Early Universe Challenges for Chameleon Gravity

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Meet the Chameleon

Scalar-Tensor Gravity: we must hide the scalar!

Chameleon Mechanism: scalar’s mass depends on environment

$\beta \frac{\rho_{\text{gal}} \phi}{M_{\text{Pl}}}$

Khoury & Weltman 2004

$\frac{V(\phi)}{M^4}$ (Potential)

$\phi \over M$ (Chameleon)

skinny chameleon = dark energy

fat chameleon = GR
Meet the Chameleon

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This is the screening mechanism employed by viable $f(R)$ gravity theories

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Khoury & Weltman 2004
Chameleon Gravity: Lightning Review

Chameleon gravity: a screened scalar-tensor theory

\[ S = \int d^4 x \sqrt{-g_*} \left[ \frac{M_{\text{Pl}}^2}{2} R_* - \frac{1}{2} \left( \nabla_\phi \right)^2 - V(\phi) \right] + S_m [\tilde{g}_{\mu\nu}, \psi_m] \]

Einstein frame: standard GR + scalar field (chameleon field)

\[ \tilde{g}_{\mu\nu} = e^{2\frac{\beta \phi}{M_{\text{Pl}}}} g_{\mu\nu} \]

Matter couples to different metric (Jordan Frame)

Chameleon equation of motion:

\[ \ddot{\phi} + 3H \dot{\phi} = - \left[ \frac{dV}{d\phi} + \frac{\beta}{M_{\text{Pl}}} \left( \rho_* - 3p_* \right) \right] \]

derivative of effective potential
Chameleon Cosmology

Fiducial Chameleon Potential:

\[ V(\phi) = M^4 \exp \left[ \left( \frac{M}{\phi} \right)^n \right] \phi \gg M \approx M^4 \left[ 1 + \left( \frac{M}{\phi} \right)^n \right] \]

Evade Solar System gravity tests and provide dark energy:

\[ M \approx 0.001 \text{ eV} \approx (\rho_{\text{de}})^{1/4} \]
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Where is the chameleon now?

\[ \rho_{\text{mat},0} = 0.3 \rho_{\text{crit},0} \]

\[ \phi_{\text{min}} = 5.9 \times 10^9 M \ll M_{\text{Pl}} \]

\[ \phi_{\text{min}} \ll M_{\text{Pl}} \text{ always!} \]

\[ \phi_{\text{min}} \lesssim M \text{ inside Earth, Sun} \]

\[ \text{and at } T \gtrsim 2 \text{ MeV} \]
Chameleon Initial Conditions

During inflation: $\rho - 3p \simeq 4\rho_{\text{infl}}$ pins chameleon $\phi \ll M$  

Brax et al. 2004

After reheating: $\rho - 3p \simeq 0$ the chameleon quickly slides down its bare potential and rolls to $\phi \gg \phi_{\text{min}}$

For $\phi \gg \phi_{\text{min}}$

$$\ddot{\phi} + 3H_* \dot{\phi} = -\frac{\beta}{M_{\text{Pl}}} (\rho_* - 3p_*) \implies \Delta \phi \simeq \frac{\phi_i}{H_i} = M_{\text{Pl}} \sqrt{6\Omega_{\phi,i}}$$

Chameleon rolls out to $\phi_{\text{min}} \ll \phi \lesssim M_{\text{Pl}}$

Hubble friction prevents the chameleon from rolling back to $\phi_{\text{min}}$
Chameleon Initial Conditions

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We have a problem!

\( \phi \) cannot vary much between now and BBN:

\[
\frac{\beta}{M_{\text{Pl}}} \Delta \phi \simeq \left| \frac{\Delta m_p}{m_p} \right| \lesssim 0.1
\]

\( \phi_{\text{BBN}} \lesssim 0.1 \frac{M_{\text{Pl}}}{\beta} \)
Kicking the Chameleon

Define the kick function:
\[ \Sigma(T) \equiv \frac{\rho - 3p}{\rho} \sim \frac{\text{force}}{\text{friction}} \]

Pressure of massive particles in thermal equilibrium

\[ p = \frac{\rho}{3} \left[ 1 - \mathcal{O} \left( \frac{m^2}{T^2} \right) \right] \]

Damour & Nordtvedt 1993; Damour & Polyakov 1994
Brax et al. 2004; Coc et al. 2006, 2009
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Every particle in the Standard Model (and beyond) kicks the chameleon.
The Old Story:

The kicks save the chameleon: $\Delta \phi \simeq -\beta M_{\text{Pl}}$ prior to BBN. \textit{Brax et al. 2004}

\begin{align*}
\Delta \phi &= -0.2 M_{\text{Pl}} \\
\phi &= \beta = 0.2 \\
\sum &\text{ (Potential)} \\
\ln(a_*) &\text{ (Chameleon)}
\end{align*}
The Old Story:

The kicks save the chameleon: $\Delta \phi \simeq -\beta M_{P1}$ prior to BBN. Brax et al. 2004

\[
\frac{\phi}{M_{P1}} = 0.2
\]

Flaws in the old story:

- underestimates chameleon motion for $\beta \gtrsim 1$
- chameleon reaches $\phi_{\text{min}}$ with a large velocity and climbs up the bare potential
- the rebound is violent enough to excite quantum perturbations with extremely high energies
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Kicking the Chameleon

\[
\phi = \frac{\beta}{M_{Pl}}
\]

\[
\Delta \phi = -1.58 \beta M_{Pl}
\]

\[
\Delta \phi / M_{Pl} \text{ due to kicks}
\]

\[
\text{chameleon coupling constant (\beta)}
\]
Kicking the Chameleon

For larger $\beta$ values:
- motion of $\phi$ affects $T_J$
- extends duration of kicks
- $|\Delta \phi| > 1.58 \beta M_{Pl}$

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The Surfing Solution

Keeping the full expression for $T_J$ reveals a new solution!

$$\phi'(p) = -\frac{M_{Pl}}{\beta}$$

keeps $T_J$ constant

and solves the chameleon equation of motion provided that

$$p = \ln(a_*)$$

and

$$\beta = \sqrt{\frac{1}{3\Sigma(T_J)}}$$

for some value of $T_J$.
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The surfing solution only exists if

$$\beta \geq \sqrt{\frac{1}{3\Sigma_{\text{max}}}}$$
Surfing Chameleons

Surfing chameleons surf all the way to the minimum of the effective potential!

\[ \varphi \equiv \frac{\phi}{M_{Pl}} \]

\[ \varphi' = -\frac{1}{\beta} \]

\[ \Sigma = \frac{1}{(3\beta^2)} \]

For every value of \( \beta > 1.82 \),

\[ \Sigma(T_{surf}) = \frac{1}{3\beta^2} \]

and the chameleon reaches \( \phi_{min} \) with a velocity:

\[ \phi = T_{surf}^2 \sqrt{\frac{\pi^2 g_*}{15(6\beta^2 - 1)}} \]
Fast-Moving Chameleons

- nearly all chameleons are kicked to $\phi \sim \phi_{\text{min}}$
- at $\phi \sim \phi_{\text{min}}, \phi \sim \text{GeV}^2 \gg M^2 \sim 10^{-24}\text{GeV}^2$
- adding particles will increase the chameleon velocity!
Now that $\phi \approx \phi_{\text{min}}$, we need to consider the chameleon potential:

$$V(\phi) = M^4 \exp \left[ \left( \frac{M}{\phi} \right)^2 \right] \quad \text{with} \quad M = 0.001 \text{ eV}$$

The chameleon doesn’t stop when it reaches the minimum of its effective potential because it’s moving too fast!

The chameleon rolls up its potential until

$$V(\phi_b) = \dot{\phi}^2 / 2$$

*Classically, the impact is a reflection!*
A Classical Impact

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Quantum Particle Production

Rapid changes in $V''(\phi)$ excite perturbations!

$$\ddot{k} + \omega_k^2(t) k = 0 \quad \omega_k^2 \equiv k^2 + V''(\bar{\phi})$$

Express in terms of Bogoliubov coefficients:

$$\phi_k(t) = \frac{\alpha_k(t)}{\sqrt{2\omega_k(t)}} e^{-i \int^t \omega_k(t') dt'} + \frac{\beta_k(t)}{\sqrt{2\omega_k(t)}} e^{+i \int^t \omega_k(t') dt'}$$

Occupation number: $n_k(t) = |\beta_k|^2$

We get particle production ($|\beta_k|^2 \gtrsim 1$) when

$$\frac{\dot{\omega}_k}{\omega_k^2} = \frac{|V'''(\phi)\dot{\phi}|}{2\omega_k^3} \gtrsim 1$$

Modes with $k \gg \frac{1}{\Delta t}$ are not excited: $\frac{\dot{\omega}_k}{\omega_k^2} \ll 1$ for $k \gg \frac{1}{\Delta t}$
Numerical Results

Numerical results confirm these expectations.

- The chameleon bounces off its bare potential.
- Perturbations are generated during the bounce, taking energy away from the background evolution.
- The perturbation energy spectrum is peaked; most of the energy is in modes with \( k_{\text{peak}} \approx (\Delta t)^{-1} \).

\[
\dot{\phi}_i = 100 \text{ GeV}^2 \\
\dot{\phi}_i = 2000 \text{ GeV}^2 \\
\dot{\phi}_i = 200 \text{ GeV}^2 \\
\dot{\phi}_i = 20 \text{ GeV}^2
\]
Numerical Surprise

The numerical results confirm our expectations... except when they don’t!

