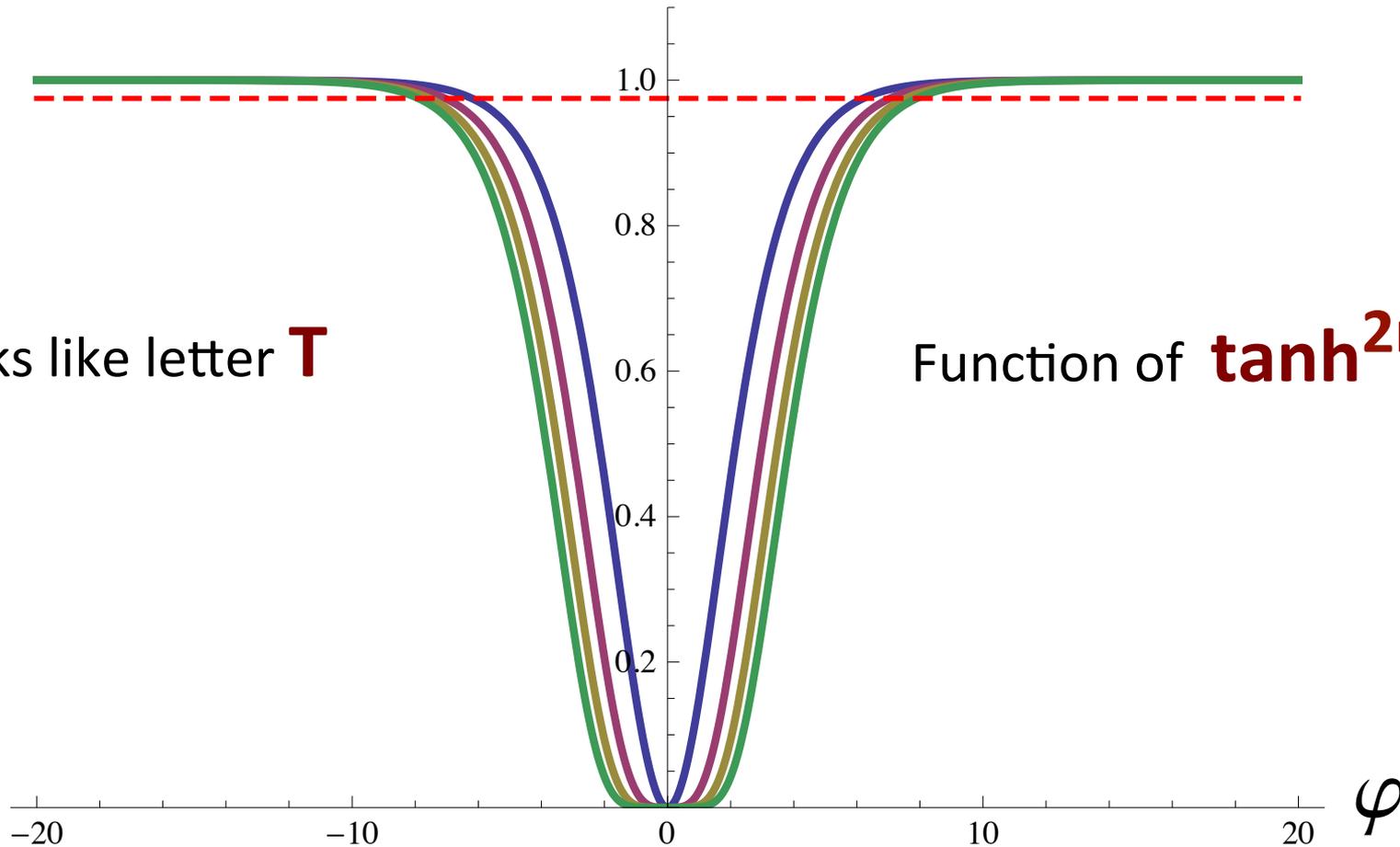
T-Model

$$F(\phi/\chi) = \lambda (\phi/\chi)^{2n} \longrightarrow V(\varphi) = \lambda_n \tanh^{2n} \frac{\varphi}{\sqrt{6}}$$

V

Looks like letter **T**

Function of **\tanh^{2n}**



T-Model:

$$F(\phi/\chi) = \lambda (\phi/\chi)^{2n}$$
$$V(\varphi) = \lambda_n \tanh^{2n}(\varphi/\sqrt{6})$$

