Early Universe Challenges for Chameleon Gravity

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PRD 89, 084074 (2014) [arXiv: 1310.5149]

COSMO-14

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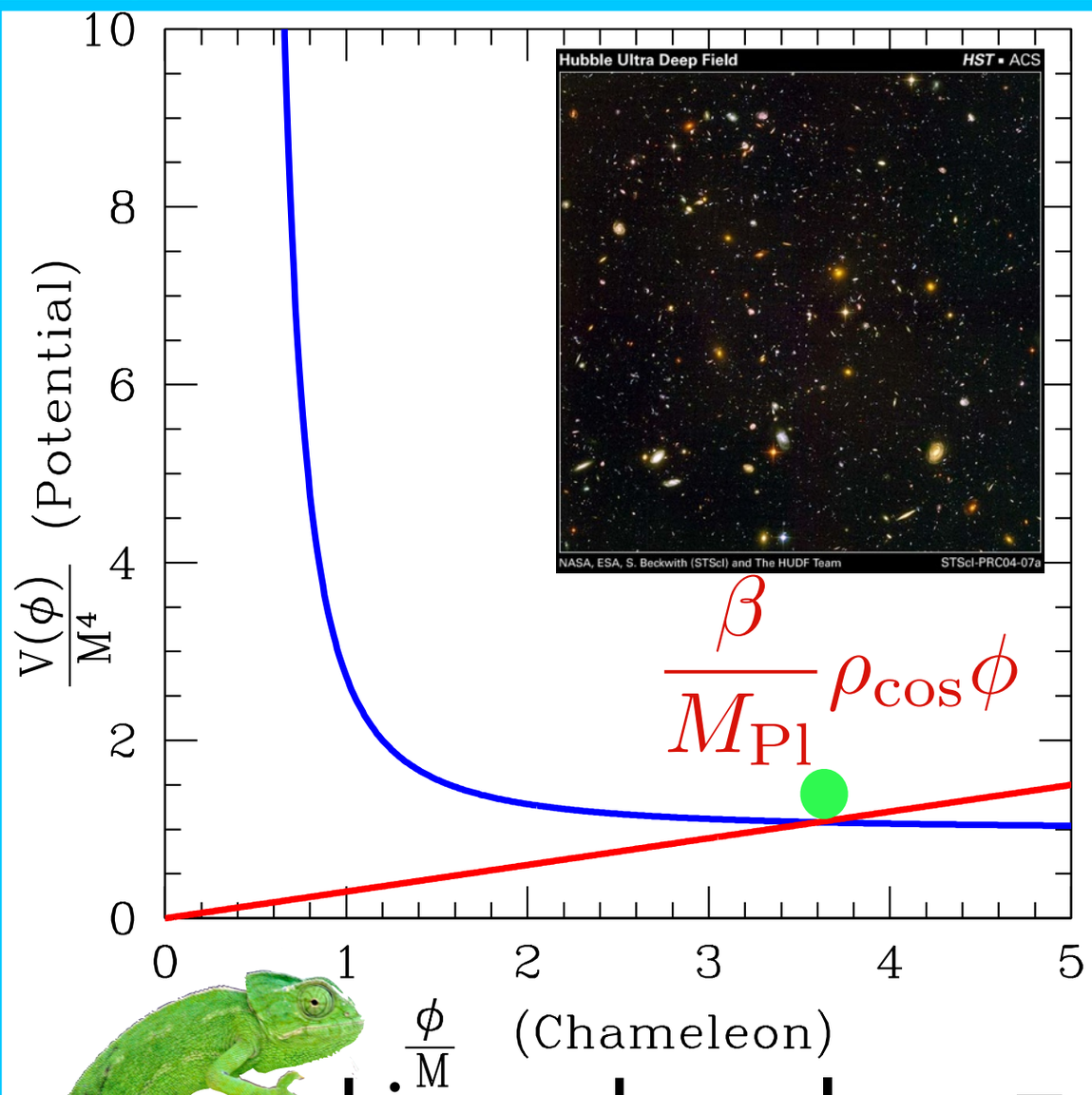
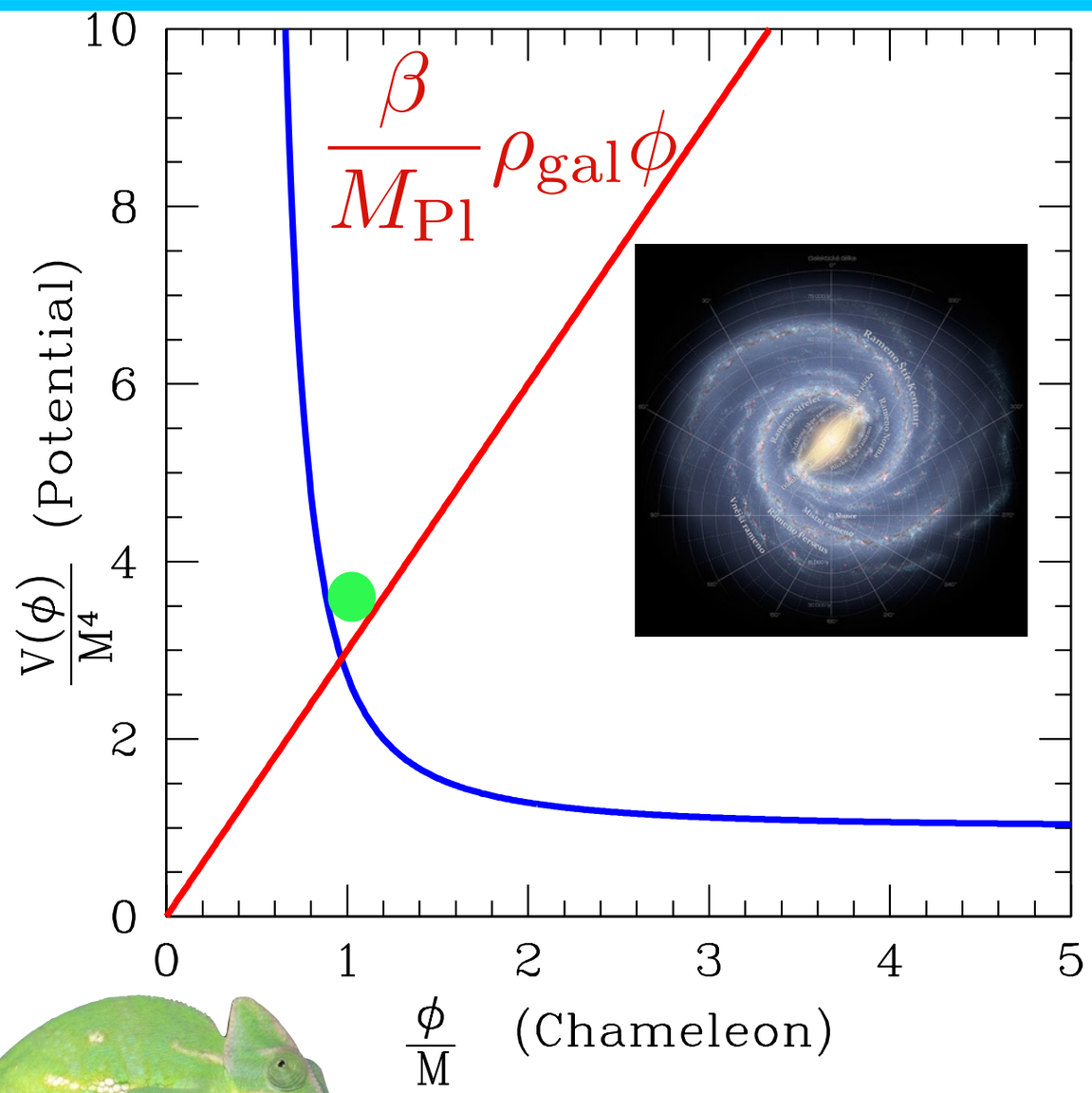
Meet the Chameleon

Scalar-Tensor Gravity: we must hide the scalar!

Chameleon Mechanism: scalar's mass depends on environment

Khoury & Weltman 2004





skinny chameleon = dark energy

fat chameleon = GR

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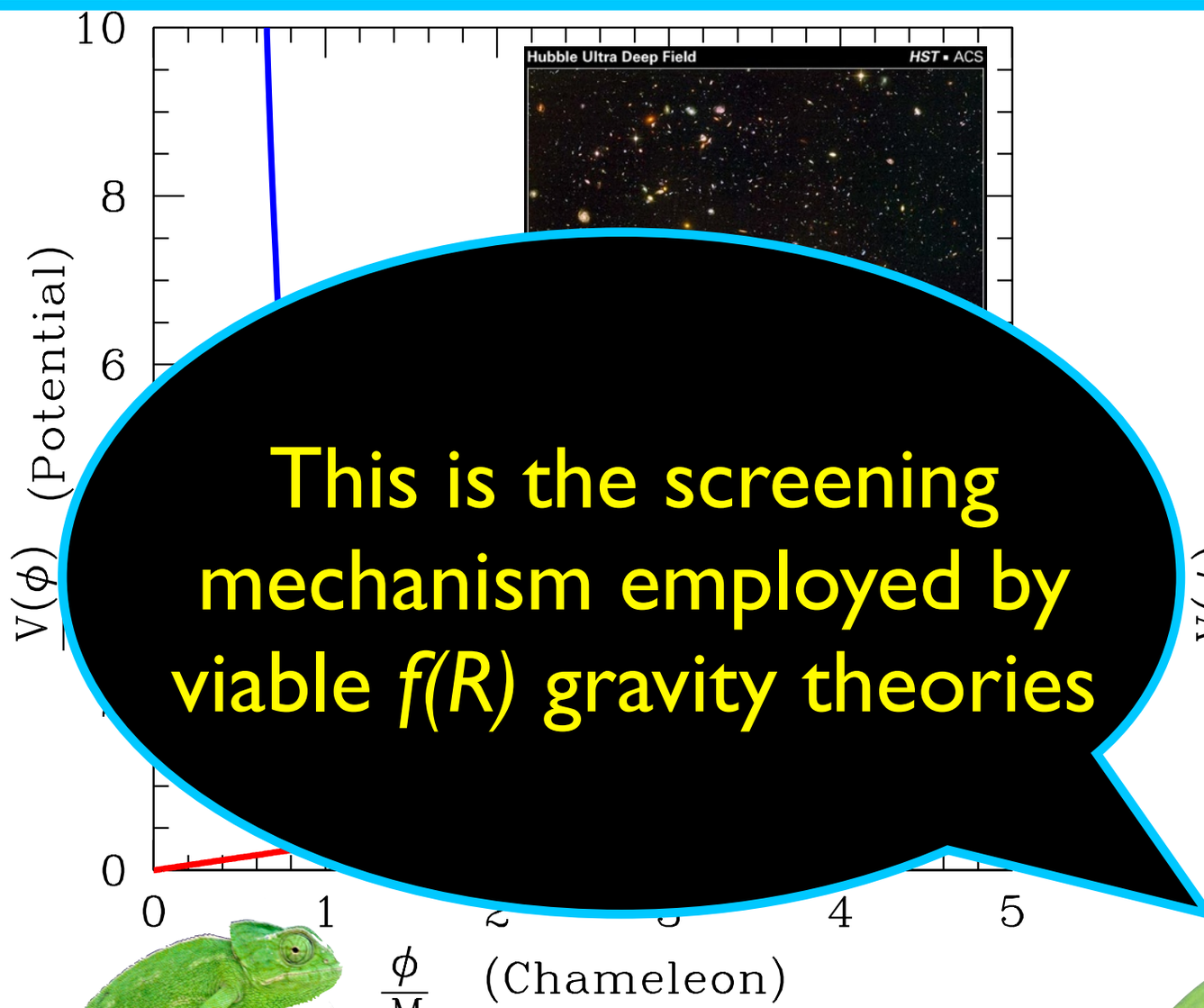
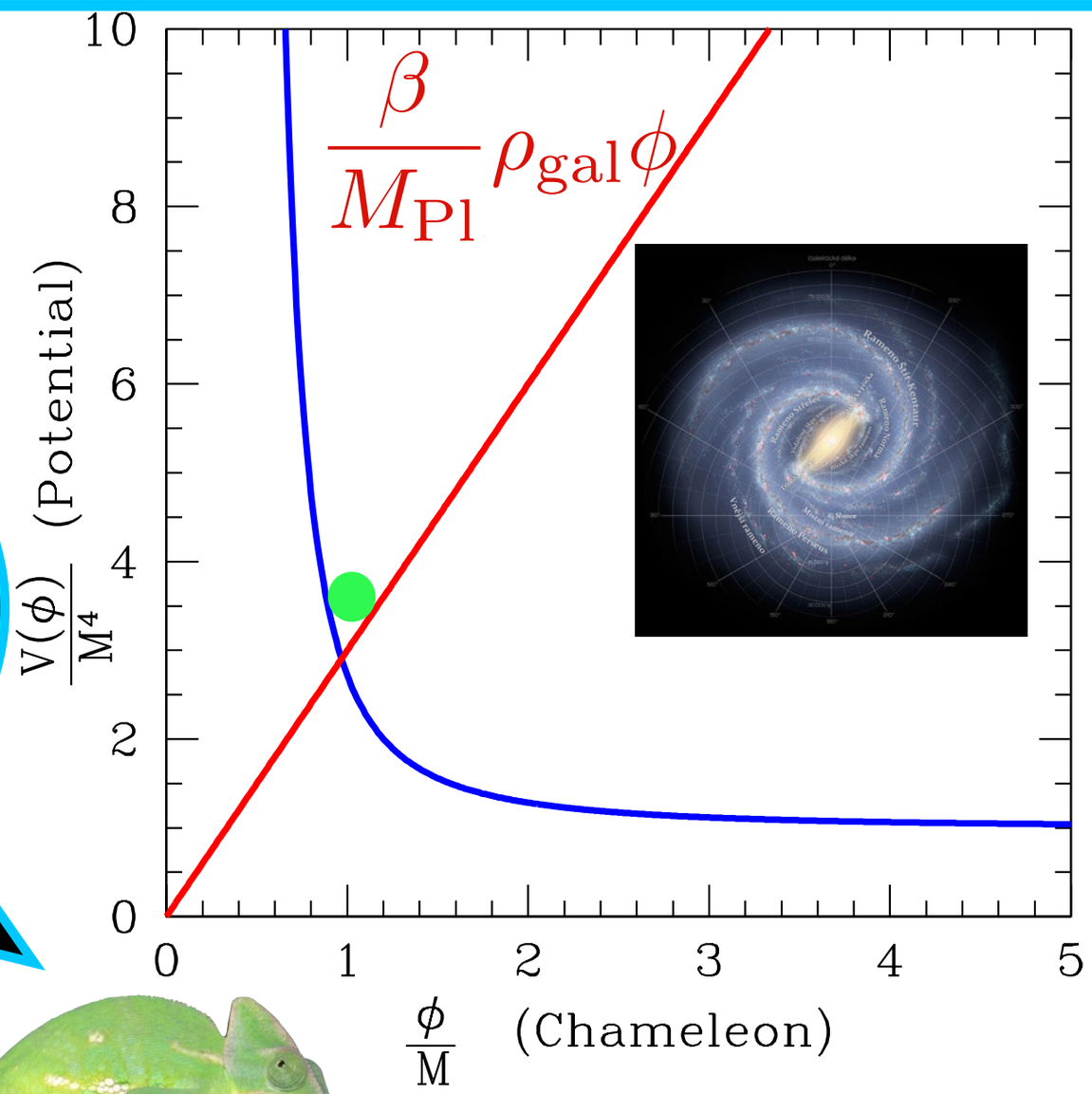


