Massive gravity and Caustics: A definitive covariant constraint analysis

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Motivation and goals

- de Rham-Gabadadze-Tolley massive gravity: no Boulware-Deser ghost
- Presence of superluminalities (cf Galileons in the decoupling limit): intense debate in the community
- First fully non-linear propagation analysis
  S. Deser, M. Sandora, A. Waldron, GZ, arXiv:1408.0561
  - Method of characteristic surfaces
  - Reliance on covariant constraint analysis
- Potential propagation pathologies...

General setting

- First order Cartan formalism in 4D
  - 4 vierbein 1-forms $e^m := e_\mu{}^m dx^\mu$ (16 fields)
  - 6 connection 1-forms $\omega^{mn} := \omega_\mu{}^{mn} dx^\mu$ (24 fields)
- dRGT action (4 fiducial vierbein 1-forms $f^m := f_\mu{}^m dx^\mu$)

$$S = -\frac{1}{4}\int \epsilon_{mnrs}\, e^m e^n \left[d\omega^{rs} + \omega^r{}_t \omega^{ts}\right] - m^2 \int \epsilon_{mnrs}\, e^m \left[\frac{\beta_0}{4} e^n e^r e^s + \frac{\beta_1}{3} e^n e^r f^s + \frac{\beta_2}{2} e^n f^r f^s + \beta_3 f^n f^r f^s\right]$$

Equations of motion

- Zero torsion condition

$$\mathcal{T}^m := \nabla e^m := de^m + \omega^m{}_n e^n \approx 0$$

- Einstein equations

$$\mathcal{G}_m := G_m - m^2 t_m \approx 0$$

  - Einstein 3-forms
    $G_m := \frac{1}{2}\epsilon_{mnrs} e^n \left[d\omega^{rs} + \omega^r{}_t \omega^{ts}\right]$
  - Mass term 3-forms
    $t_m := \epsilon_{mnrs} \left[\beta_0 e^n e^r e^s + \beta_1 e^n e^r f^s + \beta_2 e^n f^r f^s + \beta_3 f^n f^r f^s\right]$

- 40 eoms for 40 dynamical fields

Primary constraints

- Space-time decomposition of a p-form ($p < 4$)

$$\theta := \mathring{\theta} + \boldsymbol{\theta}$$

where $\mathring{\theta} \wedge dt = 0$

- Primary constraints: purely spatial part of the eoms

$$\boldsymbol{\mathcal{T}}^m = \boldsymbol{d e}^m + \boldsymbol{\omega}^m{}_n \boldsymbol{e}^n \approx 0$$

$$\boldsymbol{G}_m \approx m^2 \boldsymbol{t}_m$$

- 12+4 =16 constraints

Secondary constraints

- Symmetry constraint

$$G_{[m}e_{n]} = \frac{1}{2}\epsilon_{mnrs}e^r \nabla \mathcal{T}^s \approx 0$$

So $t_{[m}e_{n]} \approx 0$ and generically

$$\mathcal{F} := e_m f^m \approx 0$$

- Vector constraint

$$\nabla G_m = \frac{1}{2}\epsilon_{mnrs}\mathcal{T}^n \left[d\omega^{rs} + \omega^r{}_t\omega^{ts}\right] \approx 0$$

So $\nabla t_m \approx 0$ which reduces to

$$\mathcal{V} := \epsilon_{mnrs}M^{mn}K^{rs} \approx 0$$

where $M^{mn}(e^r, f^s)$ and $K^{mn} := \omega^{mn} - \bar{\omega}^{mn}(f^r)$

- 6+4=10 constraints

Tertiary constraints

- Curl of symmetry constraint

$$\nabla\mathcal{F} := K_{mn}e^m f^n \approx 0$$

where the purely spatial part is not new!

- Curl of vector constraint

$$\nabla\mathcal{V} := \epsilon_{mnrs}M^{mn}\nabla K^{rs} + \cdots \approx 0$$

  - ∇K^{rs}: no time derivatives on-shell
  - Scalar constraint:

$$\mathcal{S} := \nabla\mathcal{V} \approx 0$$

- 3+1=4 constraints
- 40-16-10-4=10 first order dofs OR 5 physical dofs

Covariant constraint analysis

Spacelike characteristic surfaces

Discussion

Spacelike characteristic surfaces

- Spacelike characteristic surfaces: hyspersurfaces which cannot be used to as initial data surfaces for the eoms (cf A. Terana's parallel session talk: Cauchy breakdown)
- Analogous problem for first order differential equations:

$$a(y,t)\dot{y} + b(y,t) = 0$$

  with $y(t_0) = y_0$
- IF $a(y_0, t_0) = 0$: impossible to evolve the differential equation for the given initial conditions at t_0!
- For mGR PDEs: non-invertibility of the coefficient of the highest order derivatives on some spacelike hypersurface Σ

Other interpretations

- Σ is the world-sheet of a superluminal shock wave-front
- Equivalently: propagation of a superluminal wave-front in the infinite frequency limit over some mean-field solution of the eoms
- Causal structure: Light-cone of the theory larger than the dynamical metric light-cone
- !Potentially dangerous (cf M. Trodden's plenary talk)