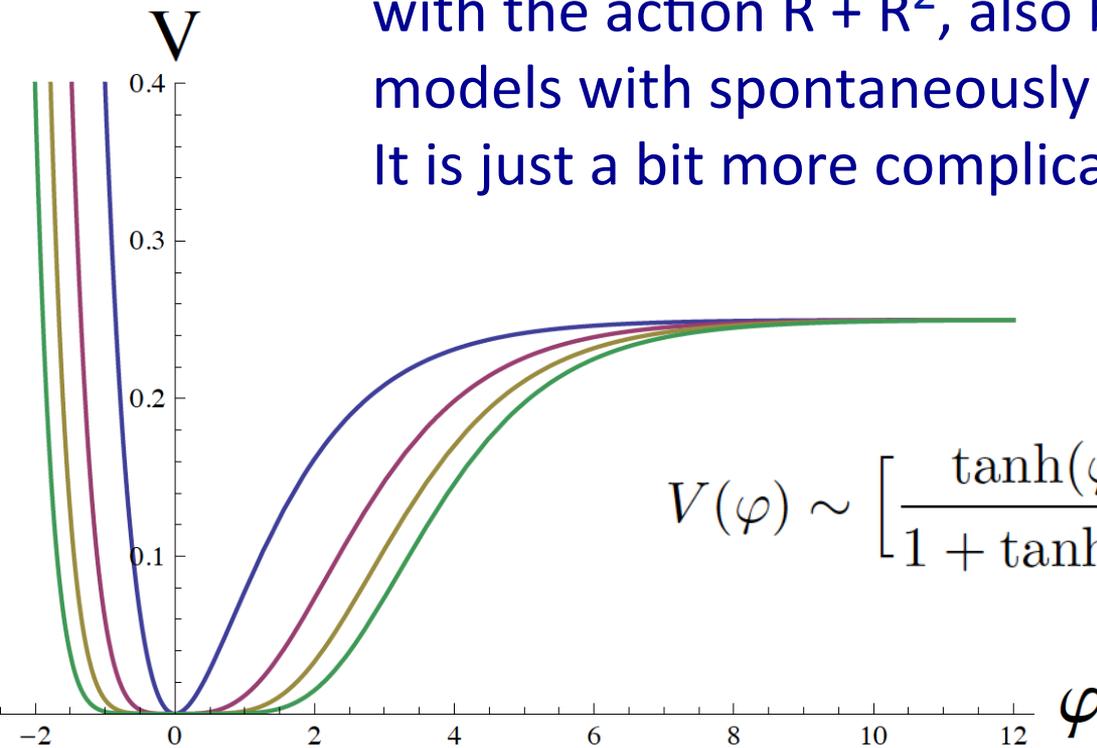
It does not look like the Starobinsky model or the Higgs inflation, but it leads to the same predictions, for any n

In ALL of these models, in the large N limit one has

$$1 - n_s = 2/N, \quad r = 12/N^2$$

ATTRACTOR

To our surprise, we found that the Starobinsky model, with the action $R + R^2$, also belongs to this broad class of models with spontaneously broken conformal symmetry. It is just a bit more complicated than the basic T-Model



