On Consistency Conditions for Primordial Perturbations

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The Consistency Relation:

Maldacena '02, Creminelli and Zaldarriaga '04

$$\lim_{q \to 0} \frac{\langle \zeta_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q}-\vec{p}} \rangle}{P_\zeta(q)} = -\left(3 + p_k \frac{\partial}{\partial p_k}\right) P_\zeta(p)$$

Follows from symmetries

Holds in all models of single-field inflation in which background is an attractor

It is interesting because it could be violated

Bunch-Davis initial state is assumed

Residual Diffs as Infinite Number of Global Symmetries:

Hinterbichler, Hui, Khoury '13

Infinite Symmetries $\longrightarrow$ Infinite Number of Consistency Conditions

$$\lim_{q\to 0} \frac{\partial^n}{\partial q^n}\left(\frac{\langle\zeta(q)\zeta(p)\zeta(p')\rangle'}{P_\zeta(q)} + \frac{\langle\gamma(q)\zeta(p)\zeta(p')\rangle'}{P_\gamma(q)}\right) \sim \frac{\partial^n}{\partial p^n}\langle\zeta(p)\zeta(p')\rangle'$$

$n = 0, 1$: 3-point functions are completely fixed by symmetries

Creminelli, Norena and Simonovic '12

$n \geq 2$: only certain combinations of derivatives are constrained

Questions:

Why are the residual gauge symmetries useful?

What if we could fix the gauge completely?

Answer:

The theory is highly constrained by the underlying gauge symmetry, even when it's fixed

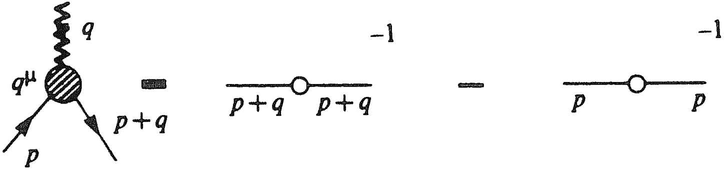
Gauge symmetries lead to Slavnov-Taylor identities Slavnov '72

Certain consequences of the gauge symmetry can be re-derived using the residual symmetries; e.g. BRST

The point is simple: the gauge-fixing term is not a problem, since it is pretty much model independent

Warming-up with QED

Slavnov '72



$$q^\mu \Gamma_\mu^{A\bar\psi\psi}(q,p,-p-q) = \Gamma_\psi(p+q) - \Gamma_\psi(p)$$

In squeezed limit

$$\Gamma_\mu^{A\bar\psi\psi}(0,p,-p) = \frac{\partial \Gamma_\psi(p)}{\partial p^\mu}$$

This relation works for fermionic as well as for scalar QED

The Most General Solution:

$$\Gamma_{\mu}^{A\bar{\psi}\psi}(q,p,-p-q) = \sum_{n=0}^{\infty} q^n \frac{\partial^{n+1}\Gamma_{\psi}(p)}{\partial p_{\mu}\partial p^n} + C_{\mu}$$

with

$$q^{\mu} C_{\mu} = 0$$

Analyticity of vertex functional $\Rightarrow \lim_{q\to 0} C_{\mu} = 0$

e.g. $C^{\mu} = q_{\nu}[\gamma^{\nu},\gamma^{\mu}]$, corresponding to the non-minimimal coupling

$$F_{\mu\nu}\bar{\psi}\gamma^{\mu}\gamma^{\nu}\psi$$

Cosmology:

$$\bar{g}_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2\,; \qquad \phi = \bar{\phi}(t)$$

Excitations of the background are introduced as

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + a^2(t)h_{\mu\nu}\,; \qquad \phi = \bar{\phi}(t) + \varphi$$

Comoving gauge is given by $\varphi = 0$

Under spatial diffeomorphisms

$$h_{ij} \quad \to \quad h_{ij} + \partial_i\xi_j + \partial_j\xi_i + \xi^k\partial_k h_{ij} + h_{ik}\partial_j\xi^k + h_{jk}\partial_i\xi^k$$

The shift and lapse are considered to be integrated out

Cosmology:

LB, Justin Khoury '13; Pimentel '13

$$q_j \left(\frac{\delta_{ij}}{3} \Gamma^{\zeta\zeta\zeta}(\vec{q}, \vec{p}, -\vec{q} - \vec{p}) + 2\Gamma_{ij}^{\gamma\zeta\zeta}(\vec{q}, \vec{p}, -\vec{q} - \vec{p}) \right) = q_i \Gamma_{\zeta}(p) - p_i \Big(\Gamma_{\zeta}(|\vec{q} + \vec{p}|) - \Gamma_{\zeta}(p) \Big)$$

