Position-dependent power spectrum of the large-scale structure:

a novel method to measure the squeezed-limit bispectrum

[1403.3411] (Wagner et al 2014 in prep, Chiang et al 2014 in prep)

Chi-Ting Chiang (蔣季庭)

with Christian Wagner, Fabian Schmidt, and Eiichiro Komatsu

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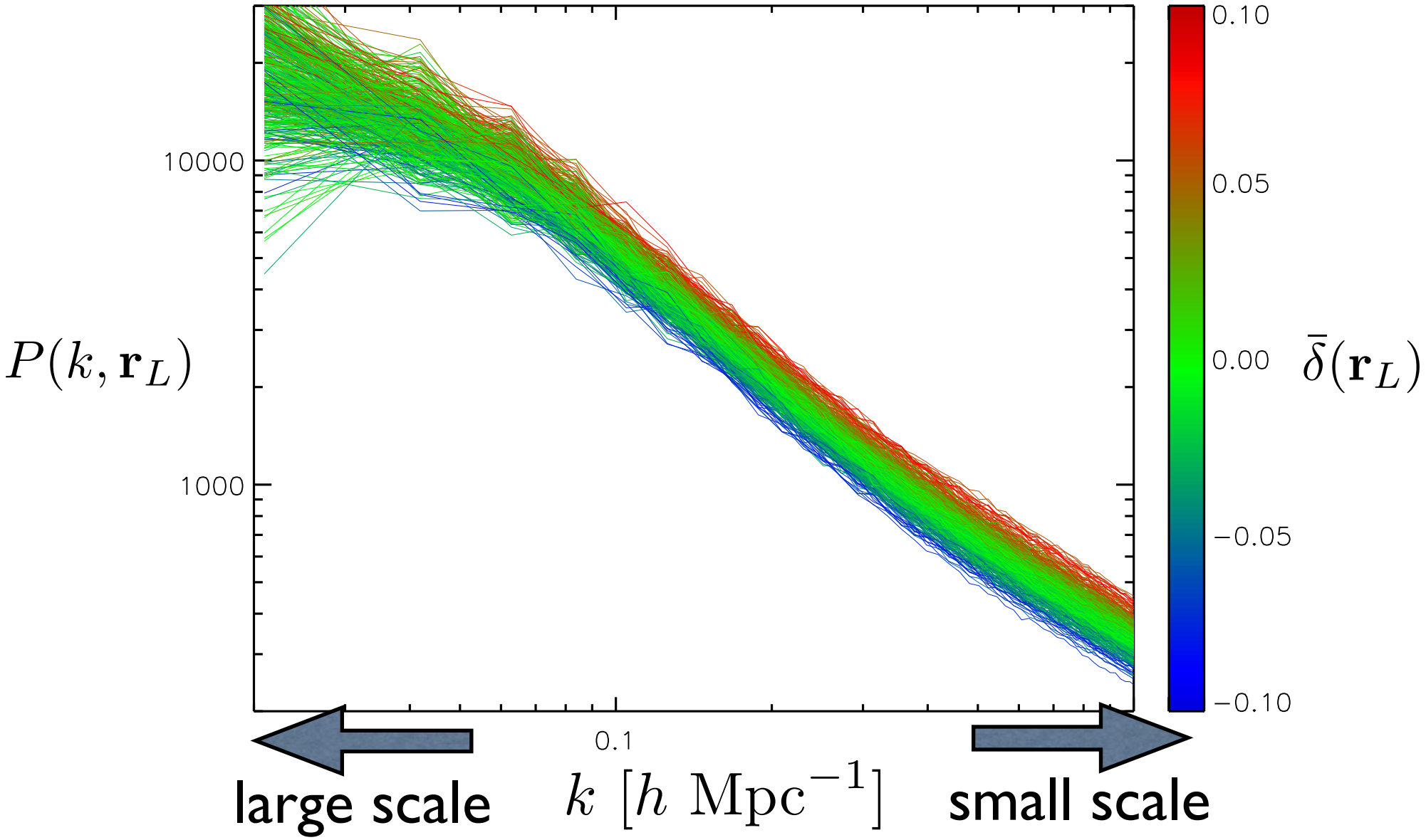
Question

- How does the small-scale power spectrum *respond* to the large-scale density fluctuations?

Answer

- The answer depends on the bispectrum (three-point function)!
- In the absence of the bispectrum, the small-scale power spectrum is independent of the large-scale density fluctuations.
- Therefore, we can measure the bispectrum by measuring the small-scale power spectrum as a function of the large-scale density fluctuations!

Small-scale power spectra depend on large-scale density fluctuations!

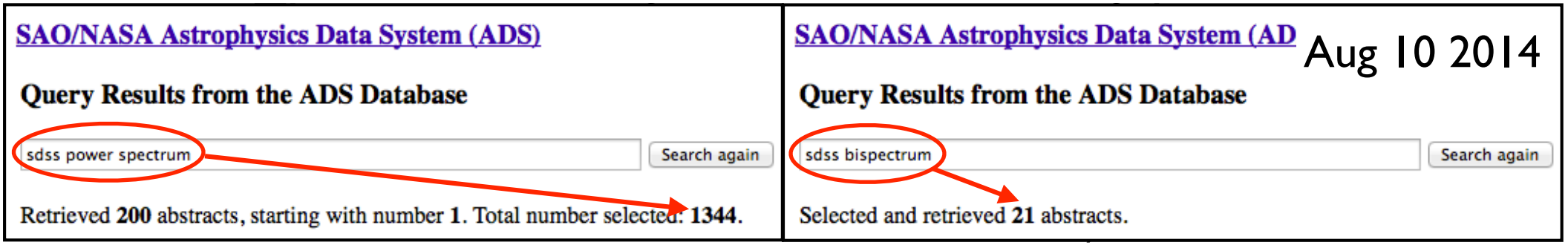


Why is it interesting to measure the bispectrum?

- If the fluctuations follow Gaussian statistics, then one only needs the power spectrum (2-point correlation function) to describe their statistical properties.
- If not, then one need higher-order statistics.
- What is the statistical properties of the primordial fluctuations?

Why study position-dependent power spectrum?

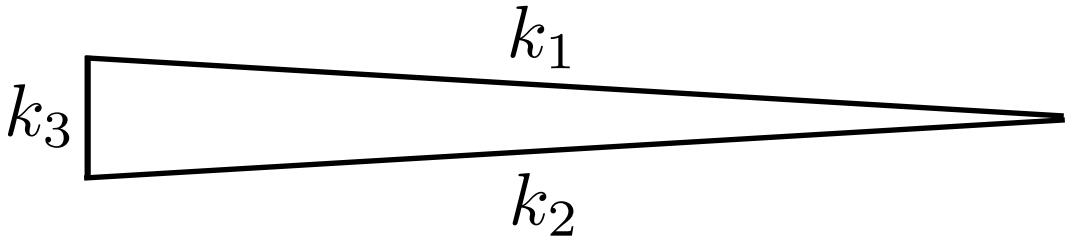
- Bispectrum is a difficult measurement when the survey geometry and selection function are not ideal. Thus, it is very difficult to extract useful information from the observed bispectrum.



- This motivates us to find a new observable which is easier to model and gives similar information.

What is the squeezed-limit bispectrum?

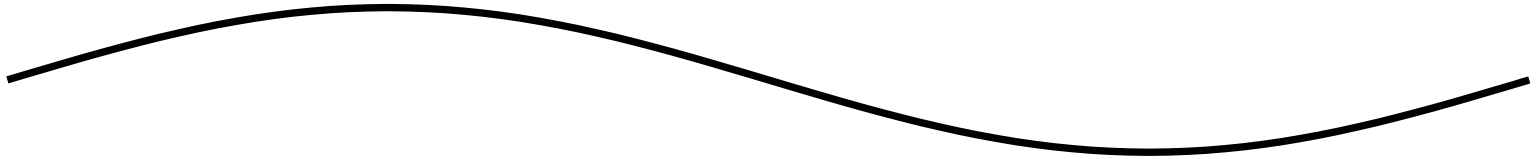
- $\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$
- squeezed limit: $k_1 \approx k_2 \gg k_3$



- Squeezed-limit bispectrum measures the correlation of one long-wavelength (small-wavenumber) fluctuation and two short-wavelength (large-wavenumber) fluctuations.