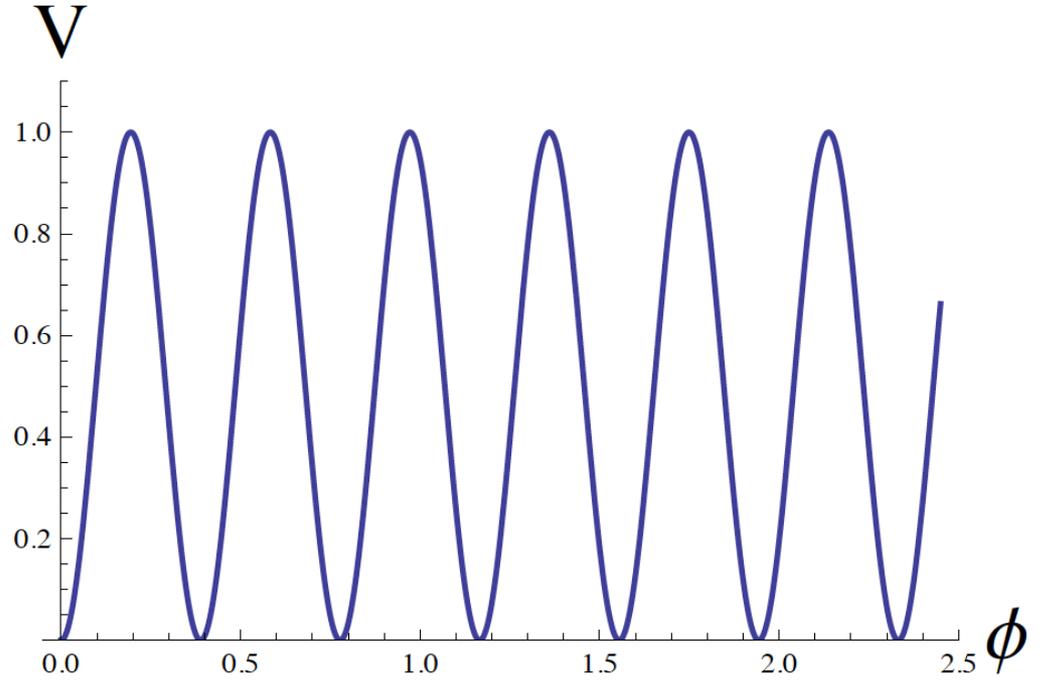
$$V(\varphi) \sim \left[\frac{\tanh(\varphi/\sqrt{6})}{1 + \tanh(\varphi/\sqrt{6})} \right]^2 \sim \left(1 - e^{-\sqrt{2/3}\varphi} \right)^2$$

Other supergravity realizations of the Starobinsky model (an incomplete list):

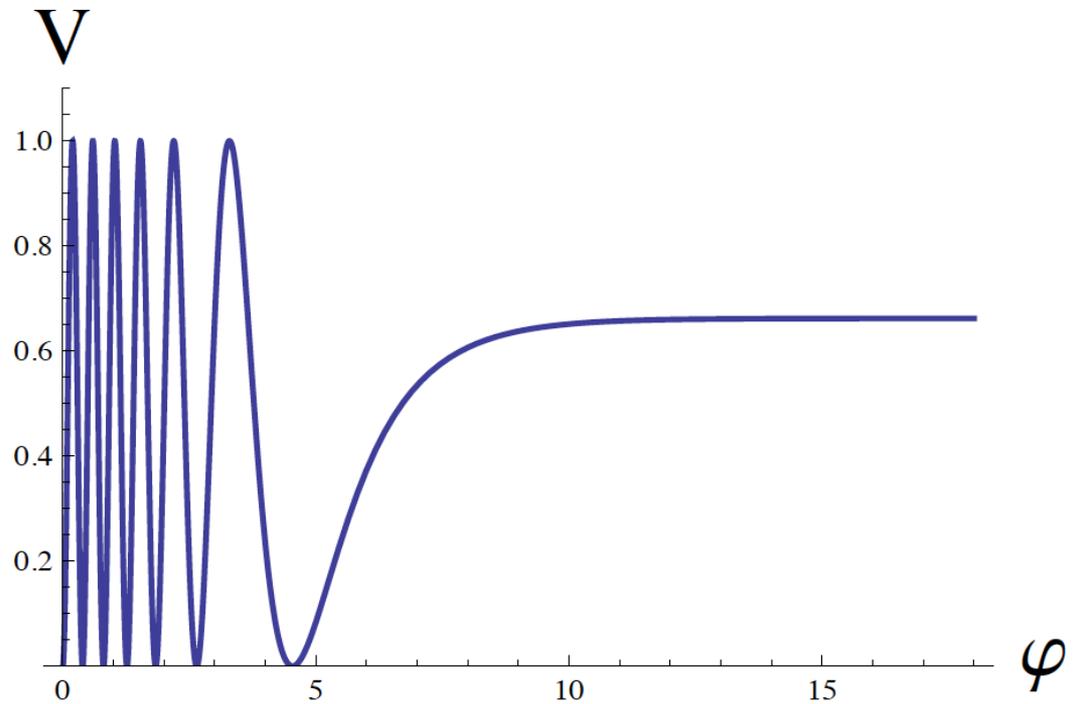
Cecotti 1987, Cecotti, Ferrara, Porrati and Sabharwal 1988, Ellis, Nanopoulos, Olive 2013, Kallosh, AL 2013, Ferrara, Kallosh, Linde and Porrati 2013, Cecotti and Kallosh 2014