This is the screening mechanism employed by viable *f(R)* gravity theories



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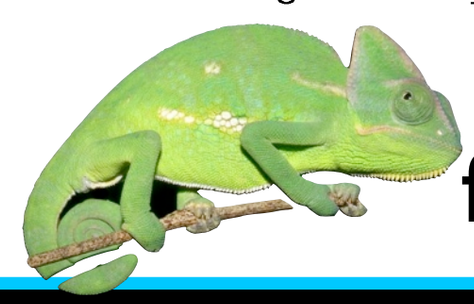


fat chameleon = GR

Chameleon Gravity: Lightning Review

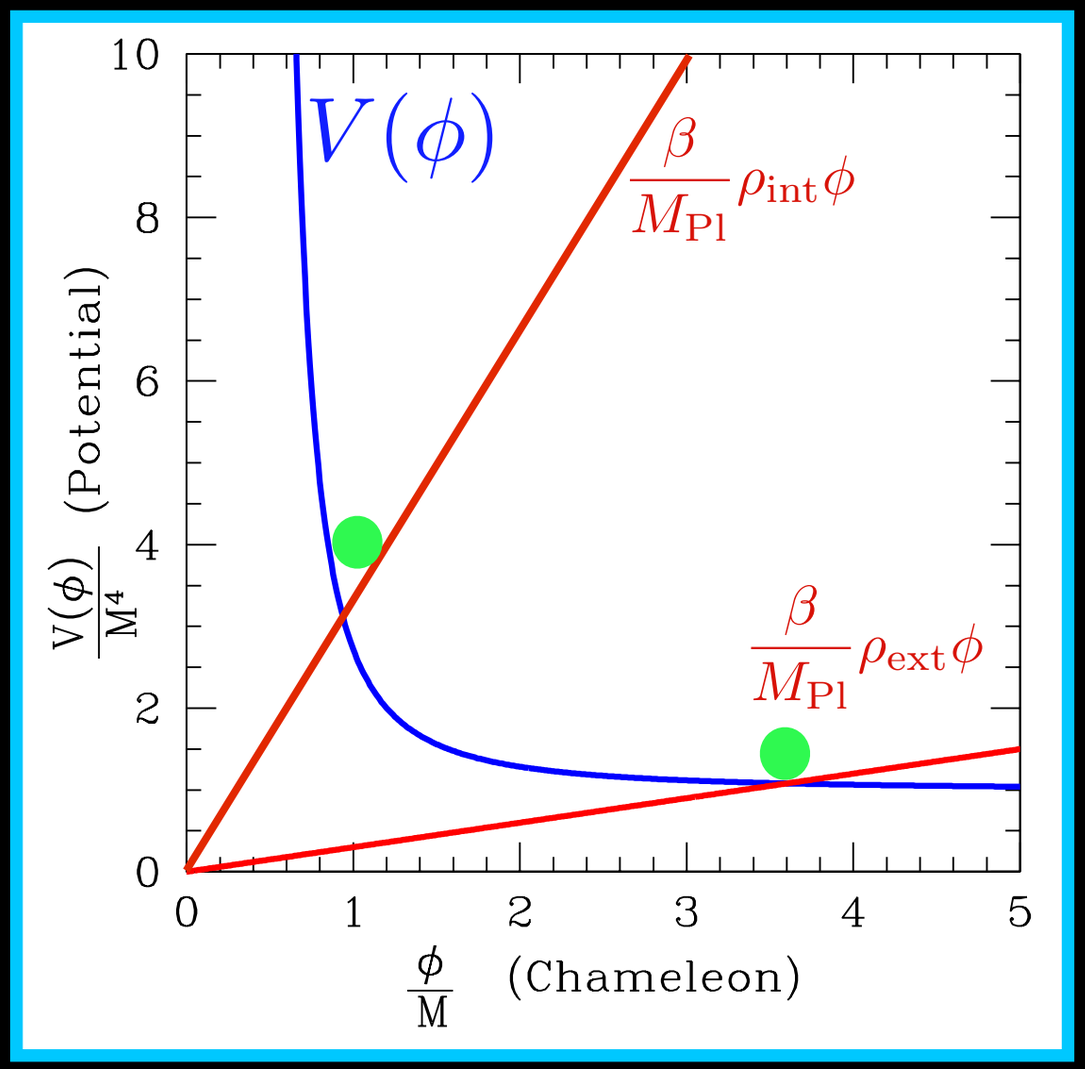
Chameleon gravity: a screened scalar-tensor theory *Khoury & Weltman 2004*

$$S = \int d^4x\sqrt{-g_*}\left[\frac{M_{\rm Pl}^2}{2}R_* - \frac{1}{2}(\nabla_*\phi)^2 - V(\phi)\right] + S_m\left[\tilde{g}_{\mu\nu}, \psi_m\right]$$

Einstein frame: standard GR + scalar field (chameleon field)

$$\tilde{g}_{\mu\nu} = e^{2\beta\phi/M_{\rm Pl}} g^*_{\mu\nu}$$

Matter couples to different metric (Jordan Frame)



Chameleon equation of motion:

$$\ddot{\phi} + 3H_*\dot{\phi} = -\left[\frac{dV}{d\phi} + \frac{\beta}{M_{\rm Pl}}(\rho_* - 3p_*)\right]$$

derivative of effective potential

Chameleon Cosmology

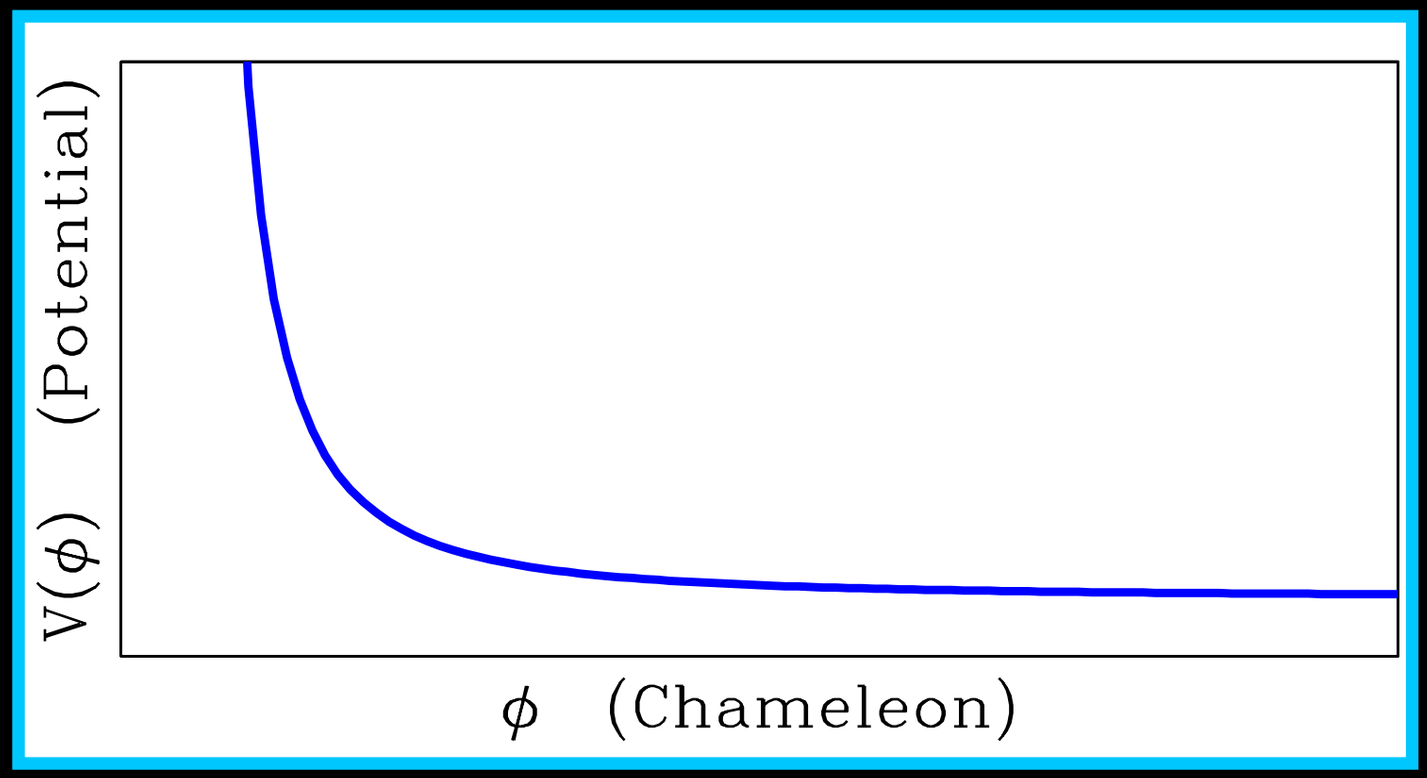
Fiducial Chameleon Potential:

Brax et al. 2004

$$V(\phi) = M^4 \exp\left[\left(\frac{M}{\phi}\right)^n\right] \overset{\phi \gg M}{\simeq} M^4 \left[1 + \left(\frac{M}{\phi}\right)^n\right]$$