How is the position-dependent P(k,r) modulated by the bispectrum?

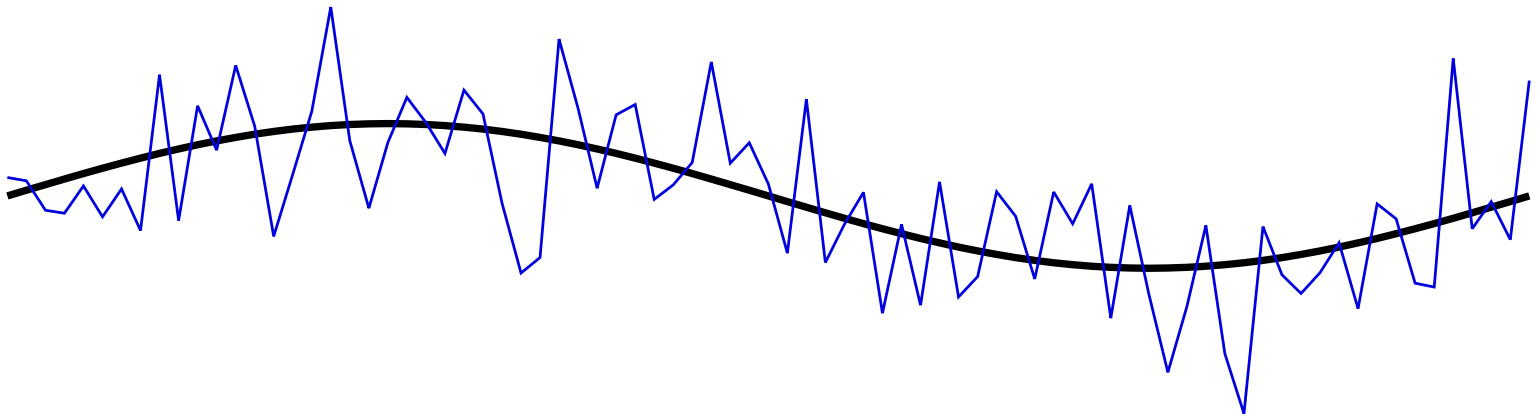
consider a long-wavelength density fluctuation...



overdensity underdensity

How is the position-dependent P(k,r) modulated by the bispectrum?

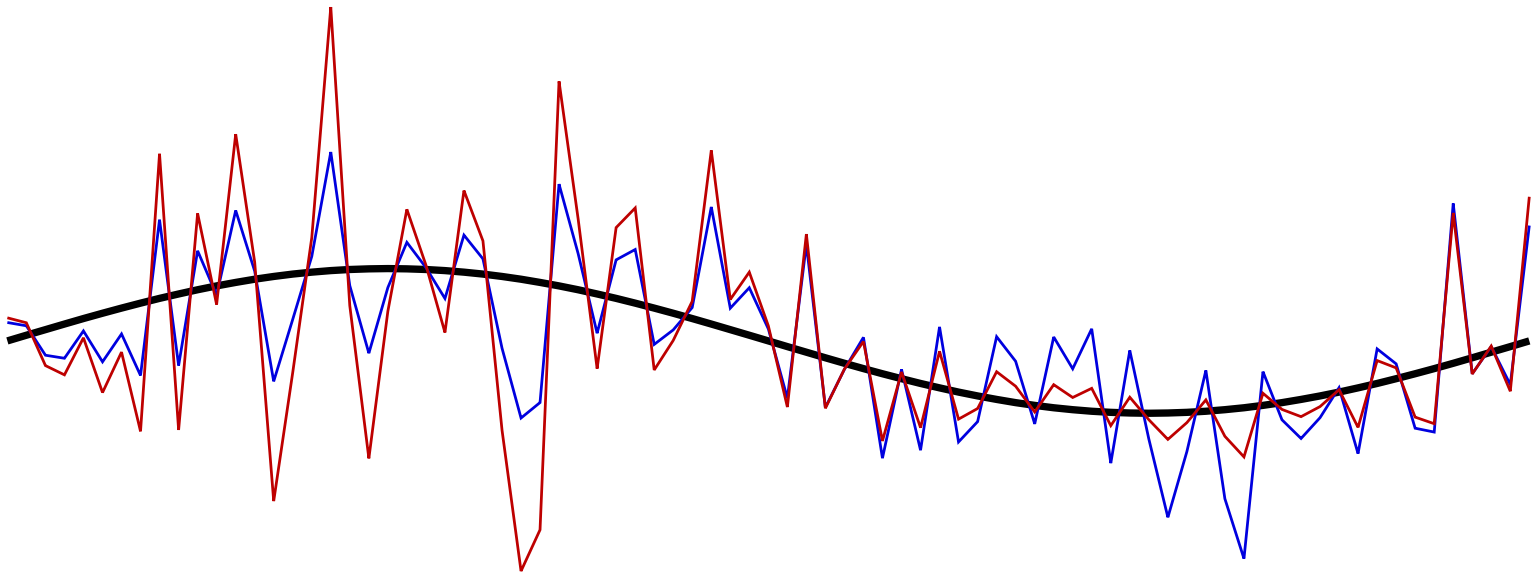
vanishing squeezed-limit bispectrum:
no correlation between power spectra and positions



overdensity underdensity

How is the position-dependent P(k,r) modulated by the bispectrum?

non-vanishing squeezed-limit bispectrum:
correlation between power spectra and positions



overdensity underdensity

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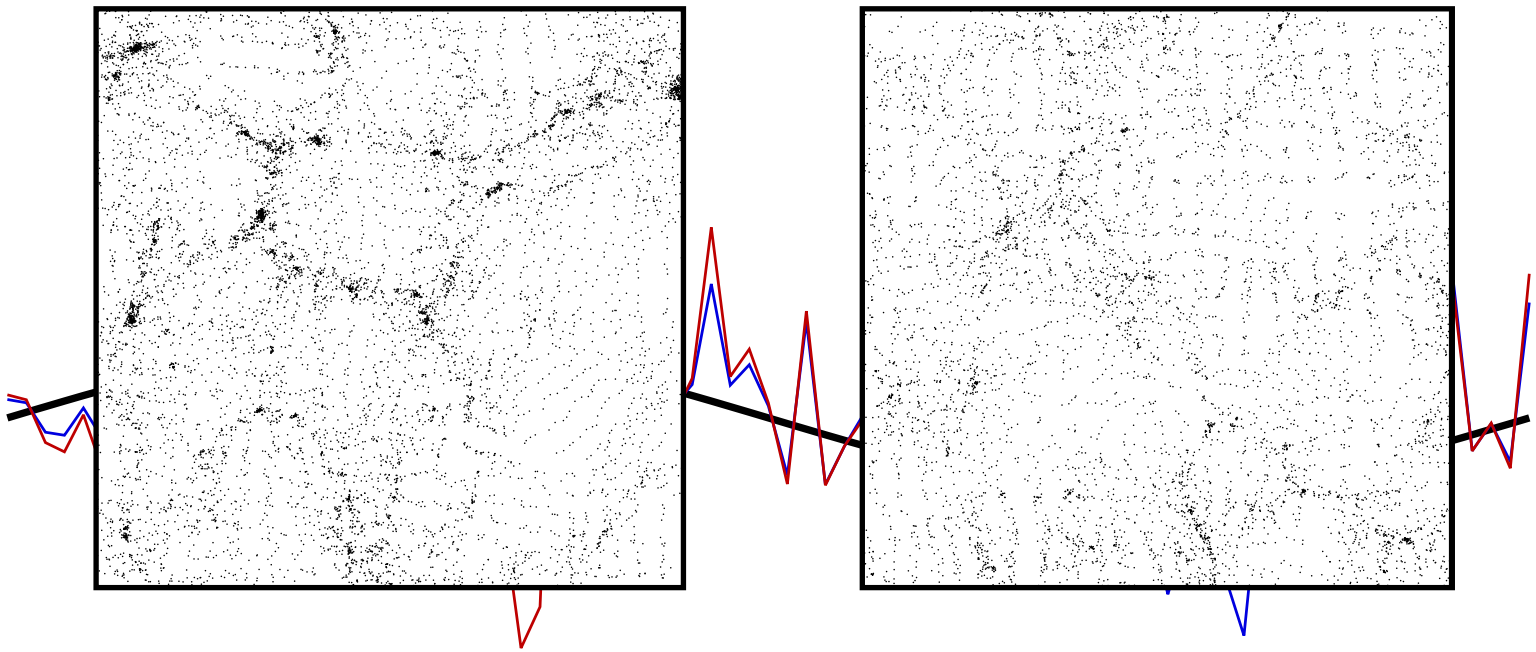