GENERALIZATIONS

Original potential

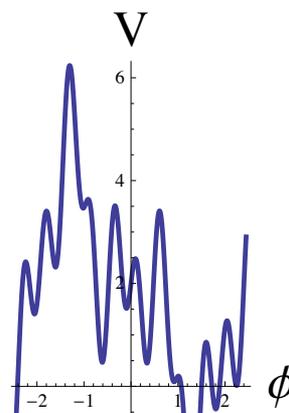


Potential in terms
of the canonical
field φ



Stretching and flattening of the potential is similar to stretching of inhomogeneities during inflation

Potential in the original variables of the conformal theory



Potential in the Einstein frame

V

-2

6

4

1

ϕ

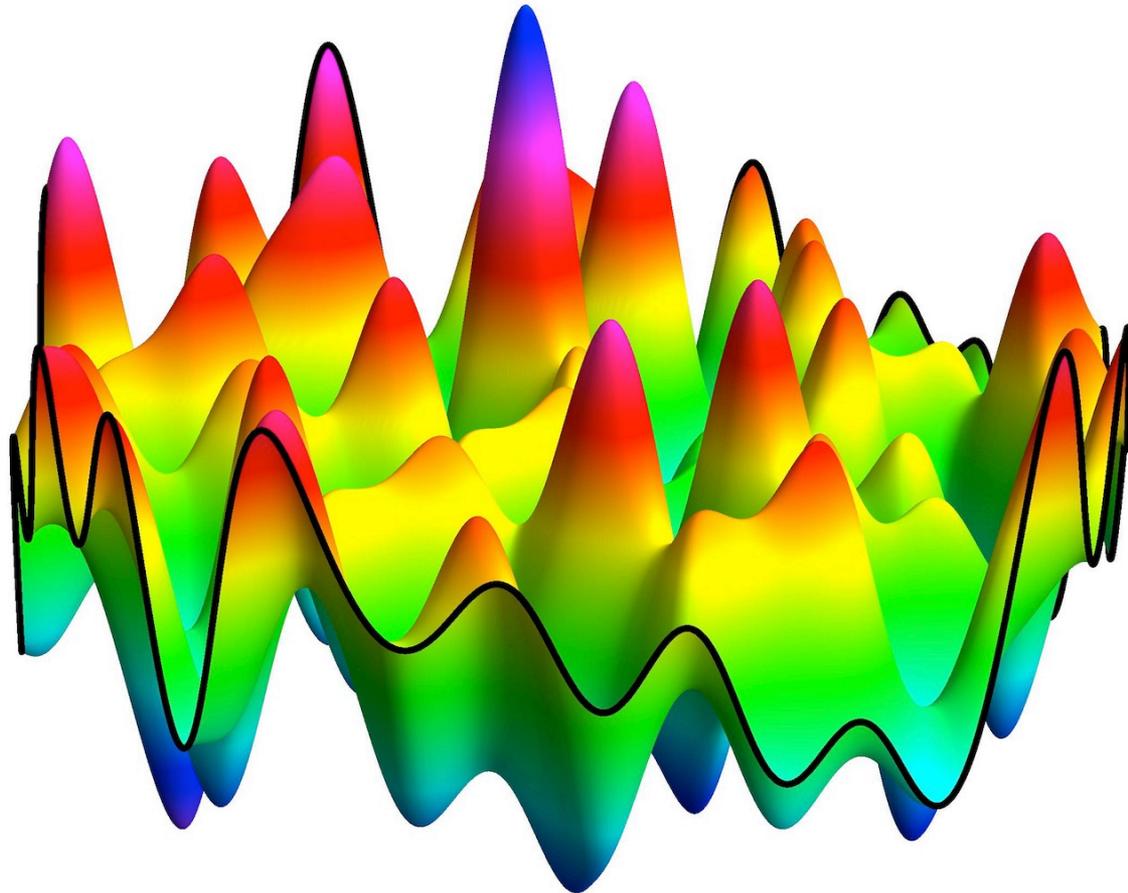
All of these models predict
 $1 - n_s = 2/N$, $r = 12/N^2$