Evade Solar System gravity tests and provide dark energy:

$$M \simeq 0.001\,\mathrm{eV} \simeq (\rho_{\mathrm{de}})^{1/4}$$



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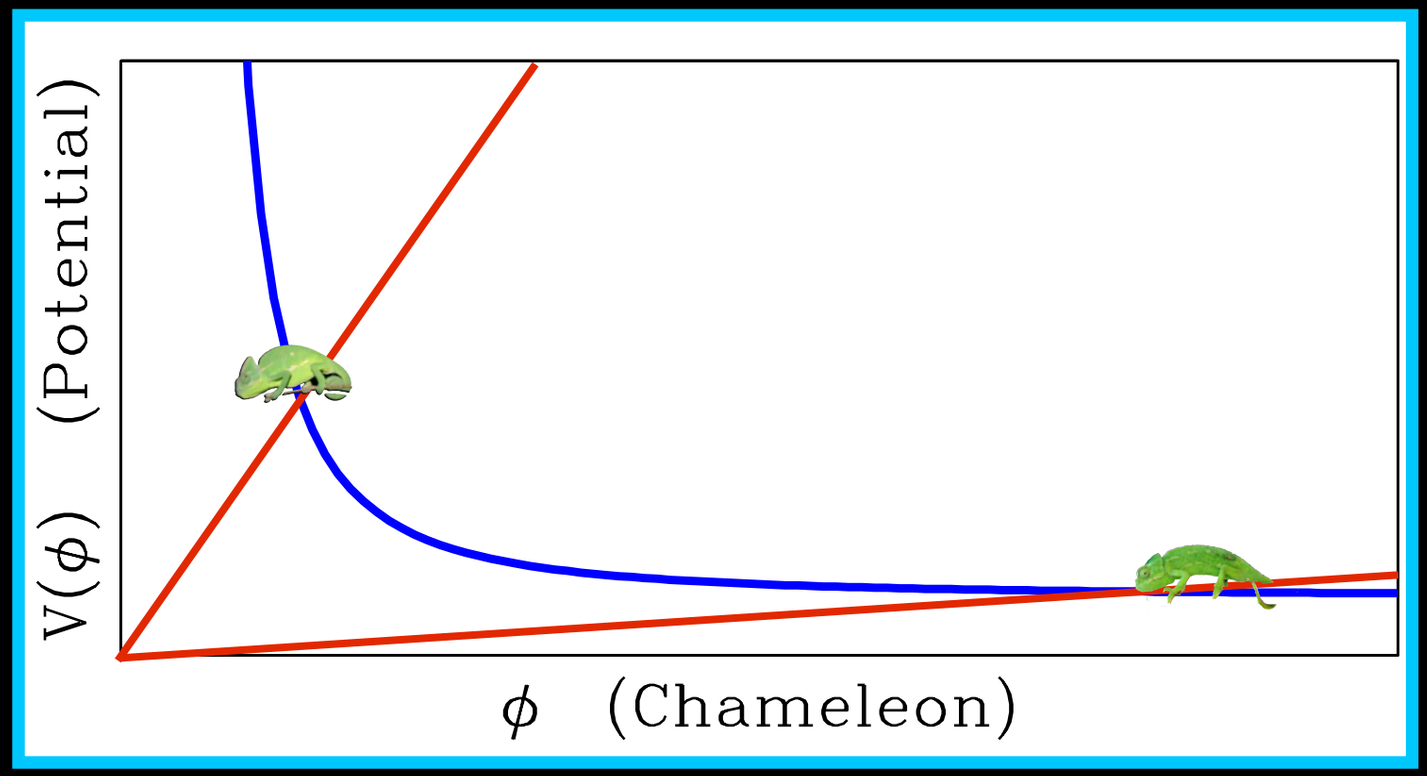
Where is the chameleon now?



$$\rho_{\mathrm{mat},0} = 0.3\rho_{\mathrm{crit},0}$$

$$\phi_{\mathrm{min}} = 5.9 \times 10^9 M \ll M_{\mathrm{Pl}}$$

$\phi_{\mathrm{min}} \ll M_{\mathrm{Pl}}$ always!

$\phi_{\mathrm{min}} \lesssim M$ inside Earth, Sun and at $T \gtrsim 2\,\mathrm{MeV}$

Chameleon Initial Conditions

During inflation: $\rho - 3p \simeq 4\rho_{\rm infl}$ pins chameleon $\phi \ll M$ *Brax et al. 2004*

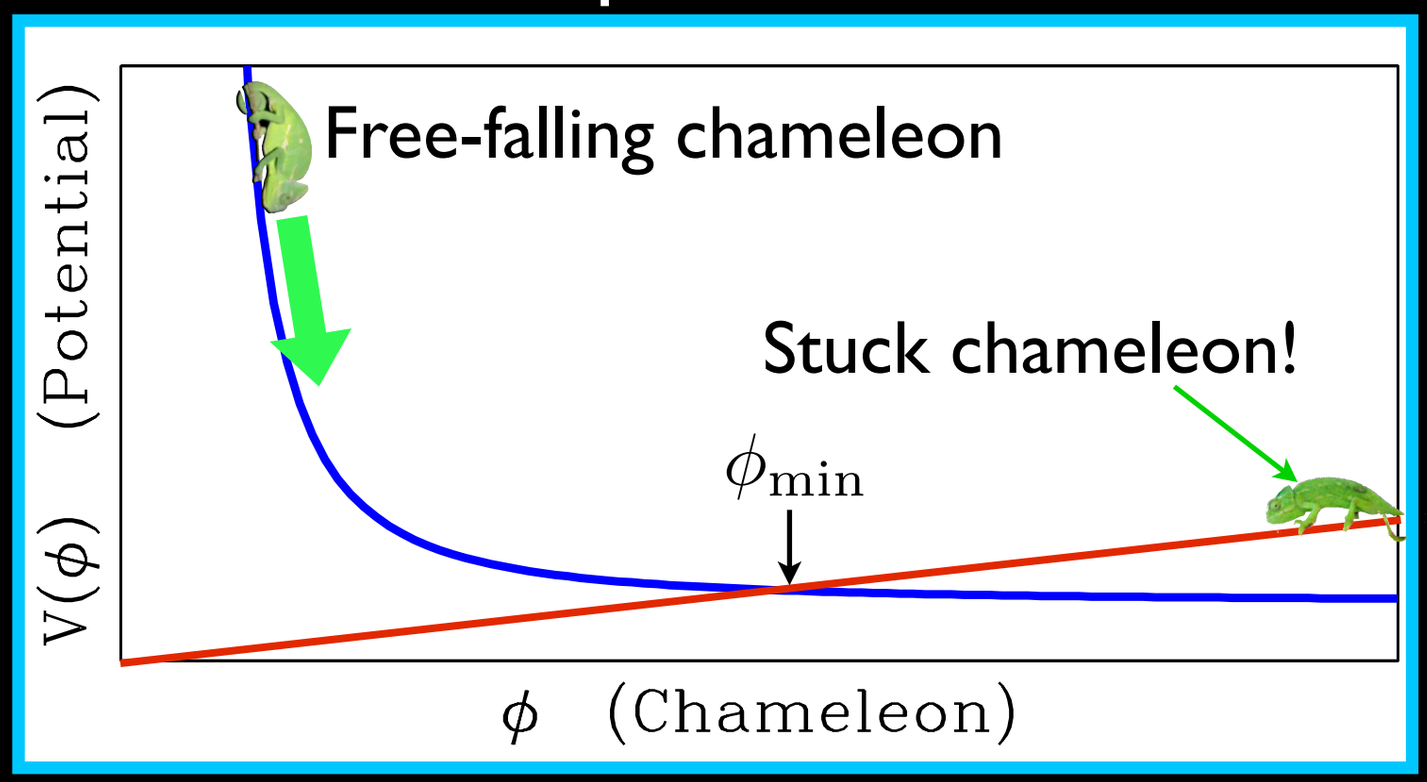
After reheating: $\rho - 3p \simeq 0$ the chameleon quickly slides down its bare potential and rolls to $\phi \gg \phi_{\rm min}$

For $\phi \gg \phi_{\rm min}$

$$\ddot{\phi} + 3H_*\dot{\phi} = -\frac{\beta}{M_{\rm Pl}}(\rho_* - 3p_*) \implies \Delta\phi \simeq \frac{\dot{\phi}_i}{H_i} = M_{\rm Pl}\sqrt{6\Omega_{\dot{\phi},i}}$$

Chameleon rolls out to $\phi_{\rm min} \ll \phi \lesssim M_{\rm Pl}$

Hubble friction prevents the chameleon from rolling back to $\phi_{\rm min}$



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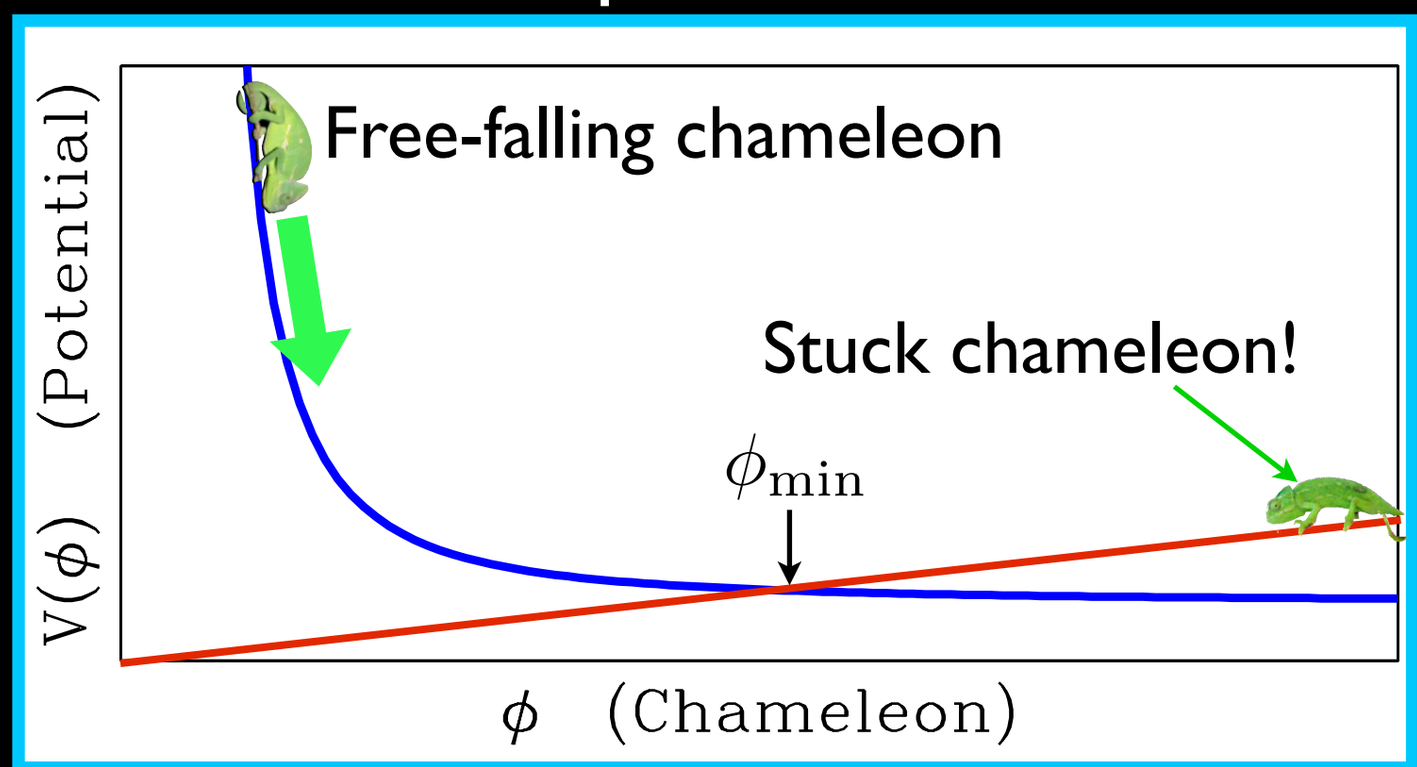


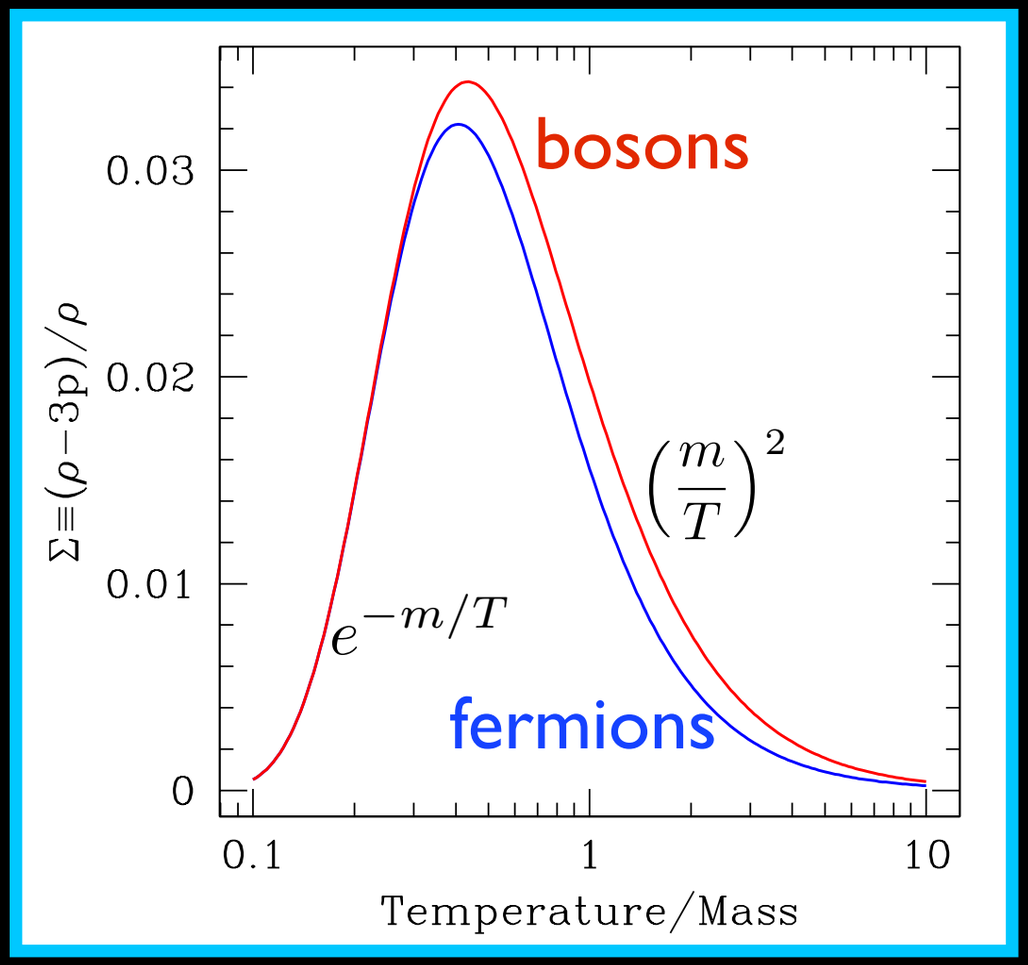
We have a problem!