overdensity underdensity

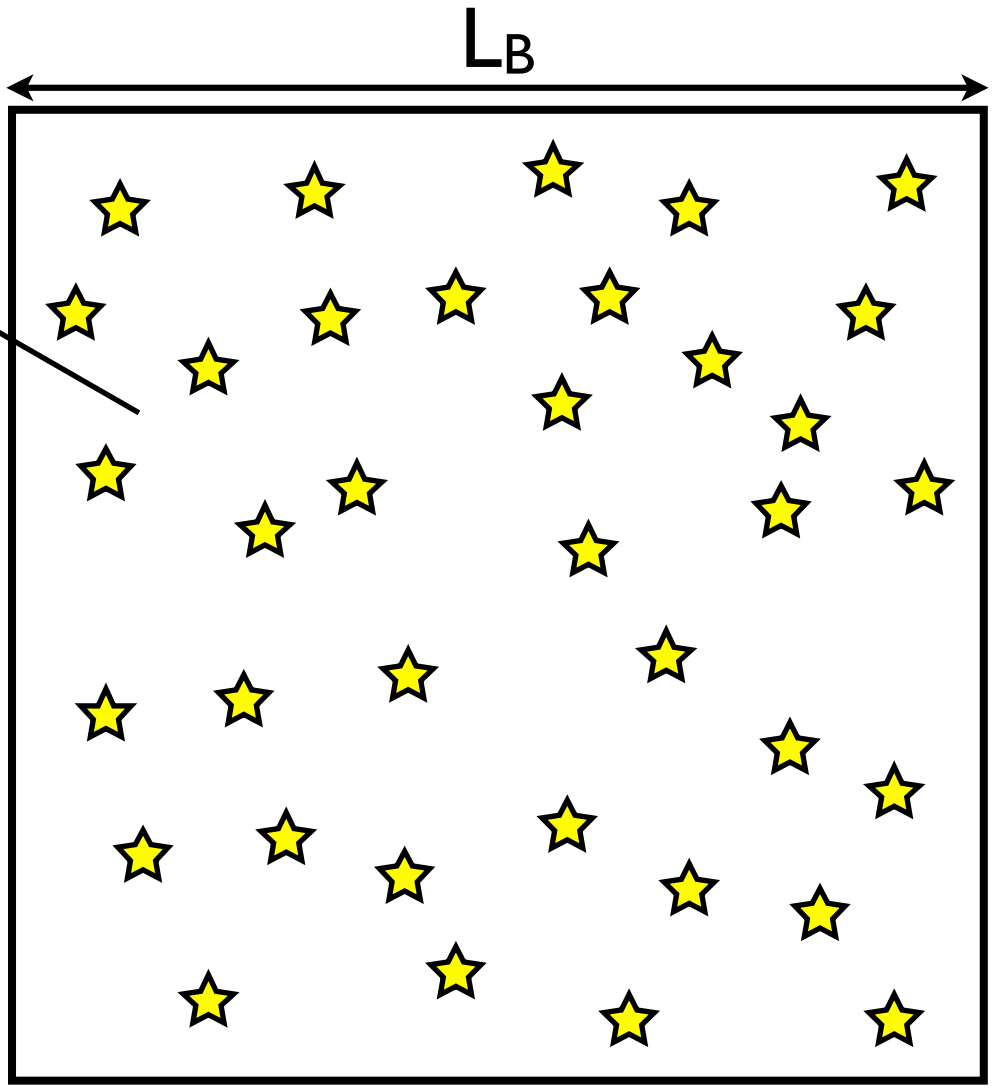
How to measure the power spectrum P(k)?

$$\delta(\mathbf{r}) = \frac{n(\mathbf{r})}{\langle n \rangle} - 1$$

Fourier transform

$$\delta(\mathbf{k}) = \int d^3r \; \delta(\mathbf{r}) e^{-i\mathbf{r}\cdot\mathbf{k}}$$

$$P(\mathbf{k}) = \frac{1}{V_B} |\delta(\mathbf{k})|^2$$



How to measure the position-dependent P(k,r)?

local mean

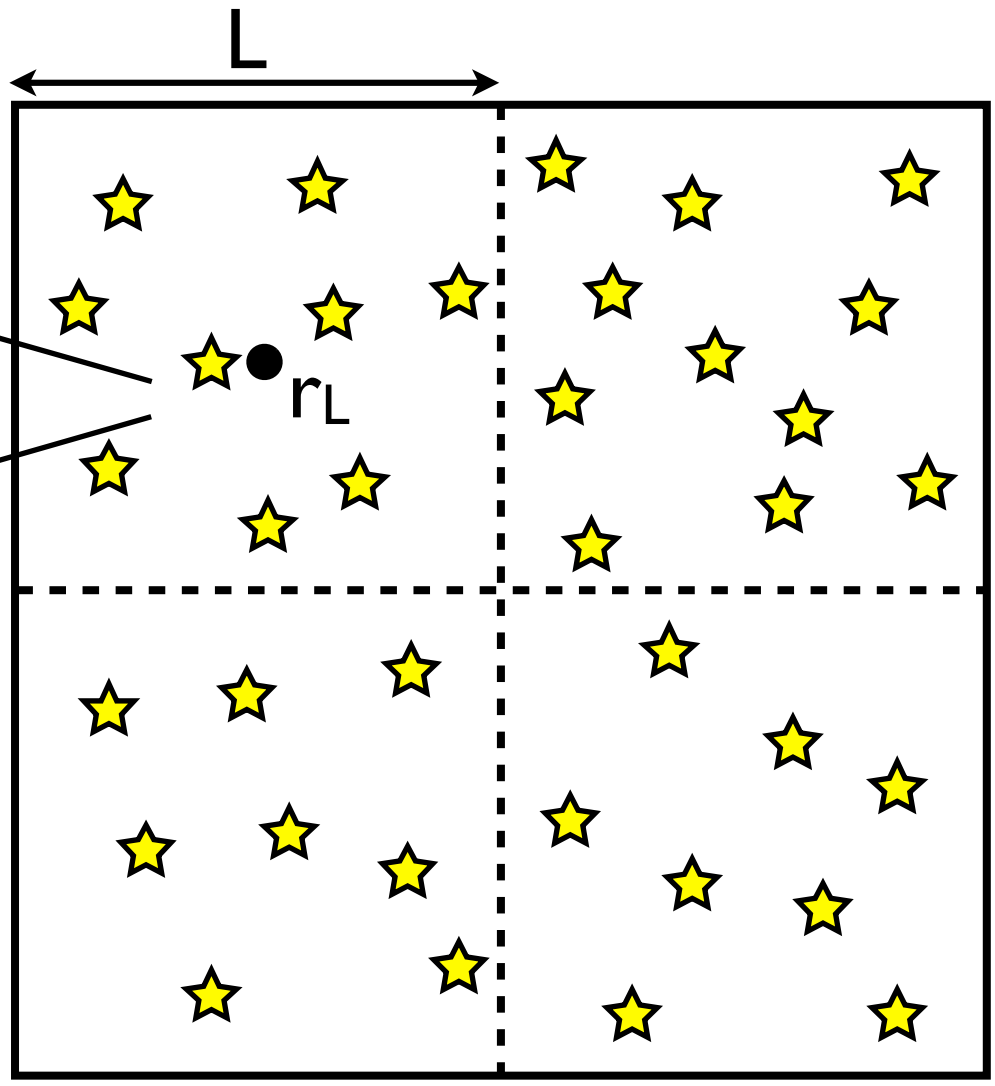
$$\bar{\delta}(\mathbf{r}_L) = \frac{\bar{n}(\mathbf{r}_L)}{\langle n \rangle} - 1$$

global mean

$$P(\mathbf{k}, \mathbf{r}_L) = \frac{1}{V_L} |\delta(\mathbf{k}, \mathbf{r}_L)|^2$$

local Fourier transform

$$\langle P(\mathbf{k}, \mathbf{r}_L) \bar{\delta}(\mathbf{r}_L) \rangle$$



Correlation - integrated bispectrum iB(k)