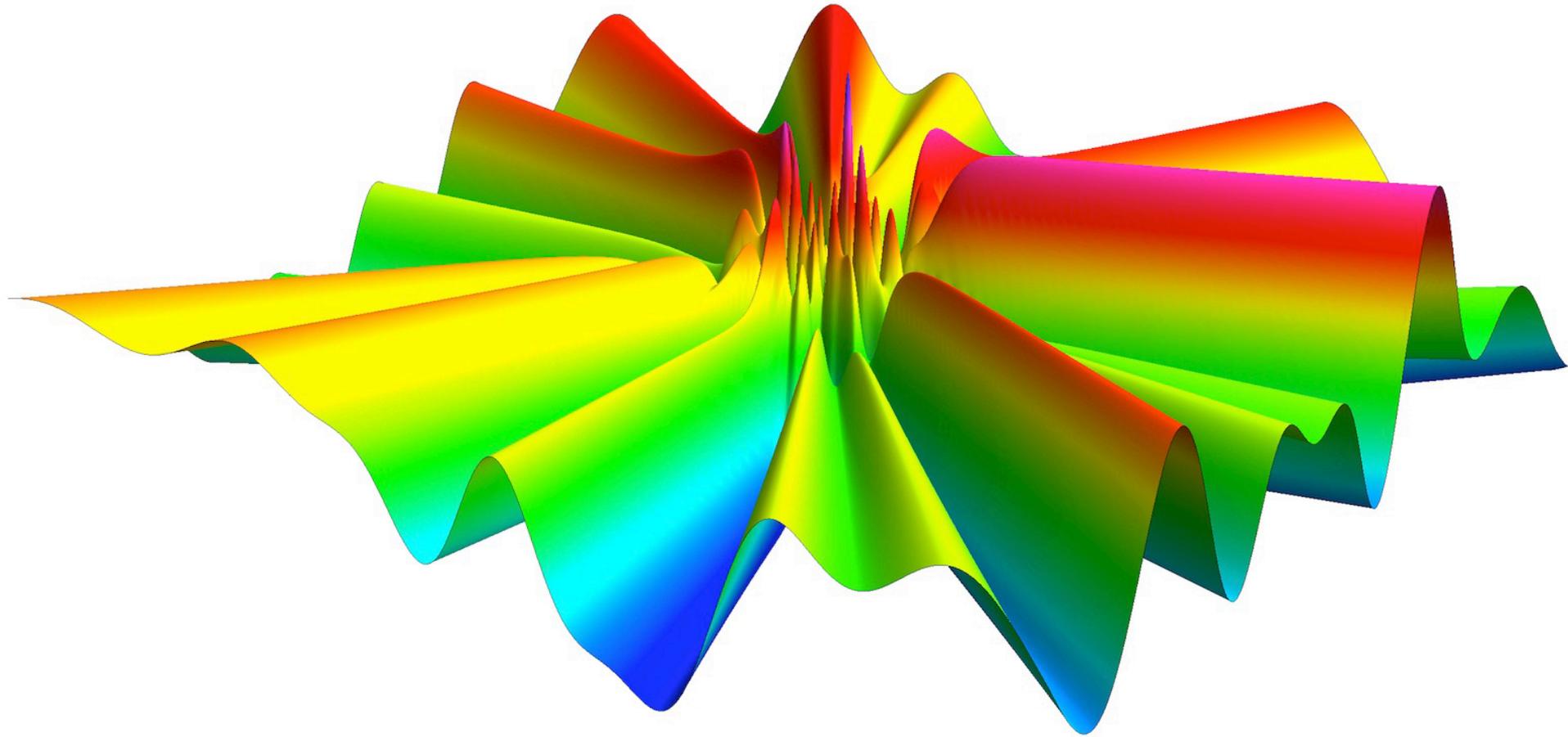
Inflation **in** the landscape is facilitated by inflation **of** the landscape

Multi-field Cosmological Attractors

More general potentials in terms of the original conformal variables. Naively, one would not expect inflation in theories with random supergravity potentials:



Stretching upon converting to canonical variables in the Einstein frame leads to inflation along dS valleys, and the same universal inflationary predictions as in the single-field models



Superconformal α -attractors

Kalosh, AL, Roest 1311.0472

Another class of cosmological attractors naturally appears in superconformal theory and supergravity. This class includes, in particular, models