ϕ cannot vary much between now and BBN:

$$\frac{\beta}{M_{\rm Pl}}\Delta\phi \simeq \left|\frac{\Delta m_p}{m_p}\right| \lesssim 0.1$$

$$\phi_{\rm BBN} \lesssim 0.1\frac{M_{\rm Pl}}{\beta}$$

Kicking the Chameleon



Define the kick function:

$$\Sigma(T) \equiv \frac{\rho - 3p}{\rho} \sim \frac{\text{force}}{\text{friction}}$$

Pressure of massive particles in thermal equilibrium

$$p = \frac{\rho}{3}\left[1 - \mathcal{O}\left(\frac{m^2}{T^2}\right)\right]$$

Damour & Nordtvedt1993; Damour & Polyakov 1994
Brax et al. 2004; Coc et al. 2006, 2009

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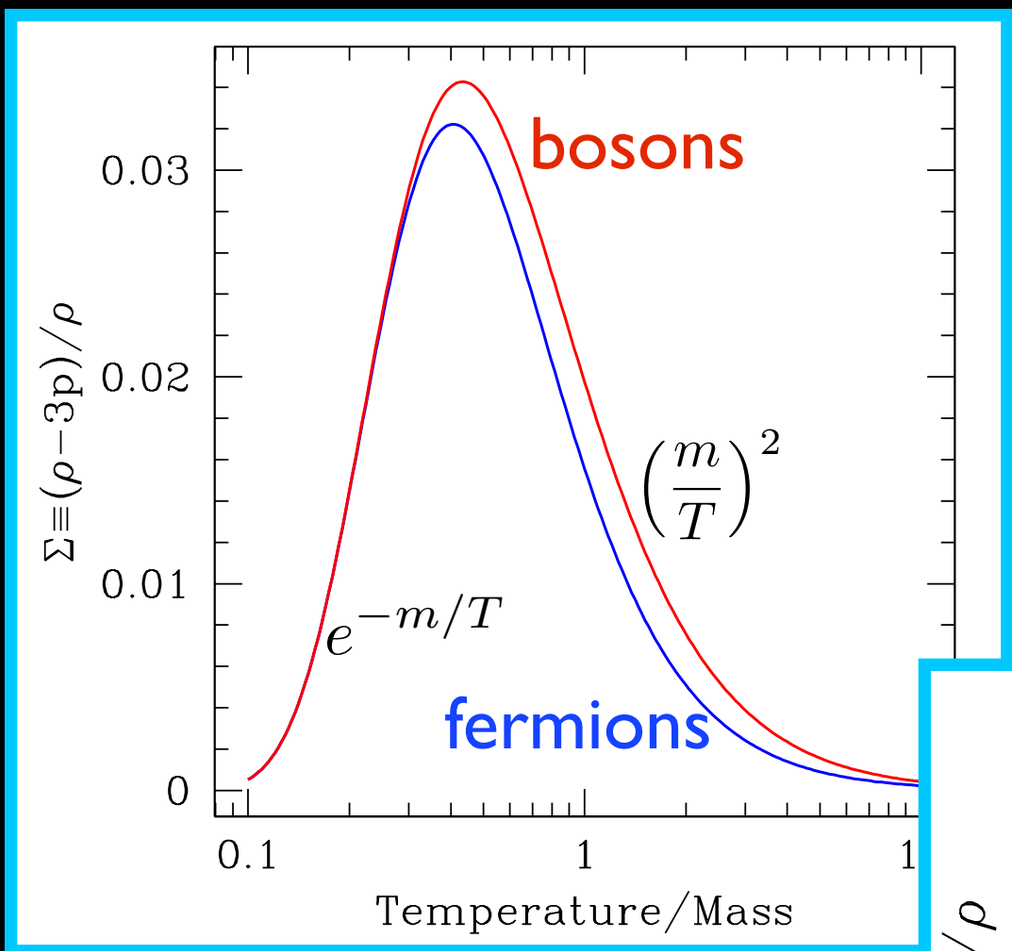

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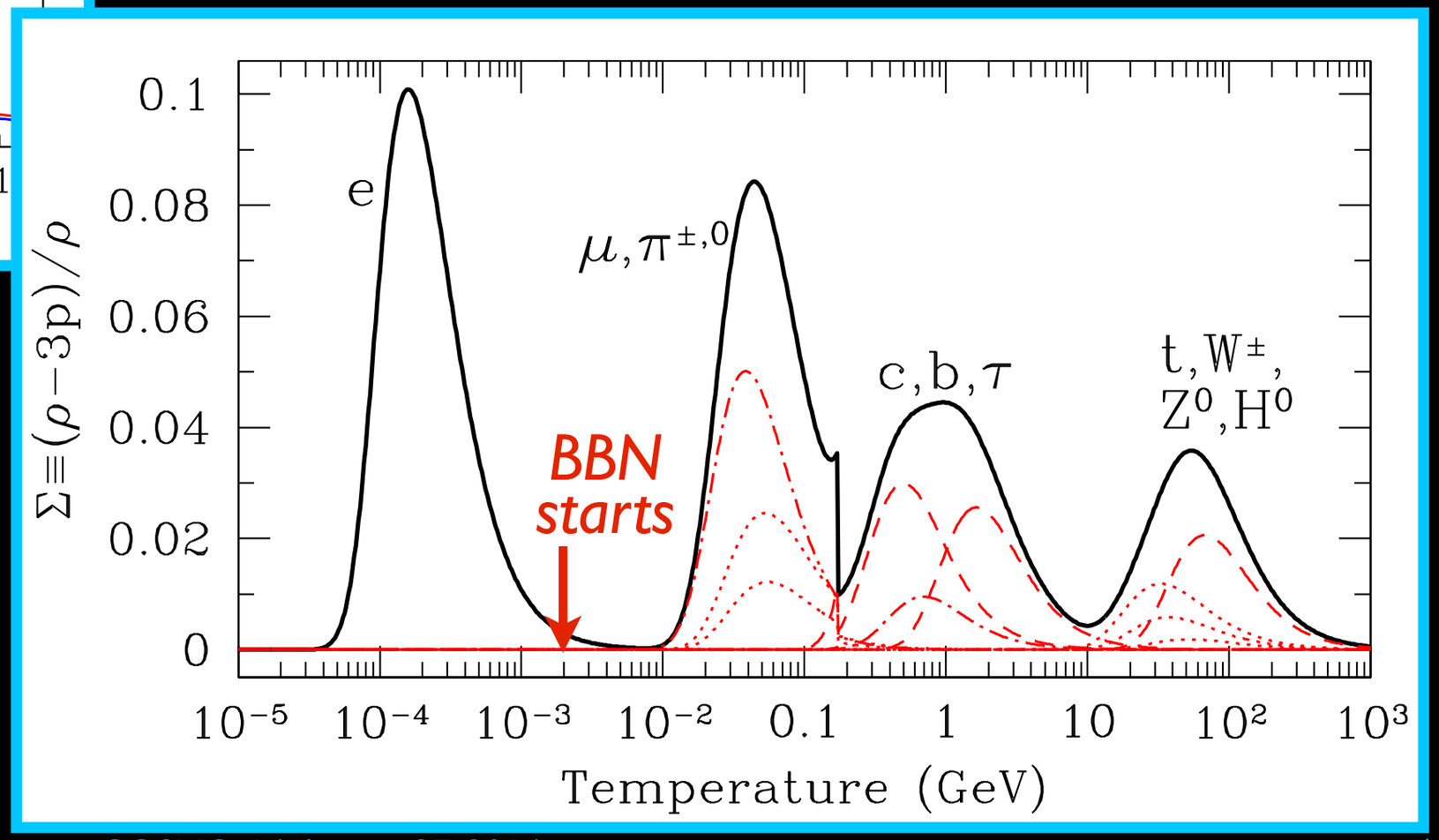
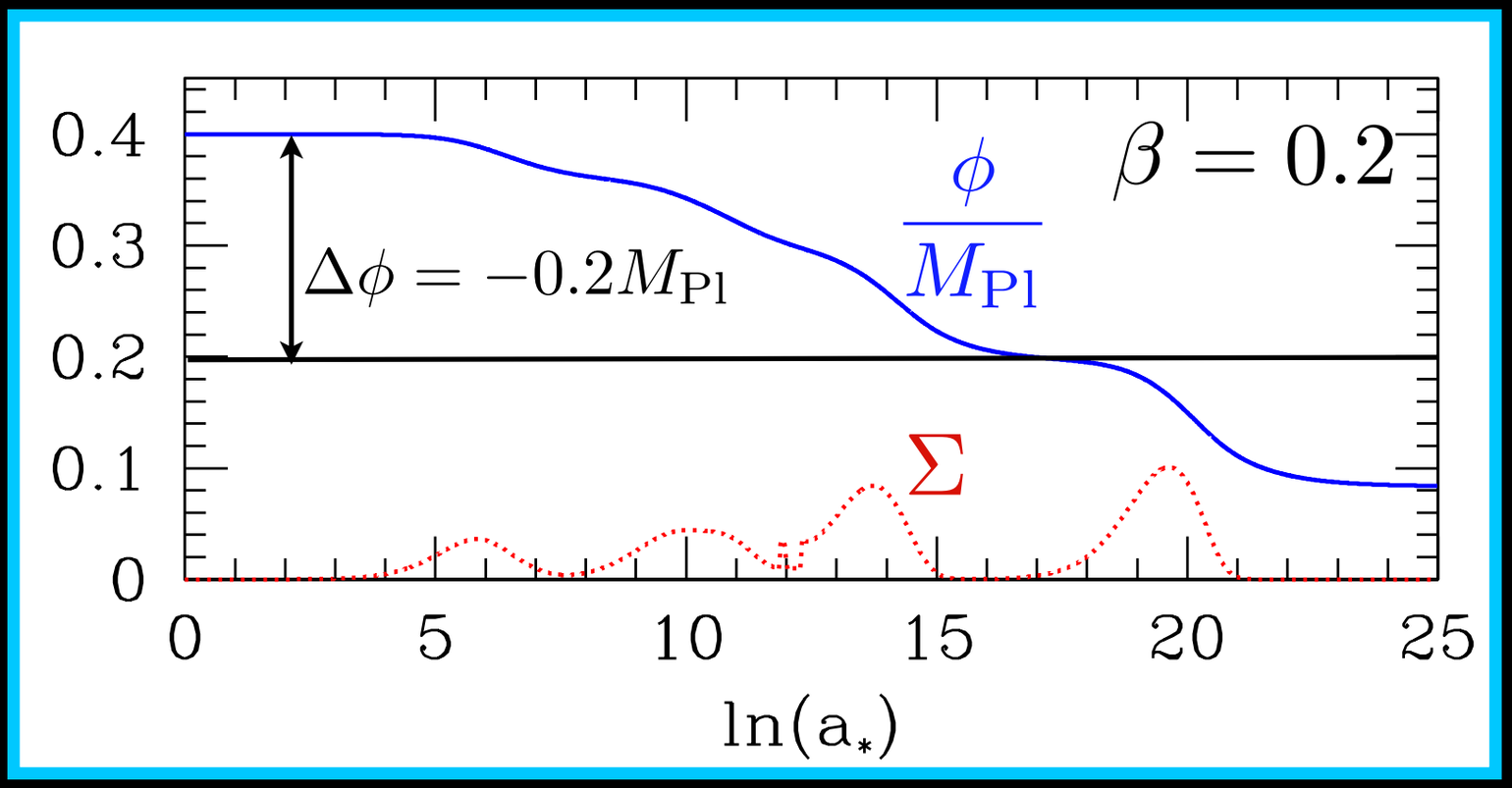
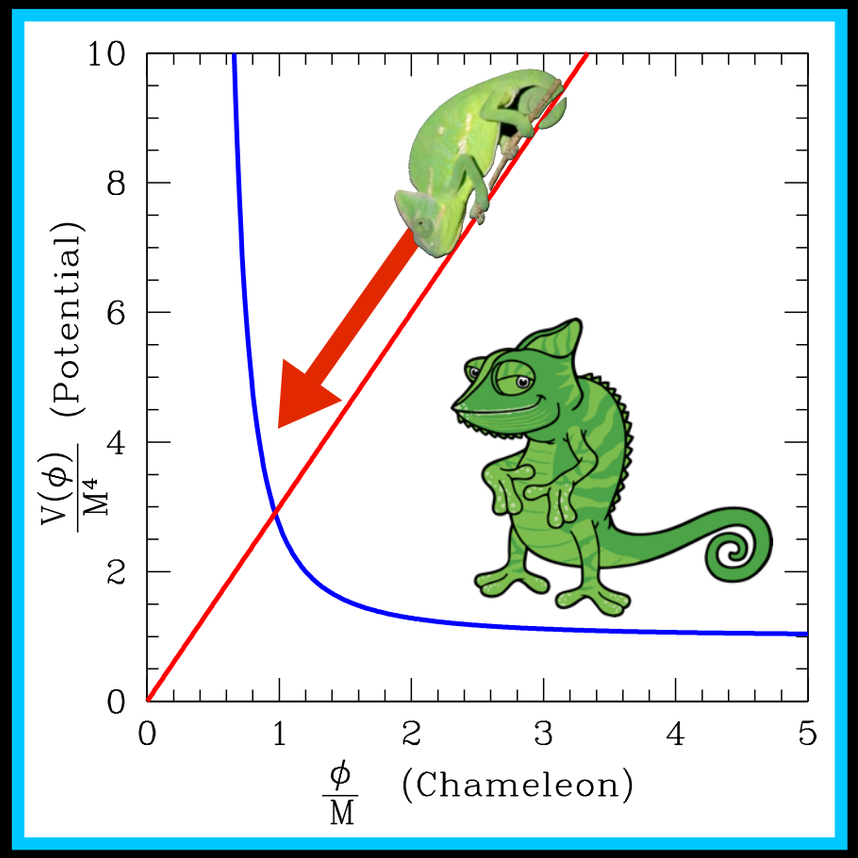


Every particle in the Standard Model (and beyond) kicks the chameleon.