- $iB(\mathbf{k}) = \langle P(\mathbf{k}, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$

$$= \frac{1}{V_L^2}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\, B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2)$$
$$\times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_2)W_L(\mathbf{q}_2)$$

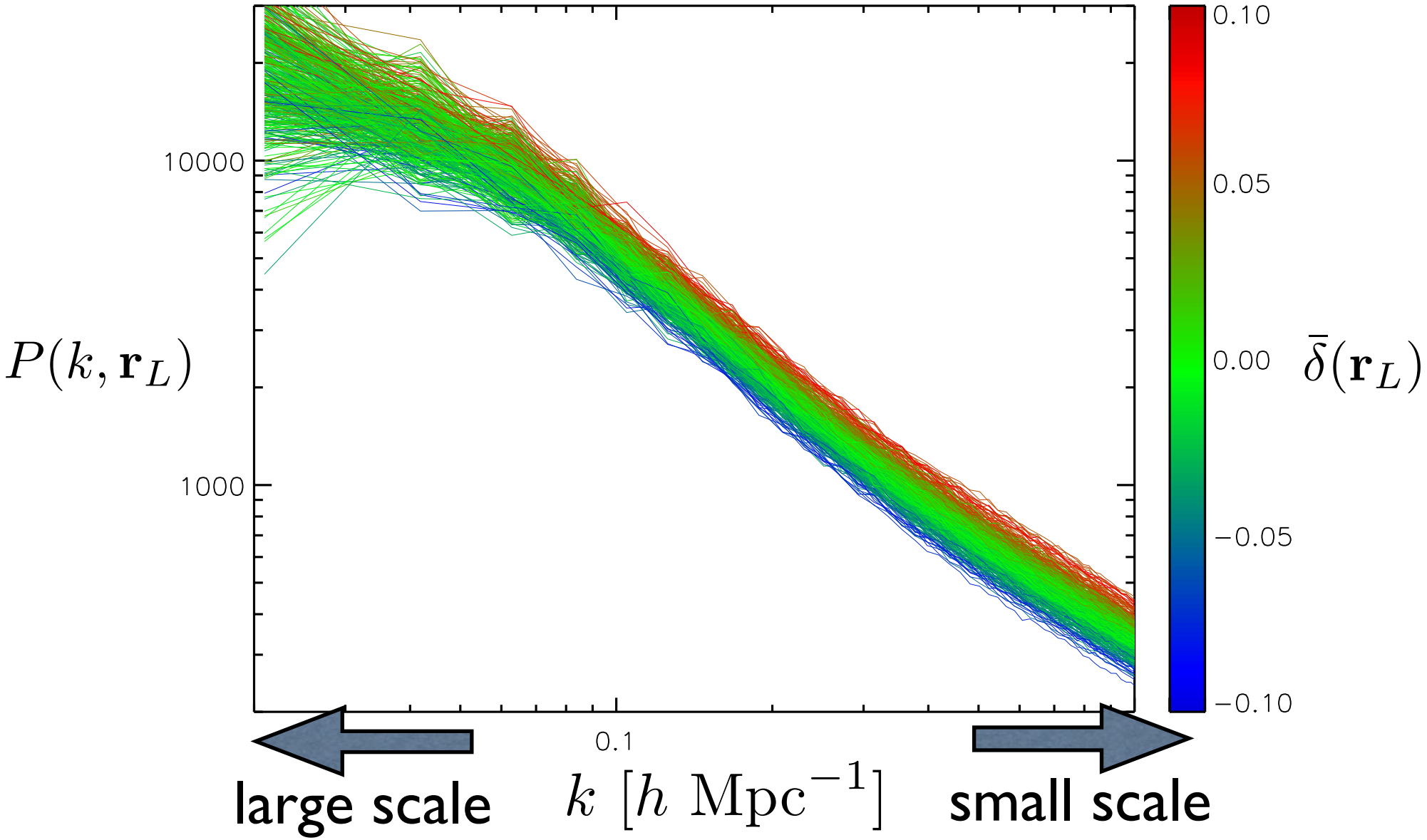
- $W_L(\mathbf{k}) = V_L\ \mathrm{sinc}\left(\frac{k_xL}{2}\right)\mathrm{sinc}\left(\frac{k_yL}{2}\right)\mathrm{sinc}\left(\frac{k_zL}{2}\right)$

- squeezed limit: $k \gg q_1 \sim q_2 \sim 1/L$ (e.g. k=0.3 h Mpc^{-1} and L=300 h^{-1}Mpc)
- focus on angular average of $\mathbf{k}$

N-body simulations

- 160 simulations of ΛCDM cosmology with Gaussian initial conditions
  - Box size: $(2400\ h^{-1}\mathrm{Mpc})^3$
  - Particle mass: $2.3\times10^{12}\ h^{-1}M_{sun}$
- Divide the simulation box into sub-volumes
  - cut=4: 64 sub-volumes of $(600\ h^{-1}\mathrm{Mpc})^3$
  - cut=8: 512 sub-volumes of $(300\ h^{-1}\mathrm{Mpc})^3$
  - cut=20: 8000 sub-volumes of $(120\ h^{-1}\mathrm{Mpc})^3$

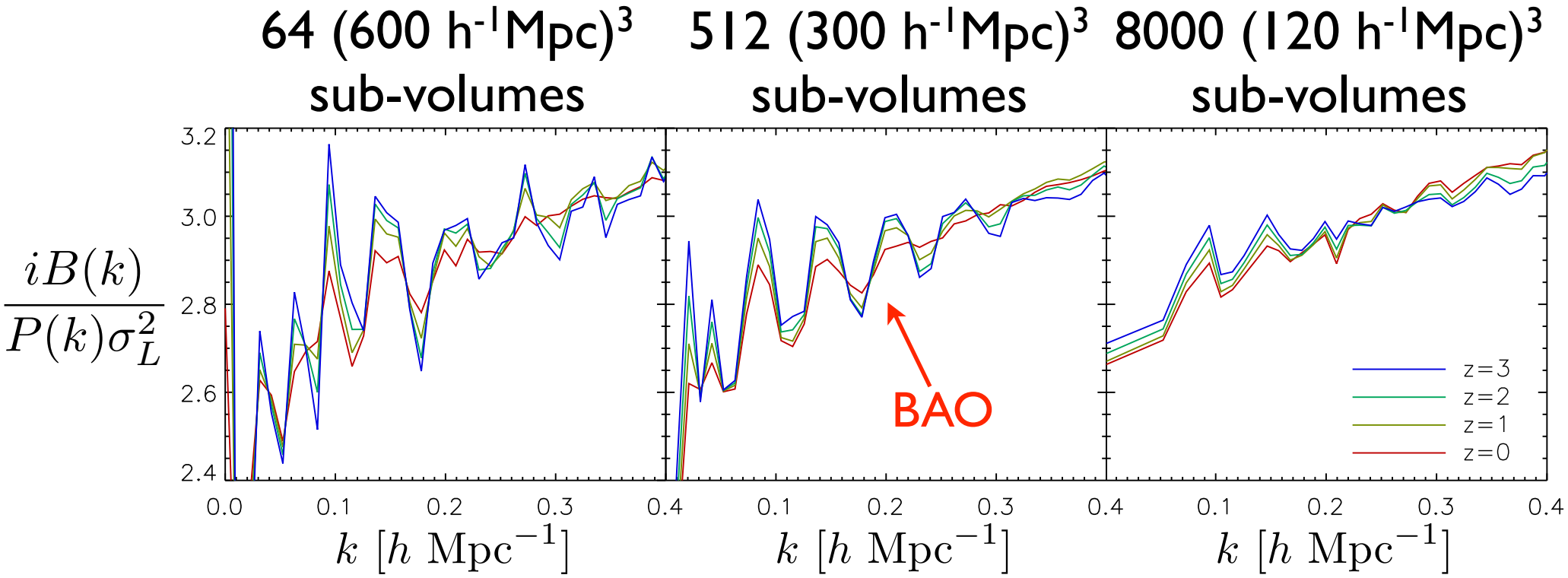
$P(k,r_L)$ measured in 512 sub-volumes of $(300\ h^{-1}\mathrm{Mpc})^3$