Analysis

- Investigation of the eoms+constraints
- Suppose Σ is spacelike characteristic surface (with timelike normal ξ_μ)
- Propagation of a shock wave-front along which first order derivatives are discontinuous

$$\partial_\mu e_\nu^m|_{\Sigma+} - \partial_\mu e_\nu^m|_{\Sigma-} := \xi_\mu \mathfrak{e}_\nu{}^m$$

$$\partial_\mu \omega_\nu^{mn}|_{\Sigma+} - \partial_\mu \omega_\nu^{mn}|_{\Sigma-} := \xi_\mu \mathfrak{w}_\nu{}^{mn}$$

- Compute the discontinuities in the eoms+constraints
- See whether the $\mathfrak{e}_\nu{}^m$ and $\mathfrak{w}_\nu{}^{mn}$ are allowed to be non-zero

Results

- Characteristic equation:

$$\chi \begin{pmatrix} \mathfrak{e}_{\nu}{}^{m} \\ \mathfrak{w}_{\nu}{}^{mn} \end{pmatrix} \approx 0$$

- Characteristic matrix χ depends on the initial conditions (compatible with the constraints) given on Σ OR equivalently on the mean-field solution of the eoms over which shocks propagate
- Invertibility is not warranted (and even dubious generically)

Results

$$l_o{}^m \boldsymbol{K}_{mn} \boldsymbol{f}^n \mathfrak{f}_{oo} + \boldsymbol{e}^m \boldsymbol{f}^n \mathfrak{w}_{omn} \approx 0$$

$$2\epsilon_{mnrs} l_o{}^m (\beta_1 \boldsymbol{e}^n + \beta_2 \boldsymbol{f}^n) \boldsymbol{K}^{rs} \mathfrak{f}_{oo} - \epsilon_{mnrs} \boldsymbol{M}^{mn} \mathfrak{w}_o{}^{rs} \approx 0$$

$$\begin{aligned}
&\epsilon_{mnrs} \left(\beta_1 \boldsymbol{e}^m \boldsymbol{e}^t - 2\beta_2 \boldsymbol{e}^{(m} \boldsymbol{f}^{t)} - 3\beta_3 \boldsymbol{f}^m \boldsymbol{f}^t\right) \left(\boldsymbol{K}^{nr} \mathfrak{w}_o{}^s{}_t - \boldsymbol{K}^s{}_t \mathfrak{w}_o{}^{nr}\right) \\
&\quad + 2\epsilon_{mnrs} \left(\beta_1 l_o{}^{[m} \boldsymbol{e}^{t]} - \beta_2 l_o{}^{(m} \boldsymbol{f}^{t)}\right) \boldsymbol{K}^{nr} \boldsymbol{K}^s{}_t \mathfrak{f}_{oo} \\
&\quad + 2m^2 \epsilon_{mnrs} l_o{}^m \Big(4\beta_0\beta_1 \boldsymbol{e}^n \boldsymbol{e}^r \boldsymbol{e}^s + 3(\beta_1^2 + 2\beta_0\beta_2) \boldsymbol{e}^n \boldsymbol{e}^r \boldsymbol{f}^s \\
&\qquad\qquad + 6\beta_1\beta_2 \boldsymbol{e}^n \boldsymbol{f}^r \boldsymbol{f}^s + (\beta_1\beta_3 + 2\beta_2^2) \boldsymbol{f}^n \boldsymbol{f}^r \boldsymbol{f}^s\Big) \mathfrak{f}_{oo} \\
&\quad - 3\epsilon_{mnrs} \beta_3 \boldsymbol{f}^m \boldsymbol{f}^n \left(\boldsymbol{\rho}^{rs} + 2m^2 \xi^{[r} \left[\boldsymbol{\tau}^{s]} - \frac{1}{2} \tau^t{}_t \boldsymbol{e}^{s]}\right]\right) \mathfrak{f}_{oo} \\
&\quad - 4\beta_2 l_o{}^m \bar{\boldsymbol{G}}_m \mathfrak{f}_{oo} - 2\epsilon_{mnrs} \beta_1 l_o{}^m \boldsymbol{e}^n \bar{\boldsymbol{R}}^{rs} \mathfrak{f}_{oo} \approx 0
\end{aligned}$$

Covariant constraint analysis

Spacelike characteristic surfaces

Discussion

How bad is this?

- Σ can be any spacelike hypersurface!
- Light-cone of the theory completely flat?
- Maybe non-invertibility occurs only in strong-coupling regimes where the theory cannot be trusted anyway...
- Maybe for some region of parameter space OR some fiducial background choice, χ is generically invertible...
- Thorough analysis difficult...

Conclusion

- First covariant constraint analysis of dRGT mGR
- Study of characteristic surfaces
- The theory has potentially pathological behavior unless...
  - Strong-coupling and quantum corrections save the day
  - Magic happens (in the characteristic matrix)