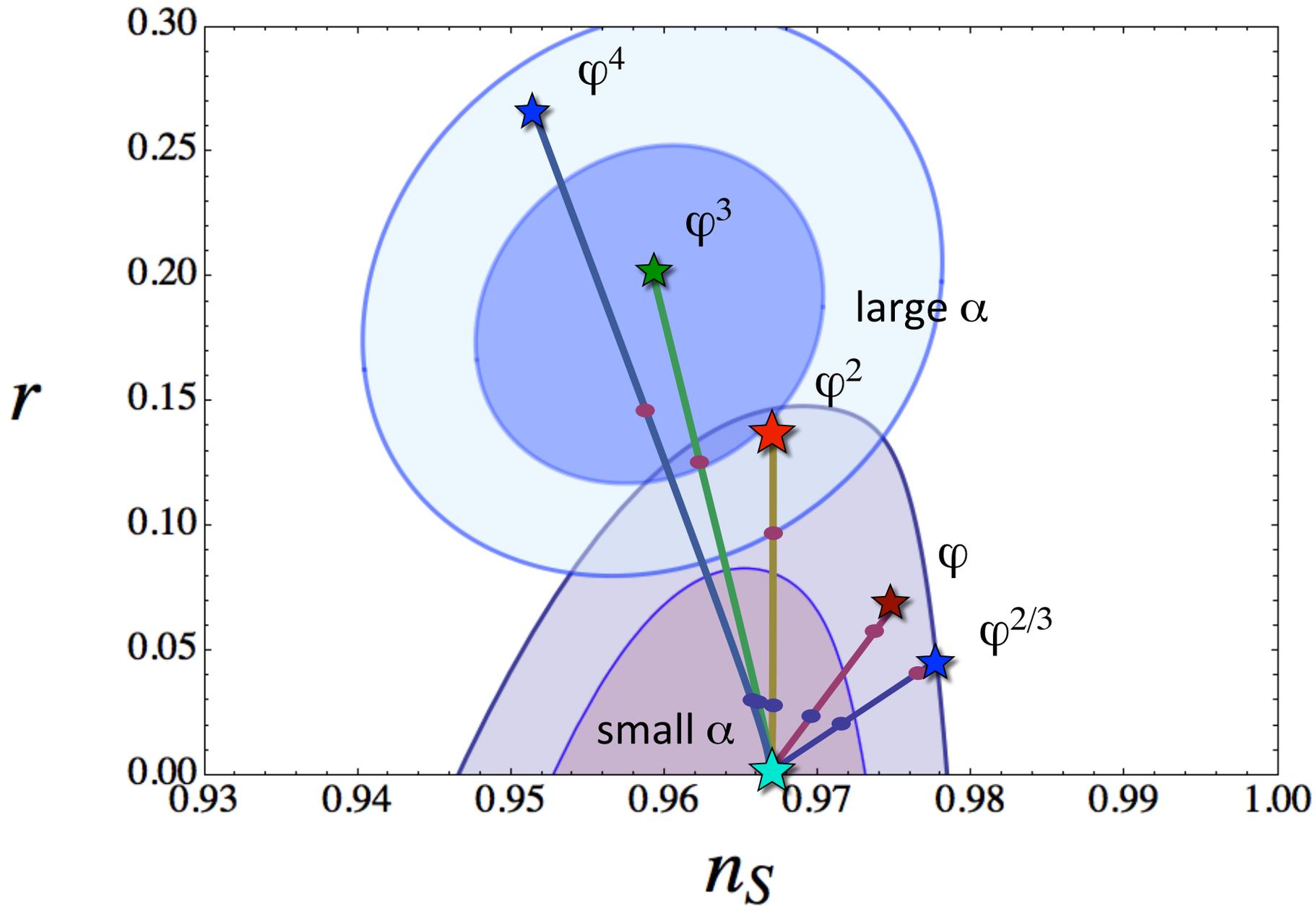
$$L = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - F\left(\tanh \frac{\varphi}{\sqrt{6\alpha}}\right) \right]$$

At $\alpha \lesssim 1$ these models have universal prediction

$$n_s = 1 - \frac{2}{N}, \quad r = \alpha \frac{12}{N^2}$$

However, for $\alpha \gg 1$ predictions depend on the choice of $F(x)$. For the simplest choice $F = x^n$, the predictions coincide with those of the simplest chaotic inflation models

$$V \sim \varphi^n$$



What is the meaning of α ?