Normalized integrated bispectrum iB(k)

- $$iB(k) = \int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\, iB(\mathbf{k}) = \langle P(k, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$$
$$= \frac{1}{V_L^2}\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\, B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2)$$
$$\times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_2)W_L(\mathbf{q}_2)$$
- in the squeezed limit ($k \gg q_1 \sim q_2 \sim 1/L$):
$$\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\, B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2) \to f(k)P(k)P(q_2)$$
- normalized integrated bispectrum:
$$\frac{iB(k)}{P(k)\sigma_L^2} \quad \text{where} \quad \sigma_L^2 = \frac{1}{V_L^2}\int \frac{d^3q_2}{(2\pi)^3}\, |W_L(\mathbf{q}_2)|^2 P(q_2)$$

Measured normalized iB(k)



- BAO is damped more with smaller sub-volume size
- Nonlinear evolution also damps BAO
- Broad-band power is slightly larger on small scales

How to model iB(k)?

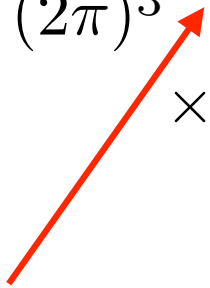
- Model the bispectrum and calculate the 8-d integral
  - standard perturbation theory
  - bispectrum fitting formula
- Separate universe approach
  - linear, halofit, Coyote emulator, halo model
- Modeling does not change with the size of the sub-volumes, so we only show the predictions for (300 h^{-1}Mpc)^3 sub-volumes.

Direct integration

$$iB(k) = \int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\, iB(\mathbf{k}) = \langle P(k,\mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$$

$$= \frac{1}{V_L^2}\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\, B(\mathbf{k}-\mathbf{q}_1,-\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2,-\mathbf{q}_2)$$

$$\times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_2)W_L(\mathbf{q}_2)$$



input whatever bispectrum you like

Standard perturbation theory

- $B_{\rm SPT}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2[F_2(\mathbf{k}_1,\mathbf{k}_2)P_l(\mathbf{k}_1)P_l(\mathbf{k}_2) + (2\ \text{cyclic})]$

  where $F_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{7} + \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{2k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2$

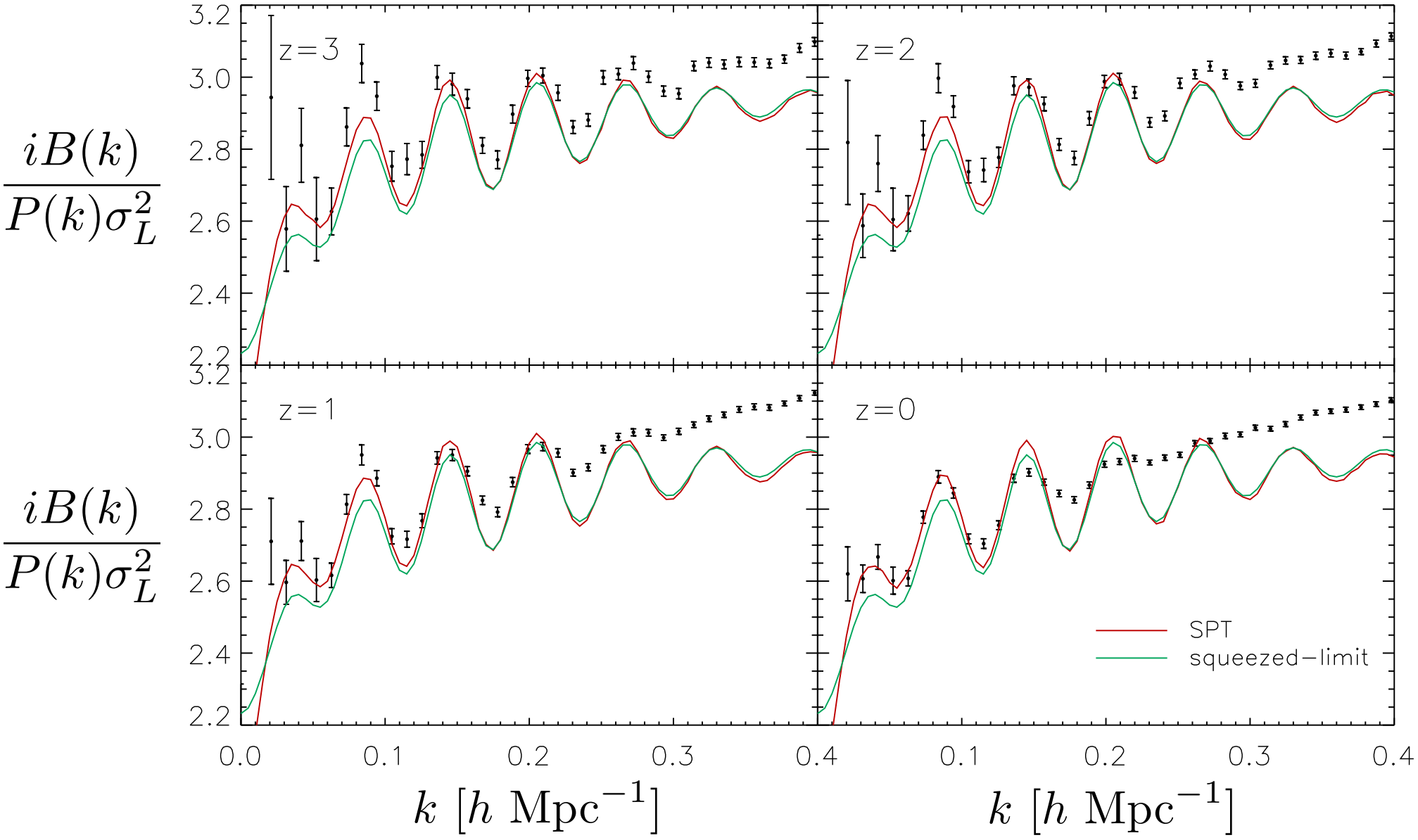
- in the squeezed limit:

$$\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\, B_{\rm SPT}(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2)$$

$$= \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k)}{d\ln k}\right] P_l(k)P_l(q_2) + \mathcal{O}\left(\frac{q_{1,2}}{k}\right)^2$$

  and thus $\frac{iB_{\rm SPT}(k)}{P_l(k)\sigma_L^2} \to \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k)}{d\ln k}\right]$