Correlation Functions:

$$
\begin{aligned}
\langle \zeta_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q}-\vec{p}} \rangle' &= P_\zeta(q) P_\zeta(p) P_\zeta(|\vec{q}+\vec{p}|) \Gamma^{\zeta\zeta\zeta}(\vec{q}, \vec{p}, -\vec{q}-\vec{p}) \\
\langle \gamma^{ij}_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q}-\vec{p}} \rangle' &= \hat{P}^{ijk\ell}(\hat{q}) P_\gamma(q) P_\zeta(p) P_\zeta(|\vec{q}+\vec{p}|) \Gamma^{\gamma\zeta\zeta}_{k\ell}(\vec{q}, \vec{p}, -\vec{q}-\vec{p})
\end{aligned}
$$

General Solution:

LB, Justin Khoury '13

$$\frac{1}{3}\delta_{ij}\Gamma^{\zeta\zeta\zeta}(\vec{q},\vec{p},-\vec{q}-\vec{p}) + 2\Gamma^{\gamma\zeta\zeta}_{ij}(\vec{q},\vec{p},-\vec{q}-\vec{p}) = K_{ij} + A_{ij}$$

K_{ij} is a Taylor series in q and is determined by the power spectrum of short modes.

A_{ij} is an arbitrary transverse and symmetric matrix

$$A_{ij} = \epsilon_{ikm}\epsilon_{j\ell n}q^k q^\ell \left(a(\vec{p},\vec{q})\delta^{mn} + b(\vec{p},\vec{q})p^m p^n \right)$$

$a(p,q)$ and $b(p,q)$ are assumed to be regular in $q \to 0$ limit

Analyticity Assumption:

Consistency relations hold if A_{ij} starts to contribute at q^2 order

Analyticity/locality is a nontrivial assumption and holds only for adiabatic modes

$$N_i \supset -a^2 \frac{\dot{H}}{H^2} \frac{q_i}{q^2} \dot{\zeta}$$

Consistency Relations:

3-point functions are determined by power spectrum up to order q

$$\frac{\langle \zeta_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q}-\vec{p}} \rangle'}{P_\zeta(q)} = -\left(3 + p_k \frac{\partial}{\partial p_k}\right) P_\zeta(p)$$
$$-\frac{1}{2} q_k \left(6 \frac{\partial}{\partial p_k} - p_k \frac{\partial^2}{\partial p_a \partial p_a} + 2 p_a \frac{\partial^2}{\partial p_a \partial p_k}\right) P_\zeta(p) + \mathcal{O}(q^2)$$

$$\frac{\langle \gamma^{ij}_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q}-\vec{p}} \rangle'}{P_\gamma(q)} = -\frac{1}{2} \hat{P}^{ijk\ell}(\hat{q}) p_k \frac{\partial}{\partial p_\ell} P_\zeta(p)$$
$$+\frac{1}{4} \hat{P}^{ijk\ell}(\hat{q}) q_m \left(p_m \frac{\partial^2}{\partial p_k \partial p_\ell} - 2 p_k \frac{\partial^2}{\partial p_\ell \partial p_m}\right) P_\zeta(p) + \mathcal{O}(q^2)$$

Higher Order Correlation Functions:

Higher order corrections are not uniquely fixed by symmetries. In particular, starting from q^2-order the consistency relation schematically looks like

$$\frac{\langle \gamma_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q}-\vec{p}} \rangle'}{P_\gamma(q)} + \frac{\langle \zeta_{\vec{q}} \zeta_{\vec{p}} \zeta_{-\vec{q}-\vec{p}} \rangle'}{P_\zeta(q)} = \text{Terms Determined by 2-point func.}$$

$$+A - term$$

Model dependent pieces need to be projected out

Recovering Consistency Conditions to all Orders:

$$\lim_{q\to 0} Proj \times \frac{\partial^n}{\partial q^n}\left(\frac{\langle\zeta(q)\zeta(p)\zeta(p')\rangle'}{P_\zeta(q)} + \frac{\langle\gamma(q)\zeta(p)\zeta(p')\rangle'}{P_\gamma(q)}\right) \sim Proj \times \frac{\partial^n}{\partial p^n}\langle\zeta(p)\zeta(p')\rangle'$$

These relations have been checked up to $n = 3$ explicitly, including those with hard tensor modes, for models with arbitrary c_s LB, Justin Khoury, Junpu Wang '14

Summary:

We have shown that our approach recovers the known consistency conditions for primordial perturbations

Our derivation makes precise the assumptions underlying the consistency relations, namely the regularity of the effective action in the $q \to 0$ limit; possible ways for violating these relations include the modified initial state LB, Justin Khoury '14

Whether our approach can be used to obtain something beyond the consistency conditions presented here remains to be seen