The Old Story:

The kicks save the chameleon: $\Delta\phi \simeq -\beta M_{\rm Pl}$ prior to BBN. *Brax et al. 2004*





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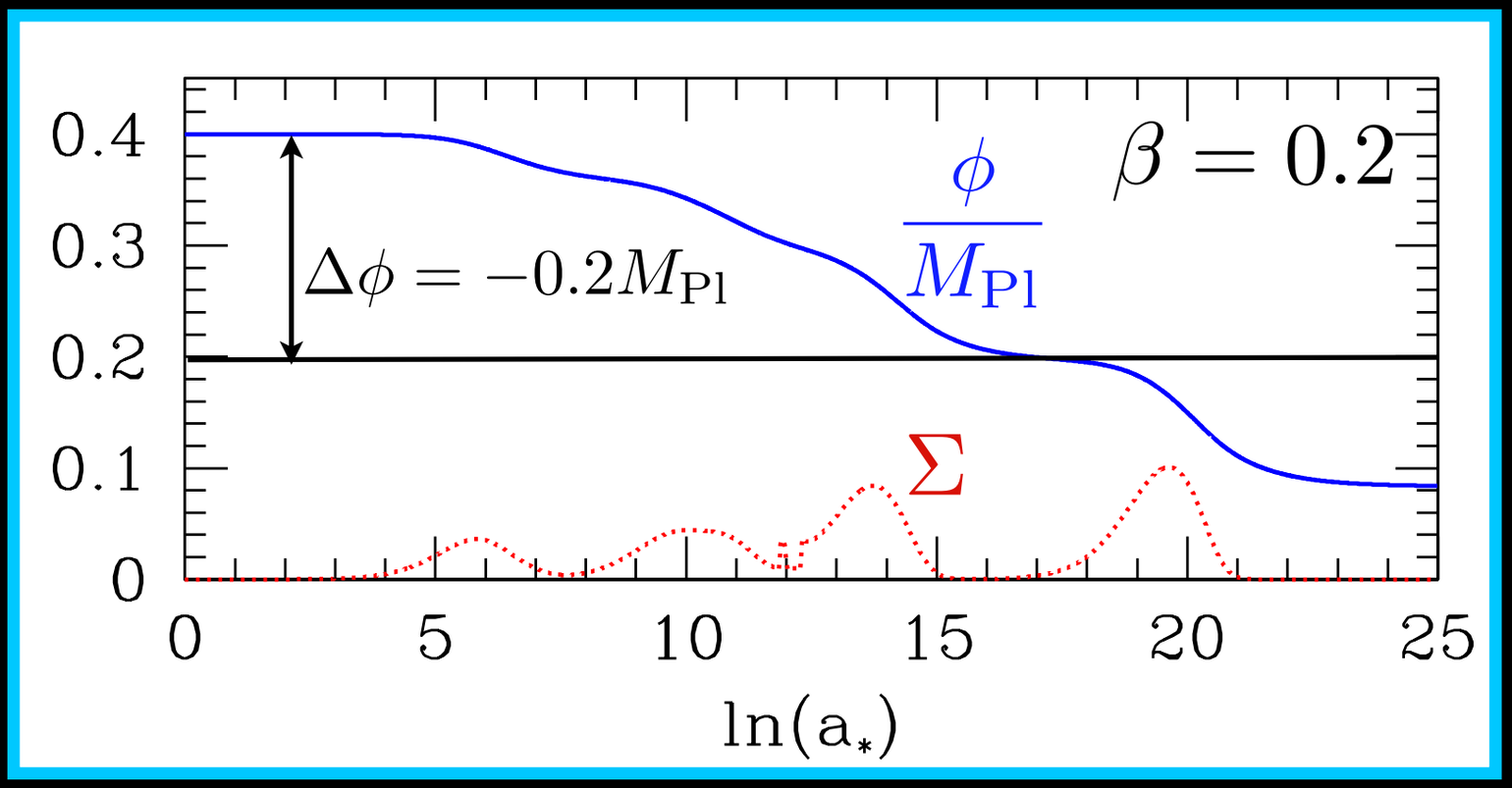

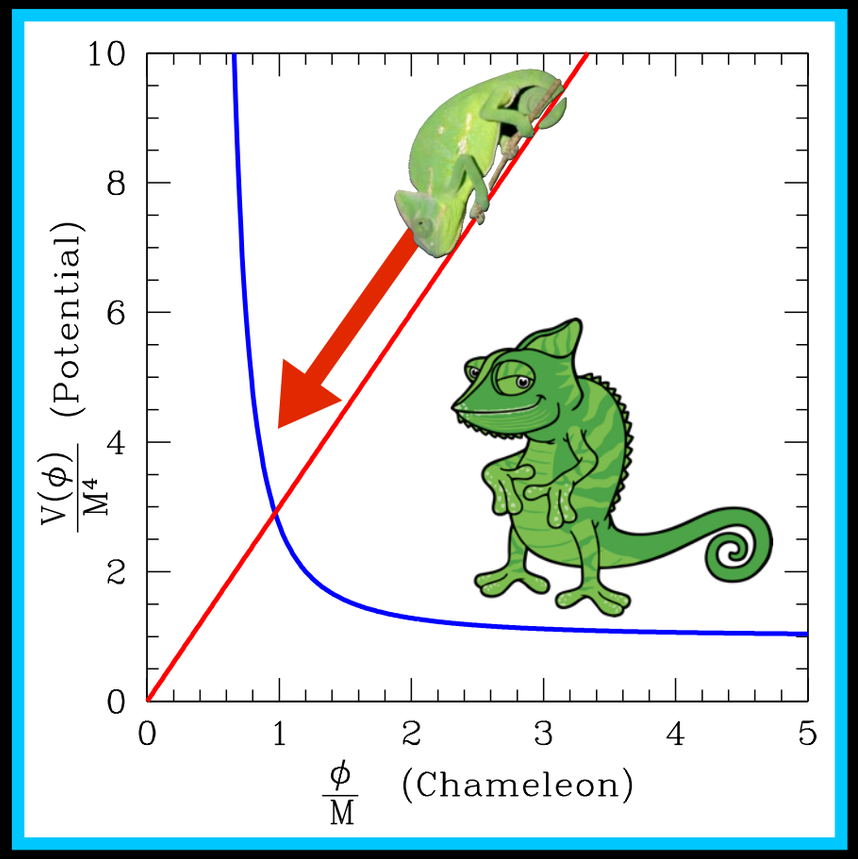

Flaws in the old story:

- underestimates chameleon motion for $\beta \gtrsim 1$
- chameleon reaches $\phi_{\rm min}$ with a large velocity and climbs up the bare potential
- the rebound is violent enough to excite quantum perturbations with extremely high energies

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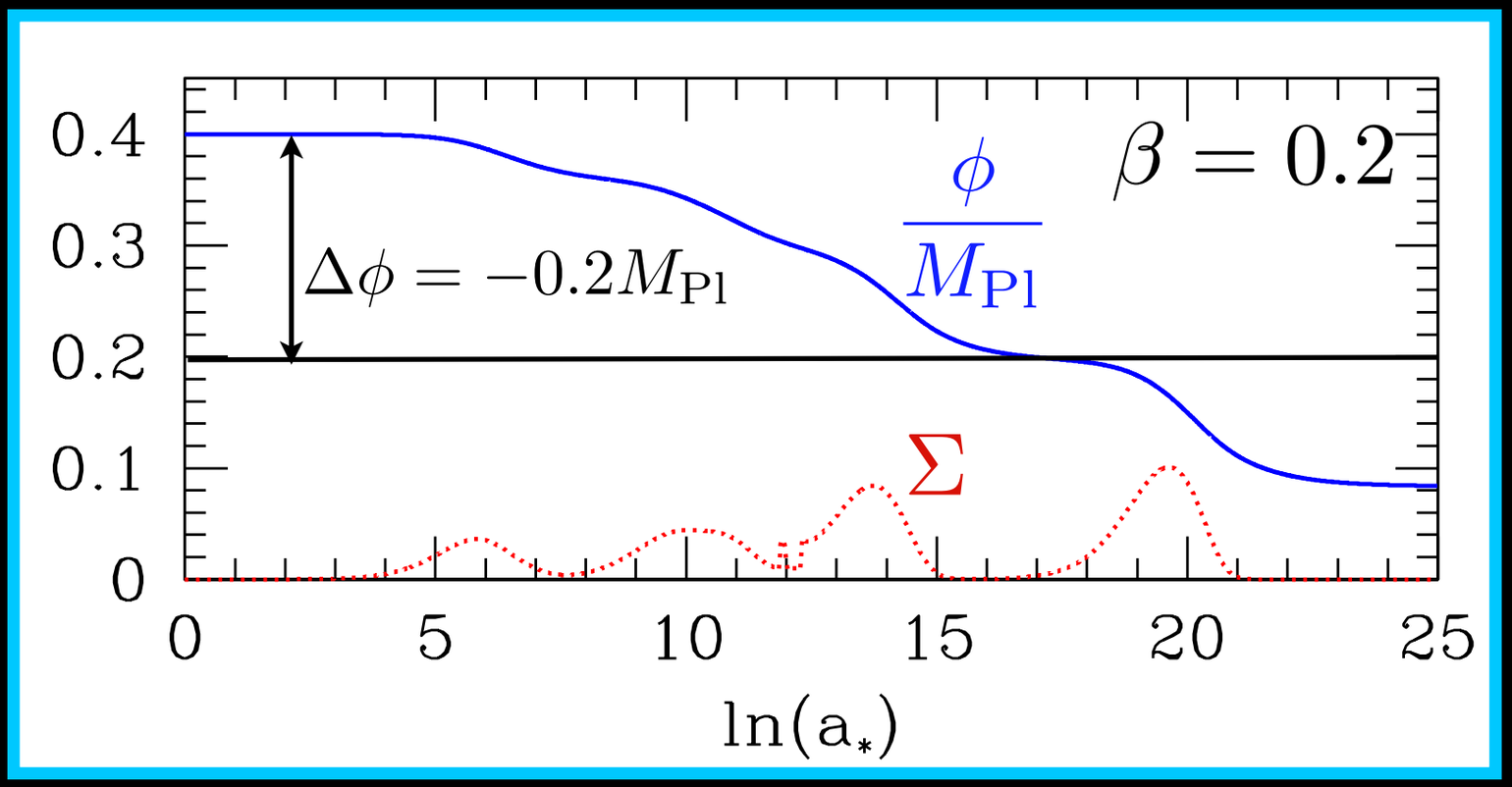