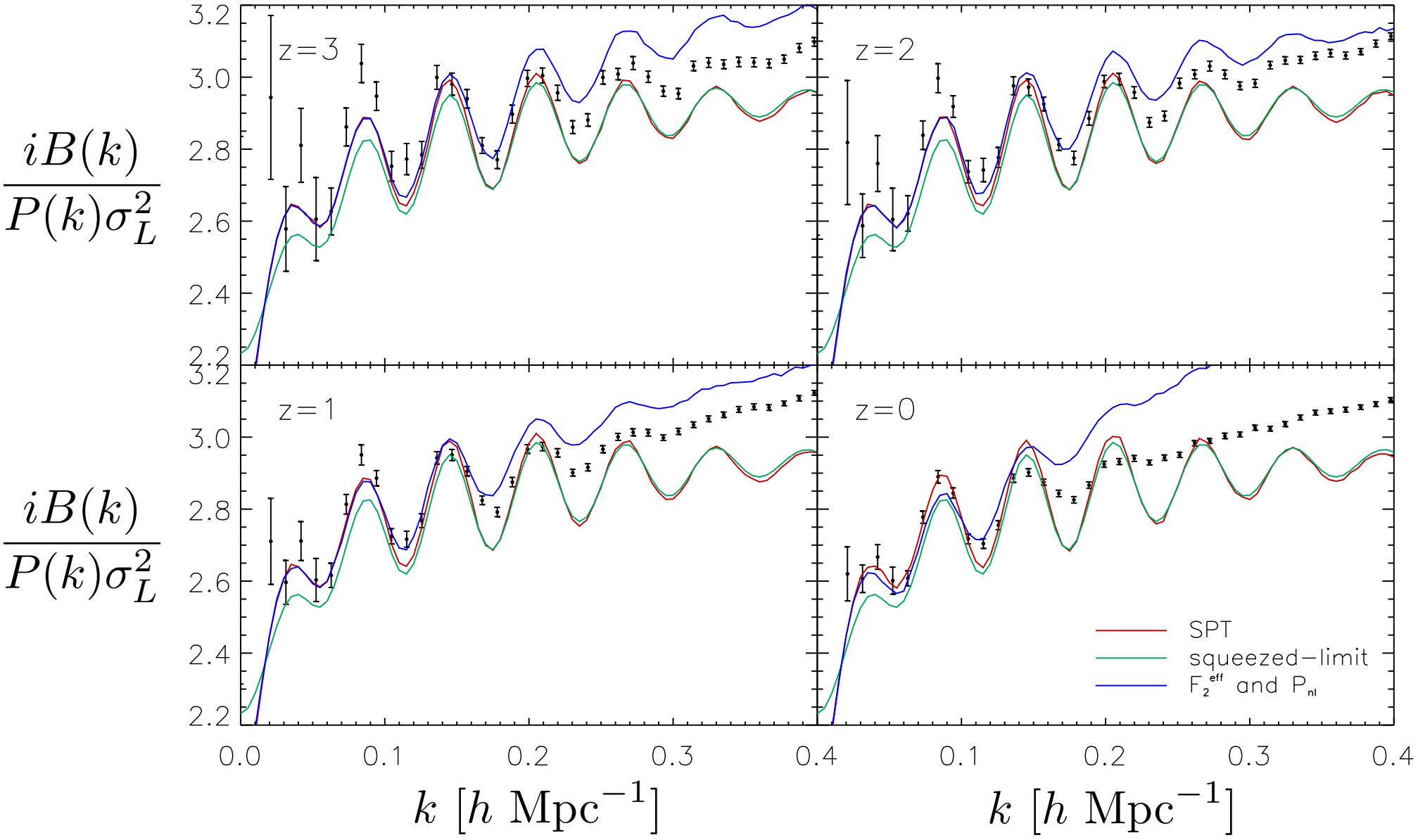
SPT modeling



Bispectrum fitting formula

- First proposed in Scoccimarro and Couchman 2001 and further improved in Gil-Marin et al. 2012
- $B_{\rm eff}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2[F_2^{\rm eff}(\mathbf{k}_1, \mathbf{k}_2) P_{nl}(\mathbf{k}_1) P_{nl}(\mathbf{k}_2) + (2 \text{ cyclic})]$
- The fitting formula contains 9 fitting parameters.

$F_2^{\rm eff}$ modeling



Separate universe approach

- Squeezed-limit bispectrum measures the correlation of large-scale fluctuations and small-scale structures.
- The power spectrum in the universe with an infinite-wavelength density perturbation δ_0 is

$$P(k|\delta_0) = P(k)|_{\delta_0=0} + P(k)\frac{d\ln P(k)}{d\delta_0}\Big|_{\delta_0=0}\delta_0 + \mathcal{O}(\delta_0^2)$$

- One can then consider the normalized integrated bispectrum as how the power spectrum responds to the density perturbation δ_0, i.e.