The chameleon turns around sooner than expected:

\[ V(\phi_b) \ll \dot{\phi}_i^2 / 2 \]
Studying the backreaction of the perturbations on the chameleon background provides insight.

\[
\dddot{\phi} + V'(\phi) + \frac{1}{2} V'''(\phi)\langle \delta \phi^2 \rangle = 0 \quad \text{background equation with backreaction}
\]

\[
\dddot{\phi} + V'(\phi) - \frac{V'''[\phi(t)]}{16\pi^2} \int_0^t V'''[\phi(t')] \dot{\phi}(t') \text{Ci} [2k_{\text{IR}}(t-t')] dt' = 0
\]

- the “dissipation” term is nonlocal; it has memory
- before the bounce, it acts like a friction term; it has the same sign as \( \dot{\phi} \) and it slows the chameleon down.
- but unlike friction, it does not decrease as the chameleon slows down. It is more like a potential, and it can turn the chameleon around!

\[
V_D(\phi) = \frac{\kappa}{2} [V''(\phi)]^2
\]

0.02 \( \lesssim \) \( \kappa \) \( \lesssim \) 0.05

calibrate using numerical results

For \( \phi \lesssim M \), \( V_D(\phi) \) dominates over the chameleon’s bare potential!
New Models for a New Potential

\[ V_D(\phi) = \frac{\kappa}{2} \left[ V''(\phi) \right]^2 \]

controls the chameleon’s motion.

Predict when the chameleon bounces: \[ V_D(\phi_b) = \frac{\dot{\phi}_i^2}{2} \]
New Models for a New Potential

\[ V_D(\phi) = \frac{\kappa}{2} \left[ V''(\phi) \right]^2 \] controls the chameleon’s motion.

Predict when the chameleon bounces: \[ V_D(\phi_b) = \frac{\dot{\phi}_i^2}{2} \]

Predict the peak wavenumber in the perturbation energy spectrum:

\[ k_{\text{peak}} \sim \frac{n|\dot{\phi}_i|}{M} \left( \ln \left[ \frac{n^4 \kappa M^4}{\frac{\dot{\phi}_i^2}{2}} \right] \right)^{\frac{n+1}{n}} \]

\[ k_{\text{peak}} = [\Delta t \mid \Delta V'' = V''(\phi_b)]^{-1} \]

\[ V(\phi) = M^4 \exp \left[ \left( \frac{M}{\phi} \right)^n \right] \]

\[ V_D(\phi_b) = \frac{\dot{\phi}_i^2}{2} \]

\[ \kappa = 0.03 \]

\[ V(\phi_b) = \frac{\dot{\phi}_i^2}{2} \]

\[ n = 2 \]

\[ n = 4 \]

\[ n = 10 \]
Power-Law Potentials

The model works equally well for power-law potentials:

\[ V(\phi) = M^4 \left( \frac{M}{\phi} \right)^n \]

Since \( \phi \) bounces at a smaller value, the timescale for the rebound is shorter, which leads to even larger \( k_{\text{peak}} \)!
High-Energy Chameleons

\[ k_{\text{peak}} = 0.7 \frac{\dot{\phi}}{M} \left( \frac{M}{\phi_b} \right)^3 \approx 0.18 \frac{\dot{\phi}}{M} \ln^{3/2} \left( \frac{2\phi_i^2}{M^4} \right) \]
Summary: A Chameleon Catastrophe

AE, Barnaby, Burrage, Huang 1304.0009 (PRL) and 1310.5149 (PRD)

What happens when you kick a chameleon?

It hits its bare potential at a fatal velocity, and then it shatters into pieces!

The chameleon’s interaction with standard model particles hurtles it toward the minimum of its effective potential.

- Chameleons with $\beta > 1.8$ surf toward $\phi_{\text{min}}$.
- At impact, $\dot{\phi} \gtrsim \text{GeV}^2$.

Because $\dot{\phi} \gg M^2$, the rebound is highly nonadiabatic, and perturbations are excited.

- Most (maybe all?) the chameleon’s energy goes into perturbations.
- The perturbations have wavenumbers $k \gtrsim 10^{13}$ GeV.
- The perturbations strongly interact with themselves and with matter: theoretical breakdown!
- Chameleons demonstrate how the presence of an extreme hierarchy of scales can challenge a theory’s stability. Are there other examples?
Perturbation Equations
Perturbation Equations

Linear perturbations with first-order backreaction:

\[
\ddot{\phi} + V'(\bar{\phi}) + \frac{1}{2} V'''(\bar{\phi}) \langle \delta \phi^2 \rangle = 0
\]

\[
\ddot{\phi}_k + [k^2 + V''(\bar{\phi})] \phi_k = 0
\]

\[
\langle \delta \phi^2 \rangle = \int \frac{d^3k}{(2\pi)^3} \left( |\phi_k|^2 - \frac{1}{2\omega_k} \right)
\]

\[
\omega_k^2 \equiv k^2 + V''(\bar{\phi})
\]