The curvature of the Kahler manifold is inversely proportional to α

Small α means high curvature, small r - good for Planck

Large α means small curvature, large r - good for BICEP2

Thus, finding n_s and r may tell us something important about the nature of gravity and geometry of superspace.

$$L = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - F\left(\tanh \frac{\varphi}{\sqrt{6\alpha}}\right) \right]$$

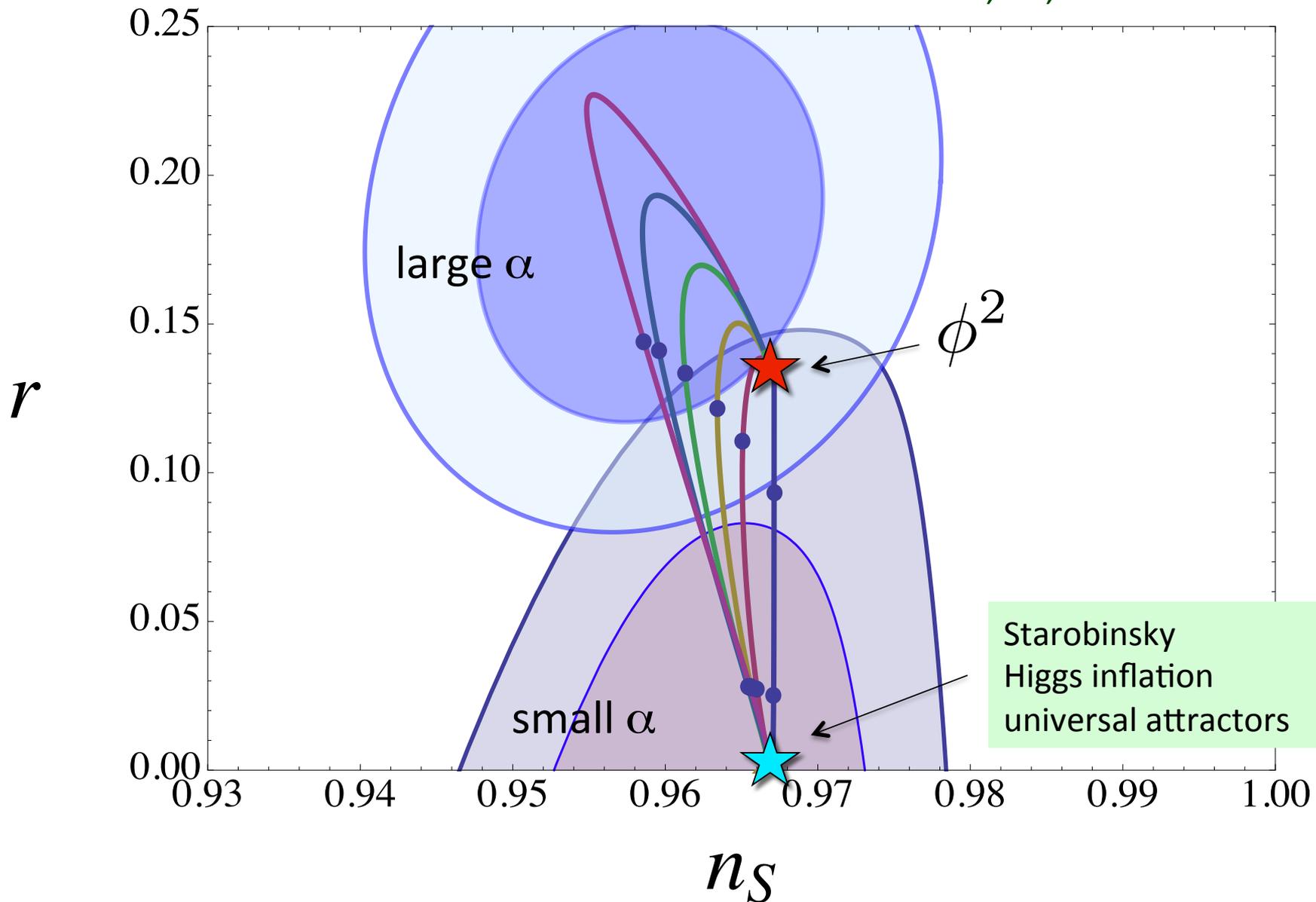
What if instead of the monomial functions $F = x^n$ one considers $F = x + x^2 + x^3 + \dots$?

Then in the large α limit, the predictions approach a single attractor point: The simplest chaotic inflation with a quadratic potential.

Kalosh, AL, Roest 1405.3646

Superconformal α -attractors

Kalosh, AL, Roest 1405.3646



Can we do it much simpler?