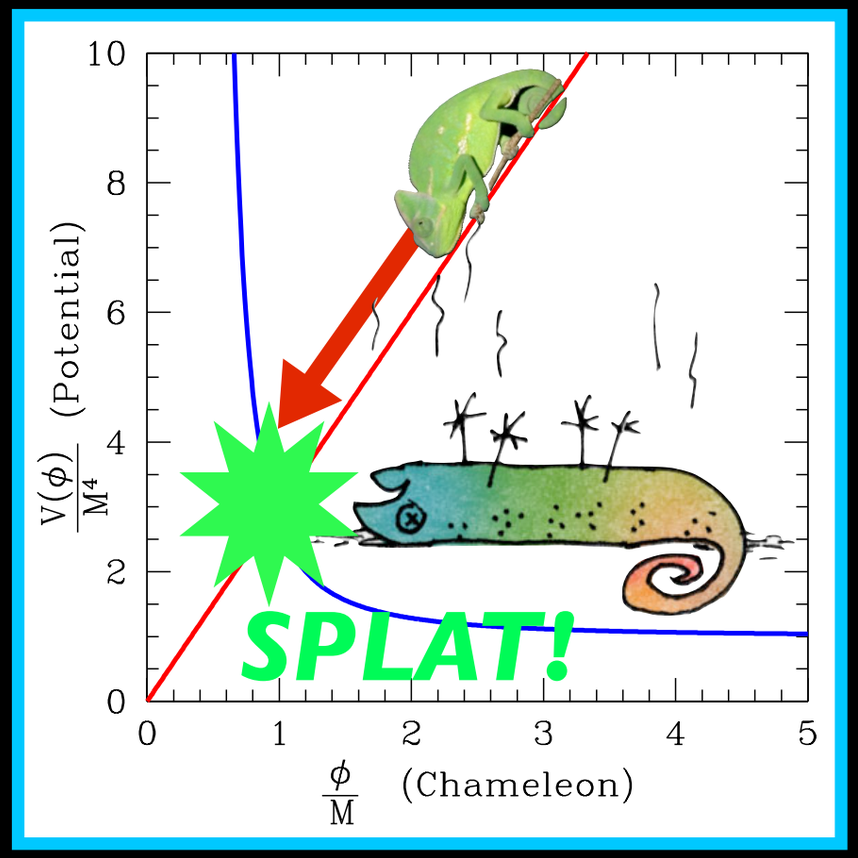

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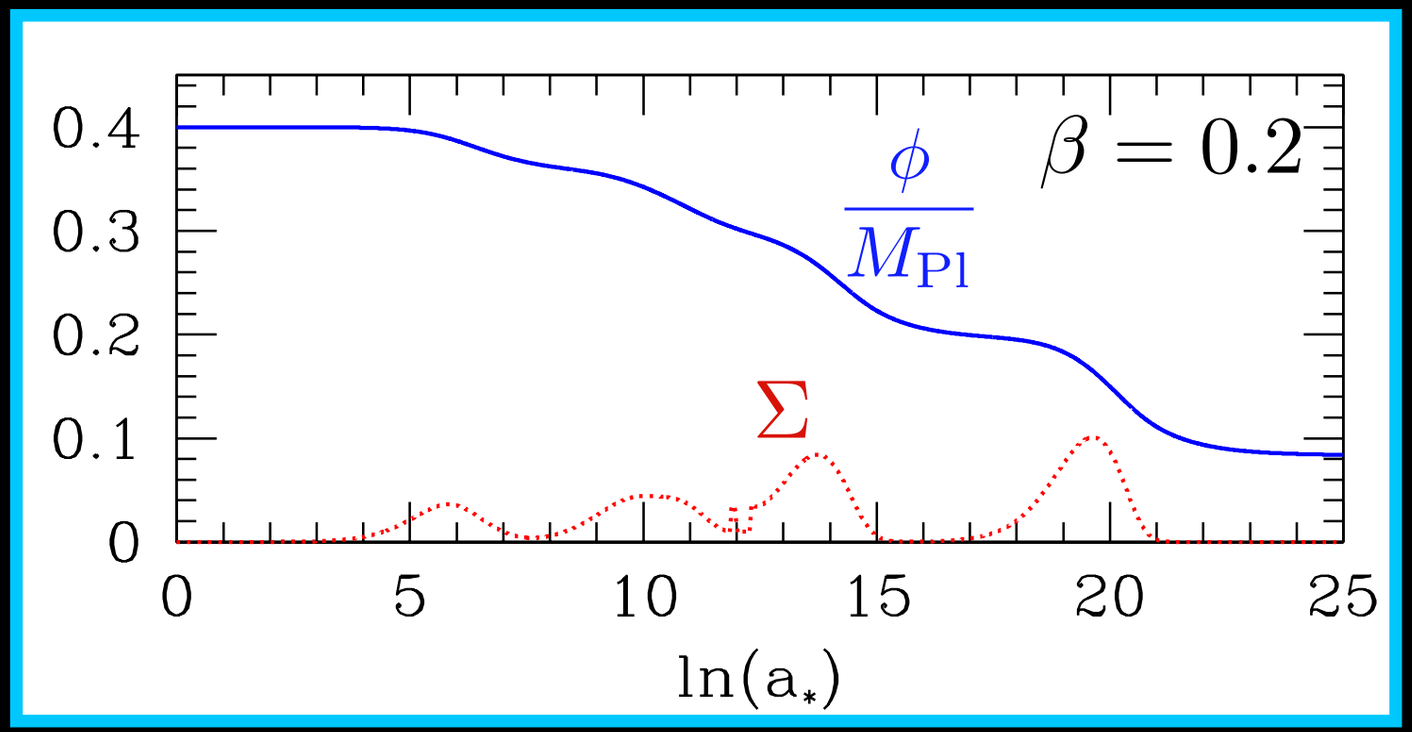

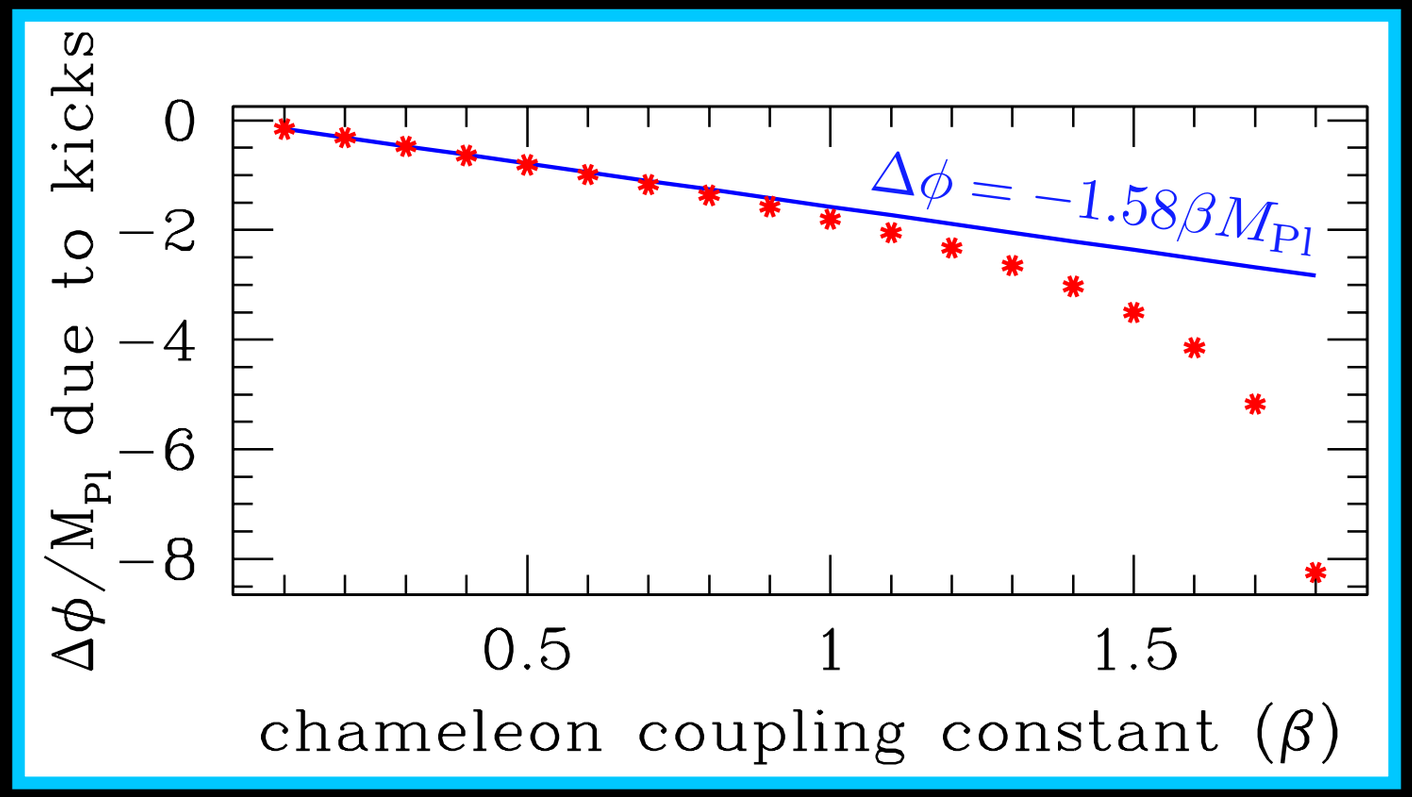

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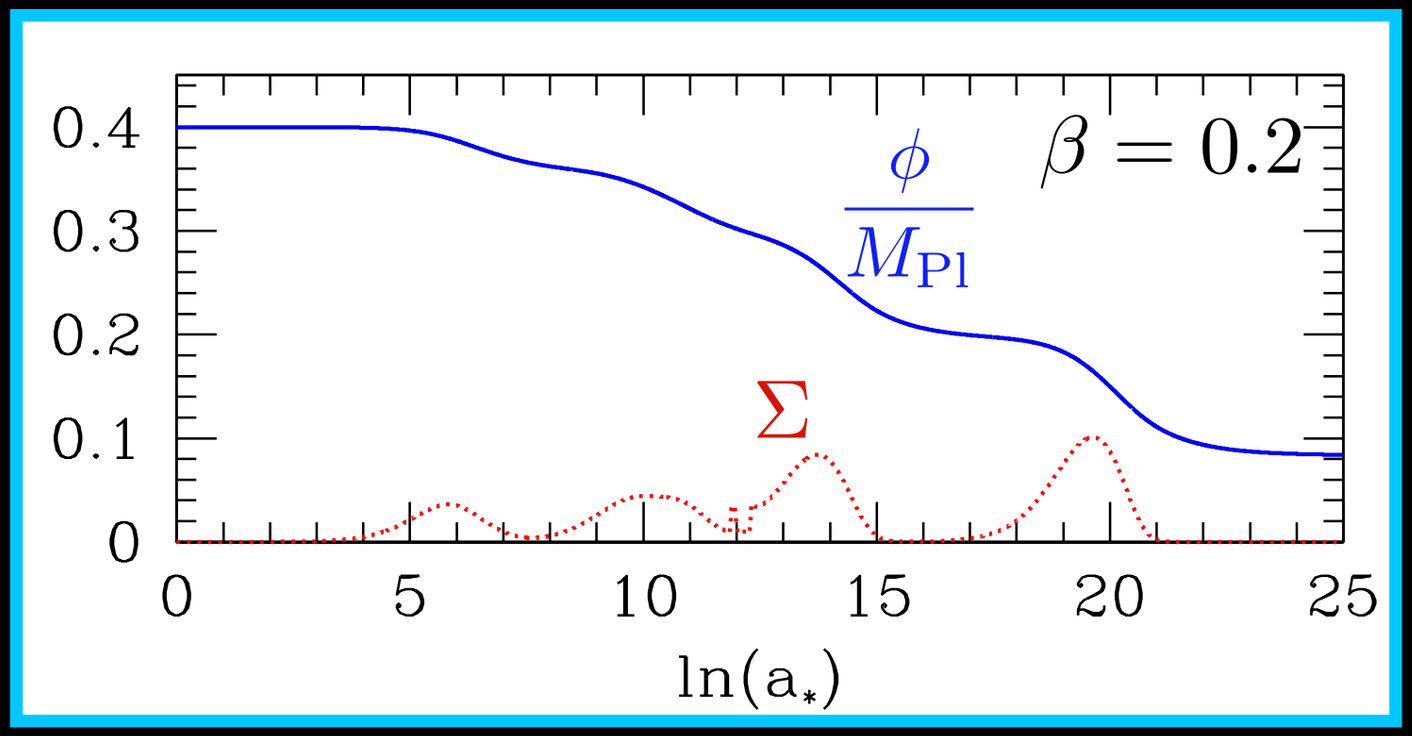
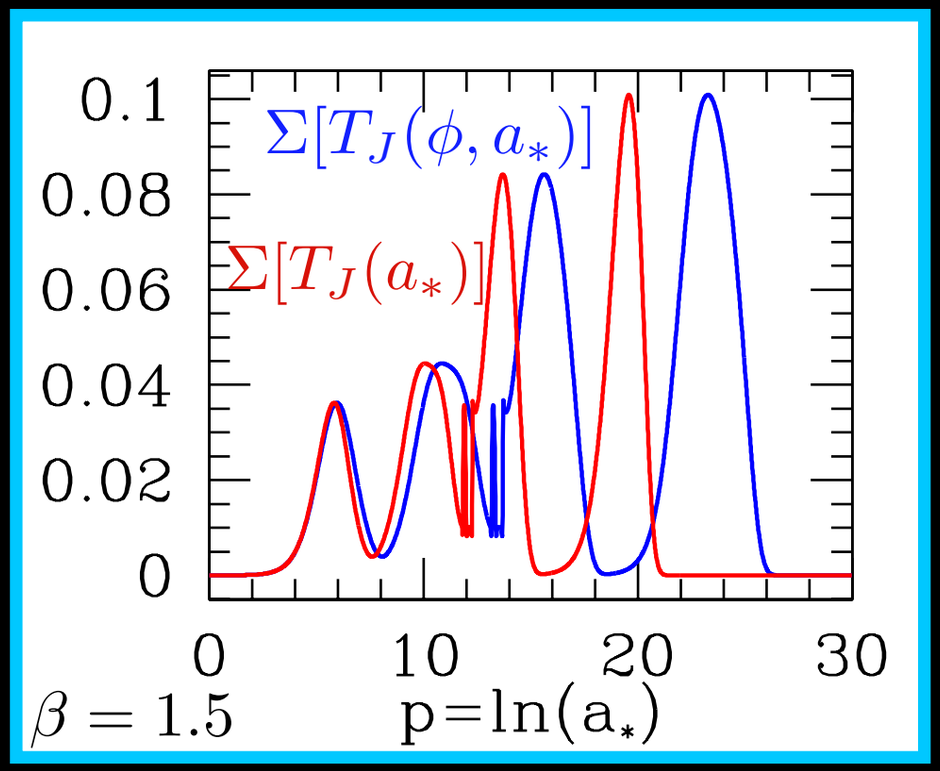
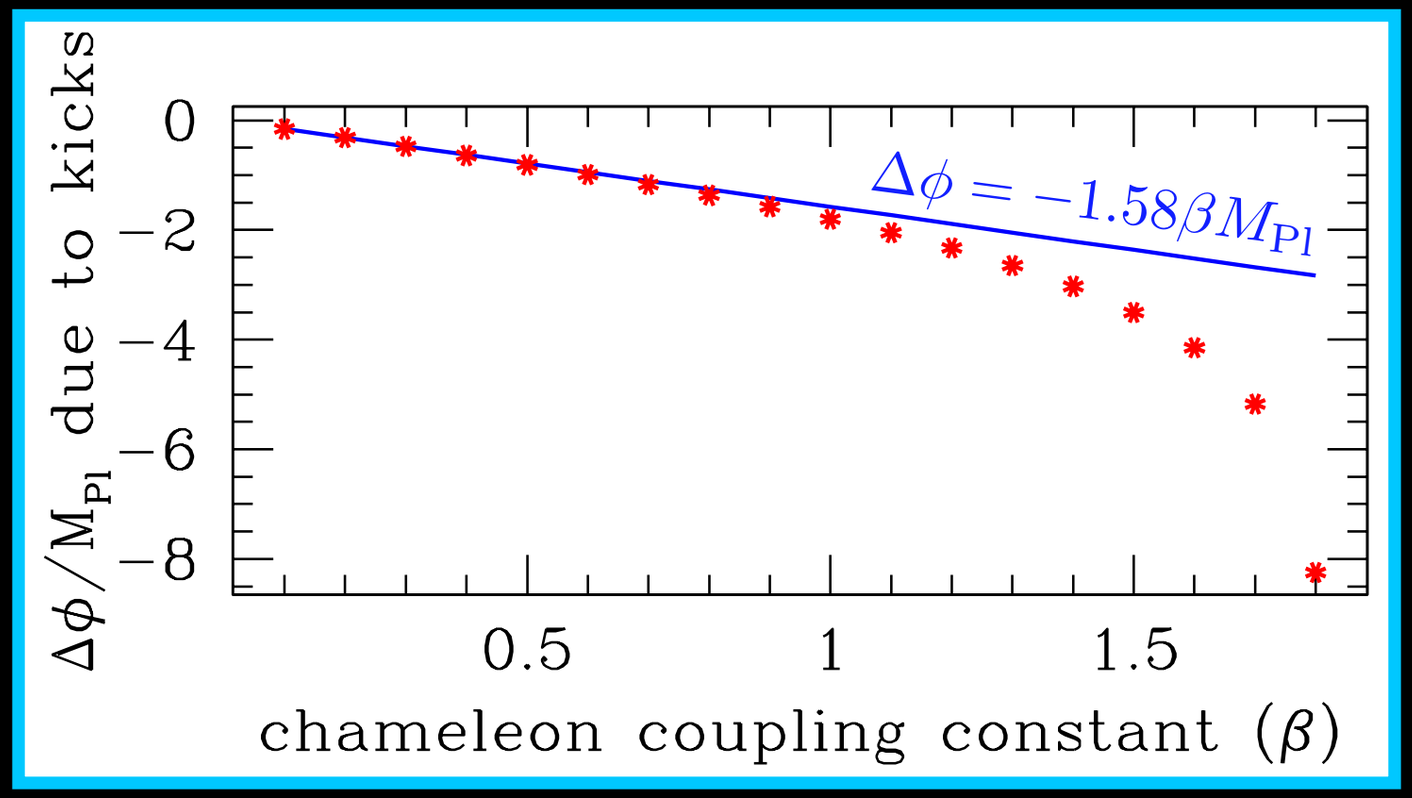