$$\frac{iB(k)}{P(k)\sigma_L^2} = \frac{d\ln P(k)}{d\delta_0}$$

Mapping the universe

Ω_m

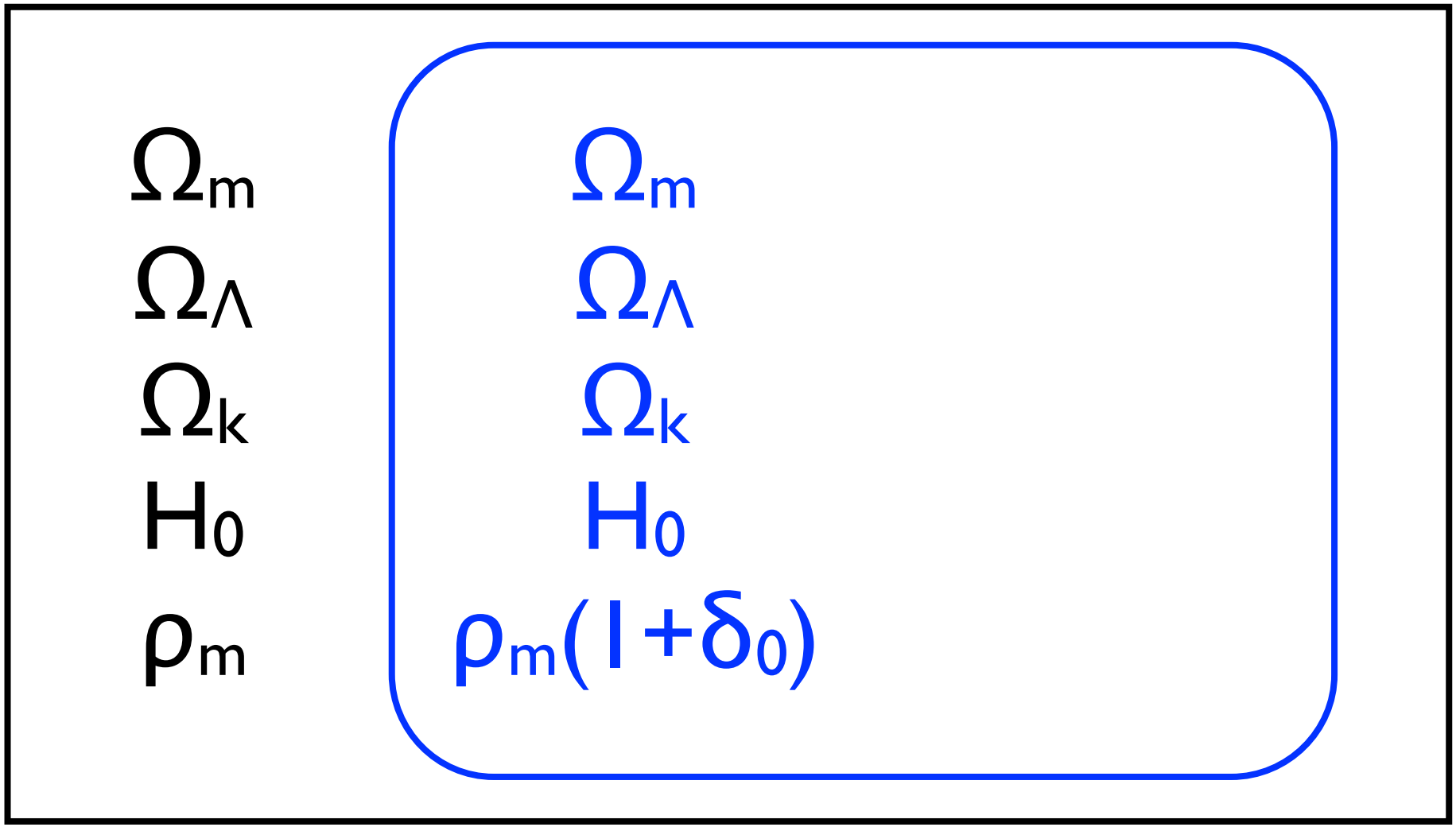
Ω_Λ

Ω_k

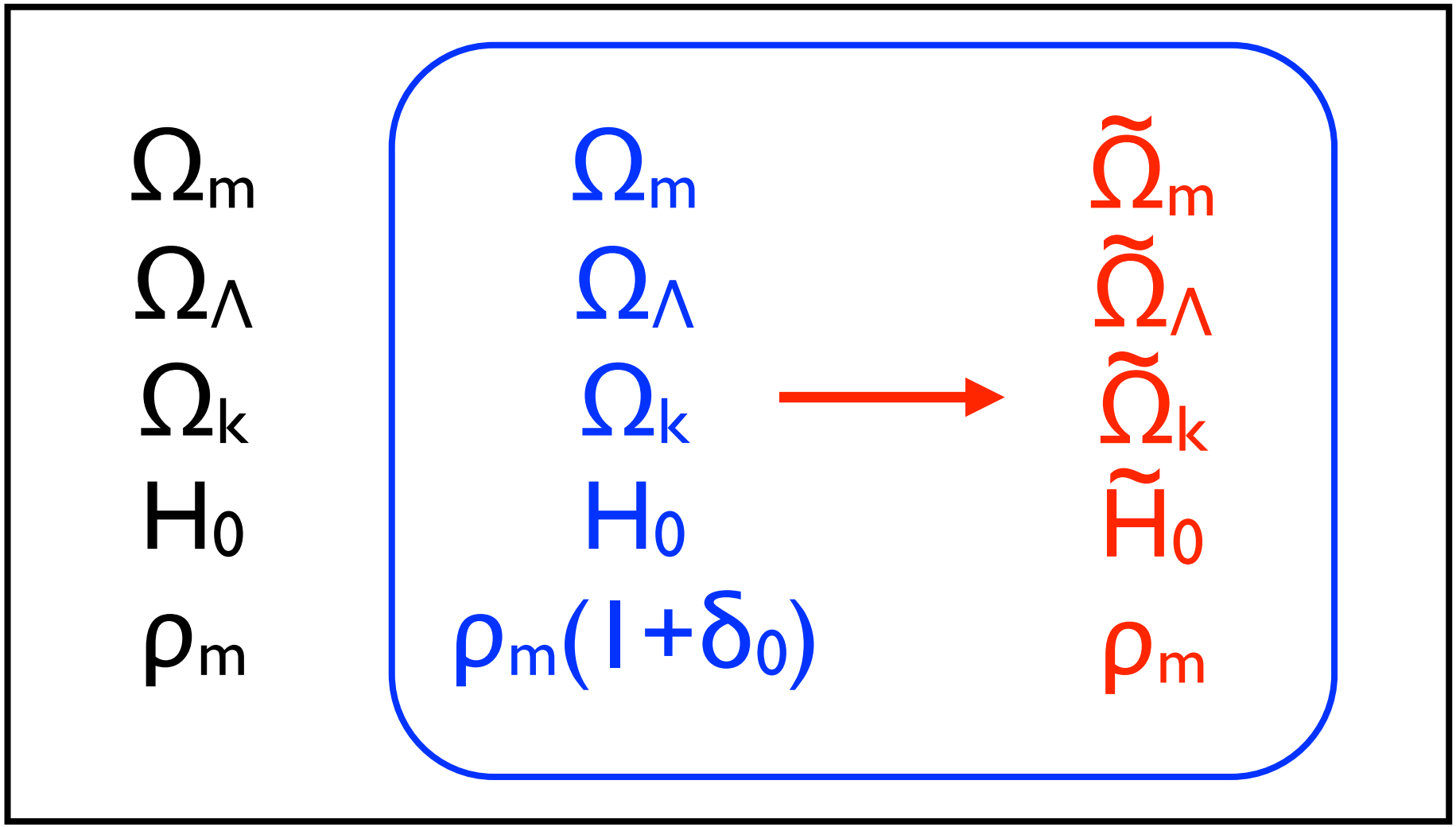
H_0

ρ_m

Mapping the universe



Mapping the universe



Flat ΛCDM universe with δ_0

change of cosmology
in linear order of δ_0

$$\tilde{H}_0 = H_0 \left[1 + \delta_H\right]$$

$$\delta_H = \left(-\frac{1}{2}\Omega_m - \frac{1}{3}f_0\right)\delta_0$$

$$\tilde{\Omega}_m = \Omega_m \left[1 - 2\delta_H\right]$$

$$\tilde{\Omega}_K = -\left(\Omega_m + \frac{2}{3}f_0\right)\delta_0$$

$$\tilde{\Omega}_\Lambda = \Omega_\Lambda \left[1 - 2\delta_H\right]$$

change of time and scale
in linear order of δ_0

$$\tilde{a}(t) = a(t)\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\delta_0\right]$$

$$\tilde{\mathbf{x}}(t) = \mathbf{x}(t)\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\delta_0\right]$$

$$\tilde{D}[\tilde{a}(t)] = D(a)\left[1 + \frac{13}{21}\delta_0\right]$$

exact only in Einstein-de Sitter universe
but depends very mildly on cosmology
(~0.1% fractional difference at z=0)

Change of power spectrum with δ_0

- change in scale:

$$\tilde{P}(k,t) \to P(k,t)\left[1 - \frac{1}{3}\frac{d\ln k^3 P(k,t)}{d\ln k}\frac{D(t)}{D(t_0)}\delta_0\right]$$

- change in scale factor (and growth rate):

$$\tilde{P}(k,t) \to P\left(k, a\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\right]\right)$$

- change in normalization ($\rho \to \rho(1+\delta_0)$):

$$\tilde{P}(k,t) \to (1+2\delta_0)P(k,t)$$

- total change:

$$\tilde{P}\left(k, a\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\delta_0\right]\right)\left[1 + \left(2 - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}\right)\frac{D(t)}{D(t_0)}\delta_0\right]$$

Change of linear power spectrum with δ_0

- linear power spectrum is proportional to D^2

$$\tilde{P}_l\left(k, a\left[1-\frac{1}{3}\frac{D(a)}{D(a_0)}\delta_0\right]\right) = \left(\frac{\tilde{D}\left(a\left[1-\frac{1}{3}\frac{D(a)}{D(a_0)}\delta_0\right]\right)}{D(a)}\right)^2 P_l(k,a)$$

- change of growth rate:

$$\tilde{D}\left(a\left[1-\frac{1}{3}\frac{D(a)}{D(a_0)}\delta_0\right]\right) = D(a)\left[1+\frac{13}{21}\frac{D(a)}{D(a_0)}\delta_0\right]$$

- finally, $\frac{d\ln P_l(k,t)}{d\delta_0} = \frac{D(t)}{D(t_0)}\left[\frac{68}{21}-\frac{1}{3}\frac{d\ln k^3 P_l(k,t)}{d\ln k}\right]$

agree with SPT (F_2 kernel) result!