Too many fields, some of them do not directly participate in inflation. Very often we must add higher corrections to the Kahler potential to stabilize these fields near their zero values...

The cure: Volkov-Akulov nonlinear realization of supersymmetry does not require fundamental scalar degrees of freedom, they are replaced by bilinear fermionic combinations with zero vev.

Terminology: Replace standard unconstrained chiral superfields by **nilpotent superfields**, which do not have dynamical scalar components.

$$S^2(x, \theta) = 0$$

Ferrara, Kallosh, AL 1408.4096

Kallosh, AL 1408.5950

Antoniadis, Dudas, Ferrara, Sagnotti 1403.3269

Supergravity with nilpotent multiplets is a new theory requiring a very sophisticated analysis, especially when fermion interactions are involved. However, the bosonic sector is much simpler than before. Here is the basic rule (see 1408.4096 and 1408.4096 for details):

New rule:

Calculate potentials as functions of all superfields as usual, and then **DECLARE that $S = 0$ for the scalar part of the nilpotent superfield**. No need to stabilize and study evolution of the S field.

Sooooooooooooo much simpler!!

De Sitter vacua and dark energy

From 2003 till recently: **De Sitter hunting in the string landscape**

Recent progress with Analytic Classes of Metastable dS vacua

Kallosch, AL, Vercoocke, Wrase, 1406.4866

We proposed a systematic procedure for building (meta)stable dS vacua in string theory STU models, or using the Polonyi type superfield.

Polonyi field was successfully used in the past in supergravity models, in particular to provide a supersymmetric version of an F-term uplifting for the KKLT de Sitter vacua. However, it was not known how to relate Polonyi to string theory.

Uplifting stringy vacua with nilpotent fields

Now suppose that the Polonyi superfield is nilpotent, $S^2 = 0$.
Take any vacuum in string theory, e.g. provided by KKLT construction or some of its generalizations.

$$W = W_{KKLT} - M^2 S, \quad K = K_{KKLT} + S\bar{S}$$
$$V = V_{KKLT}(\rho, \bar{\rho}) + \frac{M^4}{(\rho + \bar{\rho})^3}$$

Nilpotent superfields are associated with the D-brane physics in string theory, via Volkov-Akulov theory. Thus now we have a string theory motivated **supersymmetric KKLT uplifting in string theory**.

Unifying inflation and dark energy

$$K = K[(\Phi - \bar{\Phi})^2, S\bar{S}, T\bar{T}] , \quad W = Sf(\Phi) + M^2T + W_0$$

The potential as a function of the real part of Φ at $S = 0, T = 0$ is

$$V = |f(\phi/\sqrt{2})|^2 + M^4 - 3W_0^2$$

FUNCTIONAL FREEDOM in choosing inflationary potential **and** uplifting it:
inflation and the present stage of acceleration in the same theory

Previously, it was hard to do it: Once we add $M^2T + W_0$, the structure of the theory changes, the fields S and T shift, the inflaton moves differently, a mess...

Here we simply **DECLARE that $T = S = 0$** , and **uplift the potential without changing it!!!!** The field T is a Polonyi field, **but without the cosmological moduli problem associated with it: It does its job, and vanishes.**

Conclusions:

Our deep gratitude to our colleagues from BICEP, Planck , WMAP and other experiments. Over years, they presented ample evidence in favor of inflationary cosmology. The last year especially was exhilarating: Because of the stimulating pressure from observations, theorists were able to uncover many new classes of inflationary theories with incredibly interesting properties.

Conclusions:

In particular, we found a **broad class of inflationary models based on supergravity**, including models which we called **cosmological attractors**. Their observational consequences are very stable with respect to strong deformations of the inflaton potential.

Conclusions:

By changing the strength of non-minimal coupling to gravity and/or the curvature of the Kahler manifold in supergravity, one can **continuously interpolate** between **two attractor points** for these classes of **large field models**: the predictions of the simplest chaotic inflation models with large r , favored by BICEP2, and the lowest part of the n_s - r plane favored by Planck.

Conclusions:

And it seems that we even learned how to solve some of the most difficult problems of inflationary model building in supergravity:

If some scalar fields make your life difficult, make them a part of a nilpotent multiplet. They will do their job and disappear.

**It would be fantastic if we could do something similar with all other problems in our life... We will continue working on it, and return to it at
COSMO 2015**

C
H
I
C
A
G
O



C
O
S
M
O
2
0
1
4