For larger β values:

- motion of ϕ affects T_J
- extends duration of kicks
- $|\Delta\phi| > 1.58\beta M_{\rm Pl}$

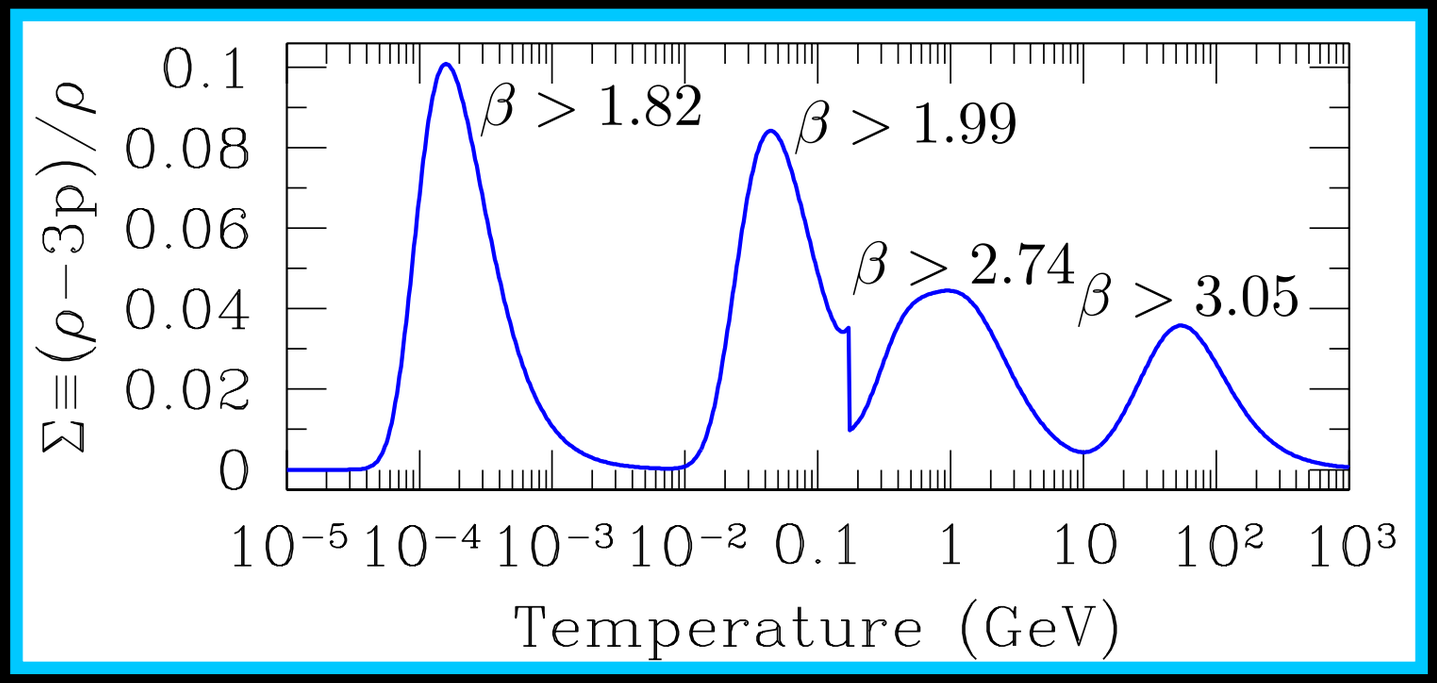
The Surfing Solution

Keeping the full expression for T_J reveals a new solution!

$$\underset{p=\ln(a_*)}{\phi'(p)} = -\frac{M_{\rm Pl}}{\beta}$$ keeps T_J constant

and solves the chameleon equation of motion provided that

$$\beta = \sqrt{\frac{1}{3\Sigma(T_J)}}$$ for some value of T_J .



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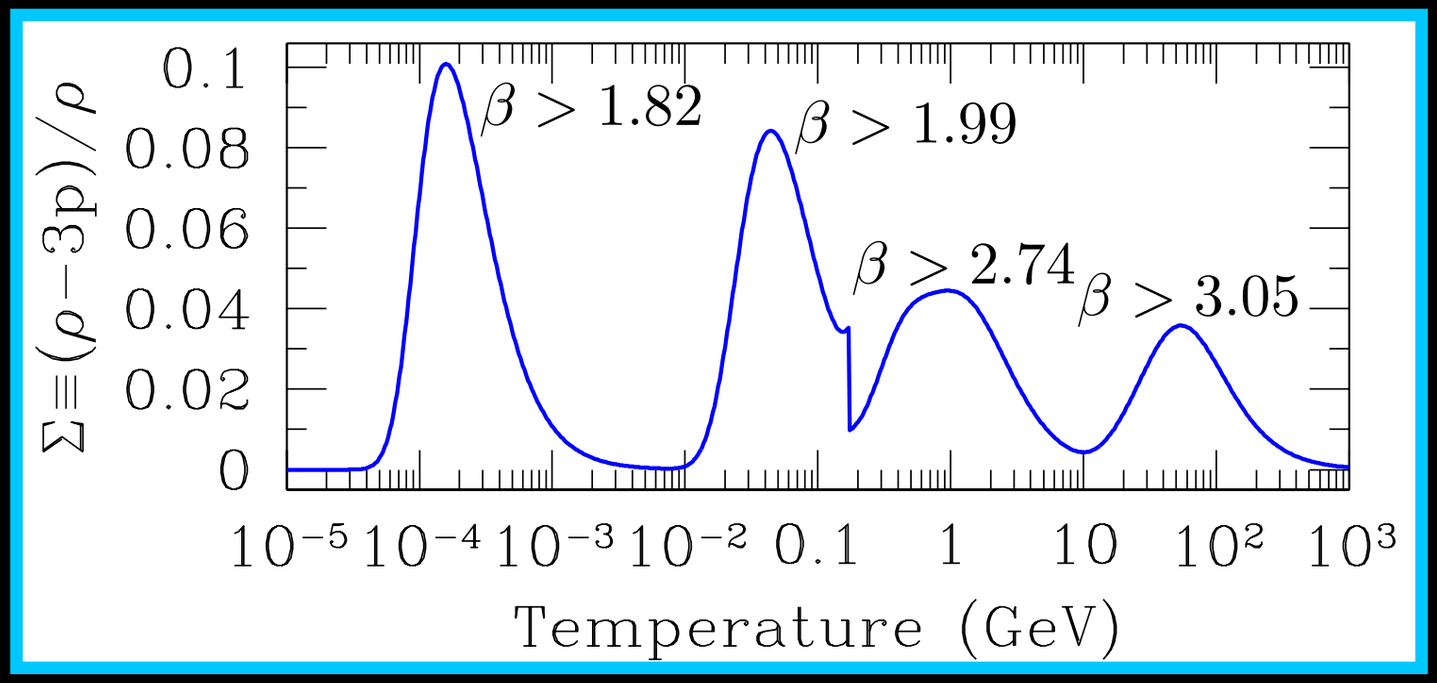
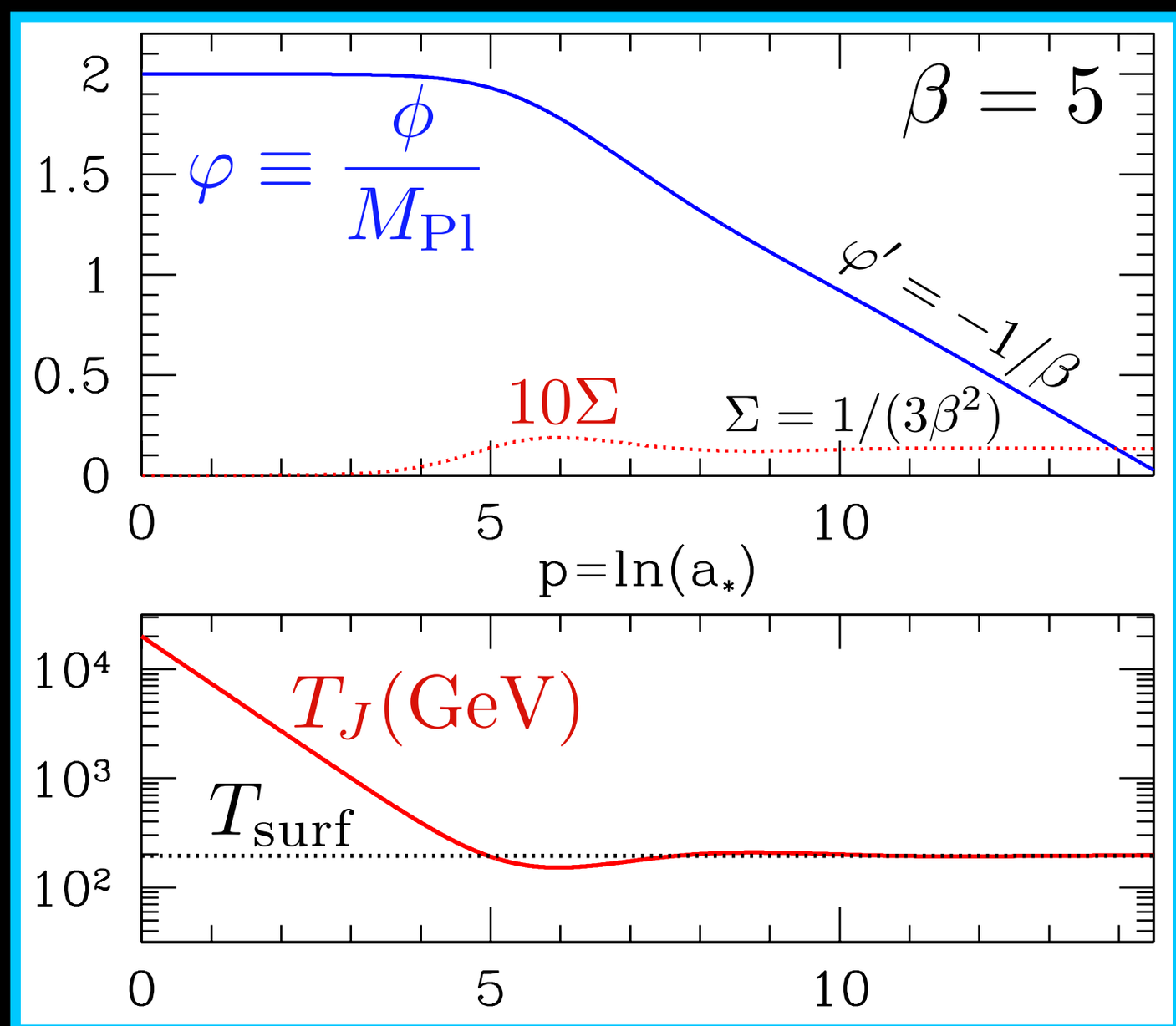
The surfing solution only exists if