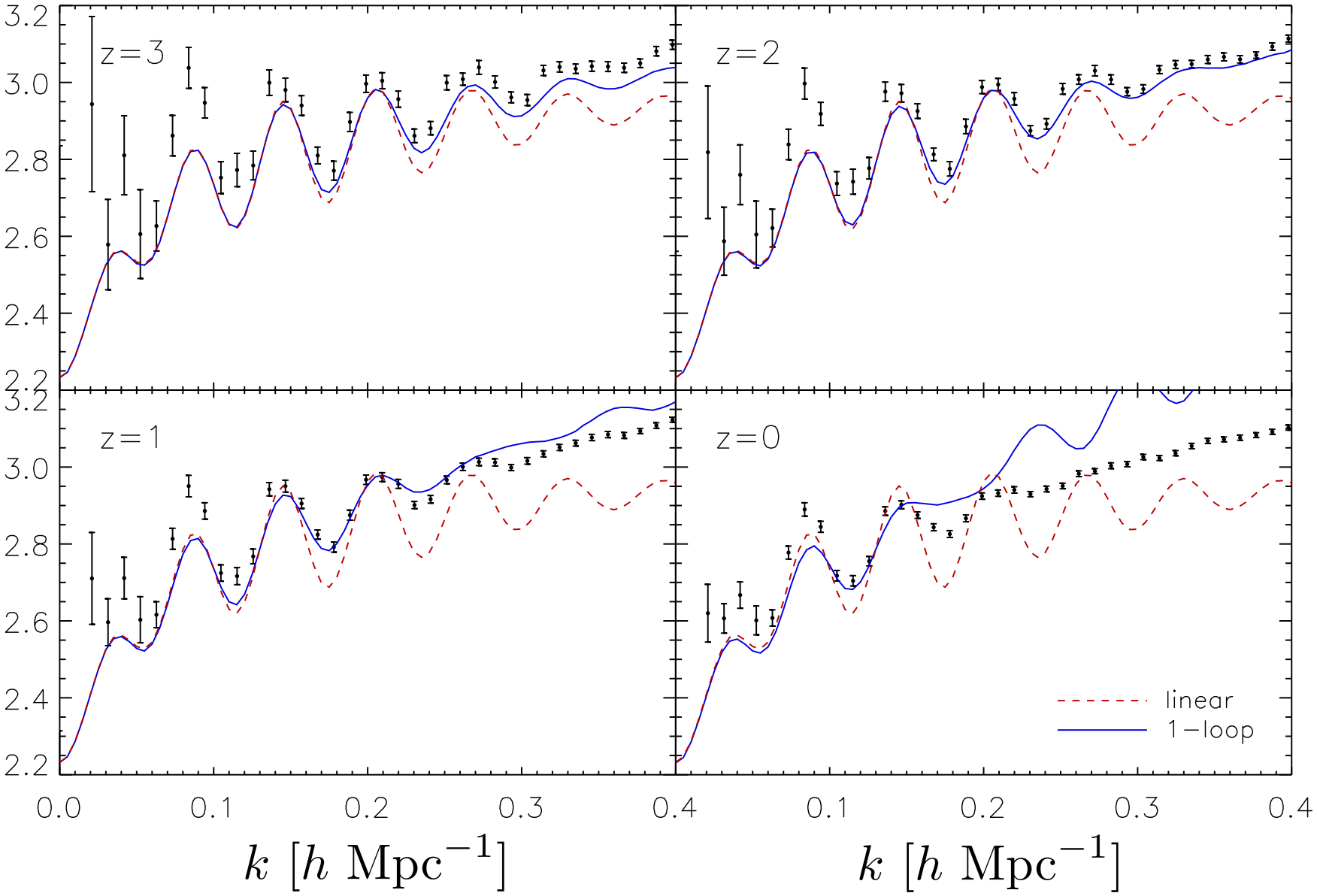
SPT 1-loop power spectrum

- $P(k,a) = P_l(k,a) + P_{22}(k,a) + 2P_{13}(k,a)$
  $= D^2(a)P_l(k) + D^4(a)[P_{22}(k) + 2P_{13}(k)]$
- $\tilde{D}[\tilde{a}(t)] = D(a)\left[1 + \frac{13}{21}\delta_0\right]$
- $\frac{d\ln P(k,a)}{d\delta(a)} = \frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}$
  $+ \frac{26}{21}\frac{P_{22}(k,a) + 2P_{13}(k,a)}{P(k,a)}$

SPT 1-loop P(k) modeling

$$\frac{iB(k)}{P(k)\sigma_L^2}$$

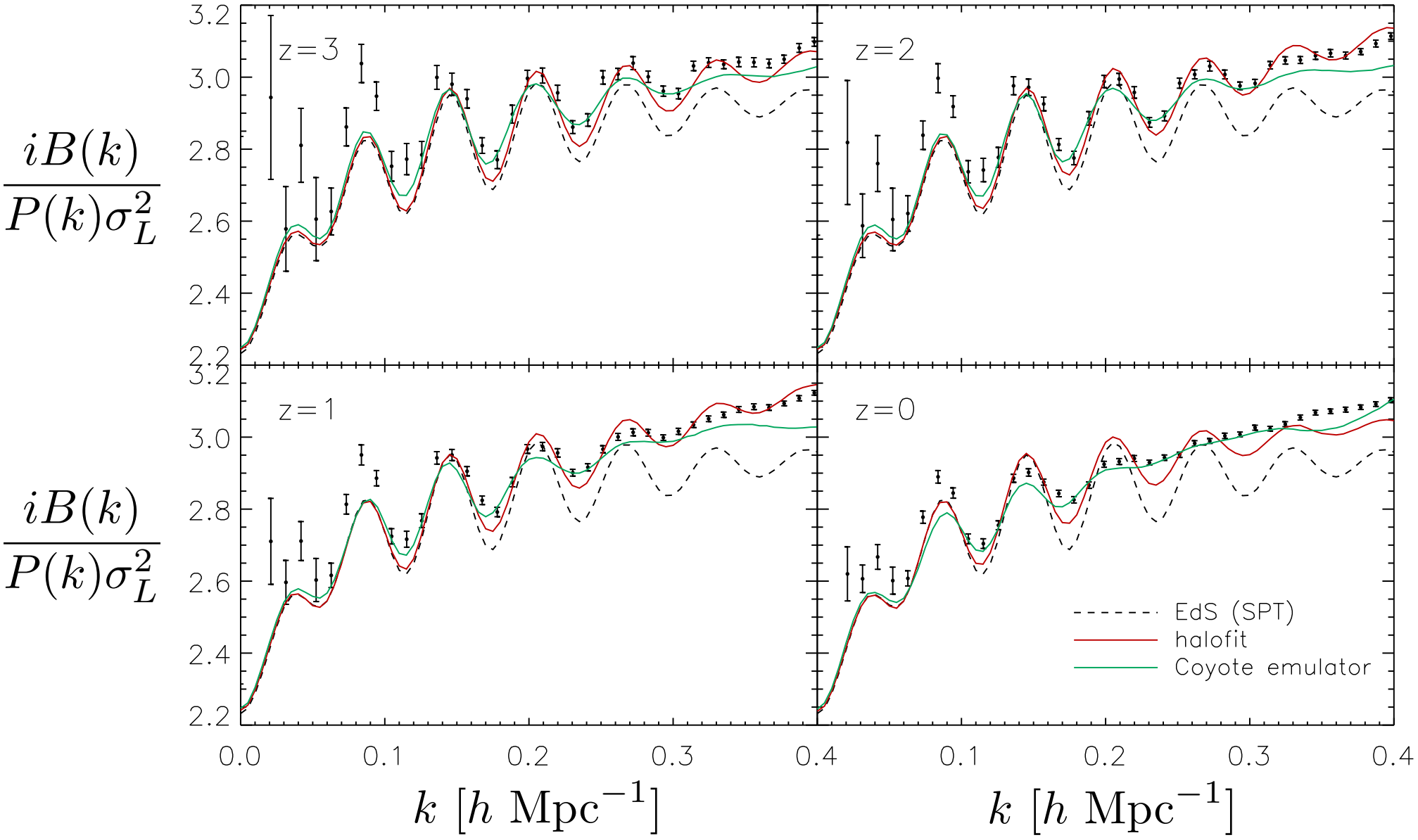
$$\frac{iB(k)}{P(k)\sigma_L^2}$$



halofit and Coyote emulator

- Mimic the effect of δ_0 by changing the cosmological parameters in the machineries of calculating non-linear power spectrum.
- halofit prescription (Smith et al. 2003) and Coyote emulator (Heitmann et al. 2013)

halofit and Coyote emulator modeling



Summary

- Without directly measuring the bispectrum, the position-dependent power spectrum P(k,r) gives similar information.
- Pros:
  - Easy to measure, especially with non-ideal survey geometry.
  - Procedures developed for power spectrum estimation can be directly applied to position-dependent power spectrum (e.g. removing window function effect).
  - iB(k) depends only on one wavenumber k (unlike the bispectrum which depends on two wavenumbers and one angle), so the covariance matrix would be easier to model.
- Cons: Use only a subset of information.
- Separate universe approach is a powerful way to model the small-scale power spectrum in the presence of δ_0.