$$\beta \geq \sqrt{\frac{1}{3\Sigma_{\rm max}}}$$



Surfing Chameleons

Surfing chameleons surf all the way to the minimum of the effective potential!



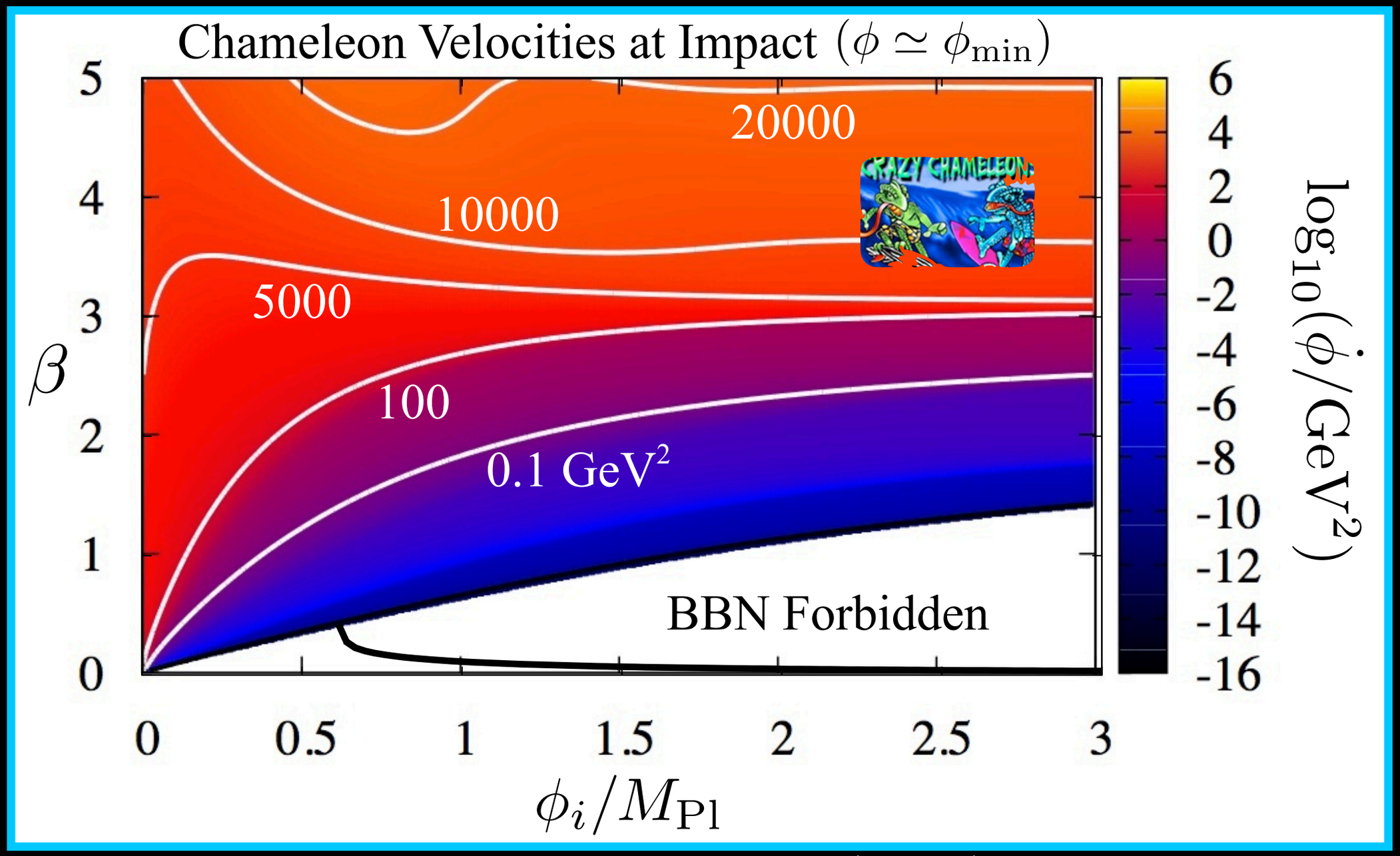
For every value of $\beta > 1.82$,

$$\Sigma(T_{\rm surf}) = \frac{1}{3\beta^2}$$

and the chameleon reaches $\phi_{\rm min}$ with a velocity:

$$\dot{\phi} = T_{\rm surf}^2 \sqrt{\frac{\pi^2 g_*}{15(6\beta^2 - 1)}}$$

Fast-Moving Chameleons



- nearly all chameleons are kicked to $\phi \simeq \phi_{\rm min}$
- at $\phi \simeq \phi_{\rm min}$, $\dot{\phi} \sim {\rm GeV}^2 \gg M^2 \sim 10^{-24}{\rm GeV}^2$
- adding particles will increase the chameleon velocity!

A Classical Impact

Now that $\phi \simeq \phi_{\rm min}$, we need to consider the chameleon potential:

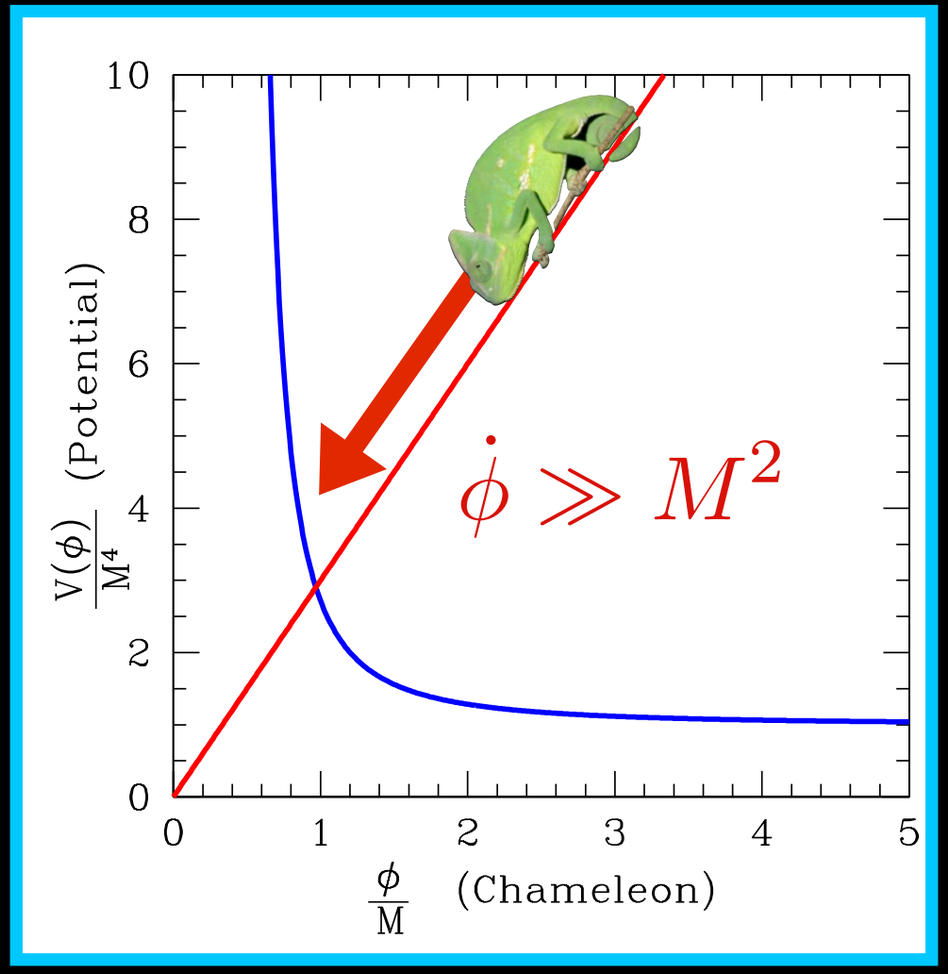
$$V(\phi) = M^4 \exp\left[\left(\frac{M}{\phi}\right)^2\right] \quad \text{with} \quad M = 0.001\,\text{eV}$$

The chameleon doesn't stop when it reaches the minimum of its effective potential because it's moving too fast!

The chameleon rolls up its potential until

$$V(\phi_b) = \dot{\phi}^2/2$$

Classically, the impact is a reflection!



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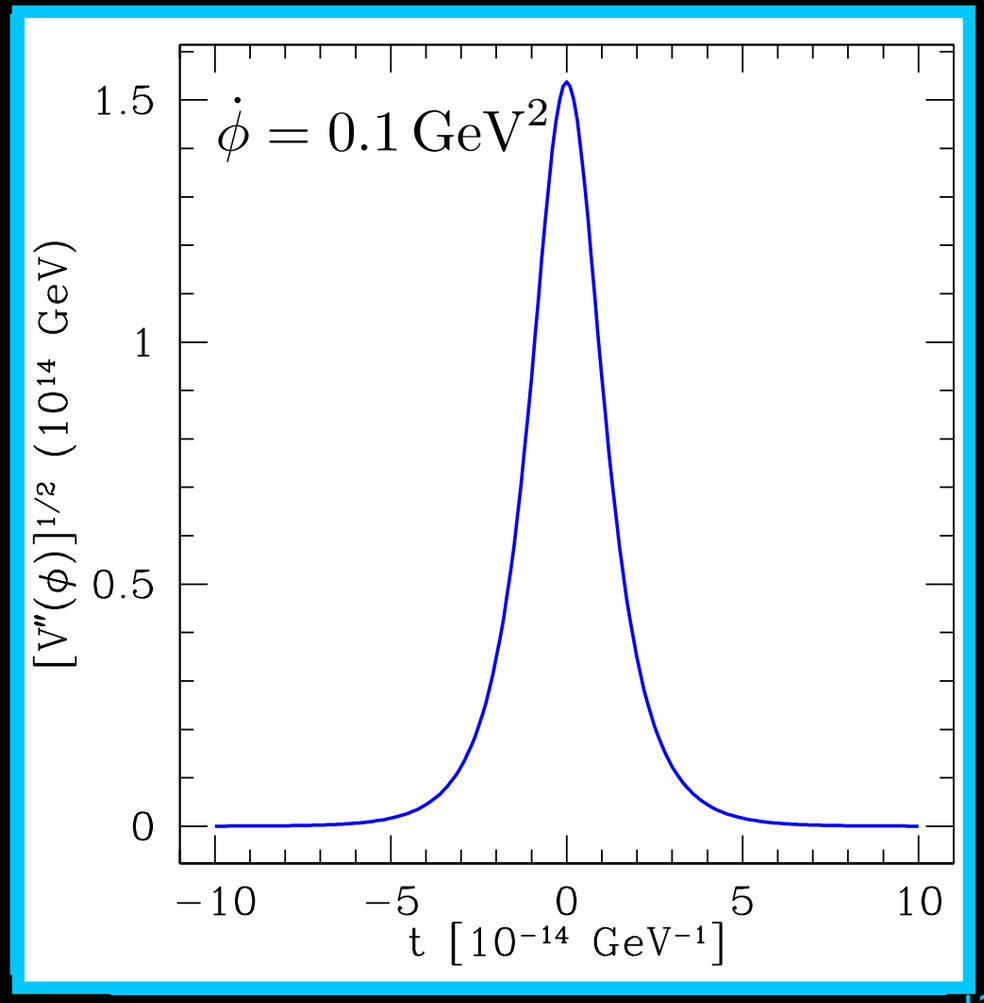


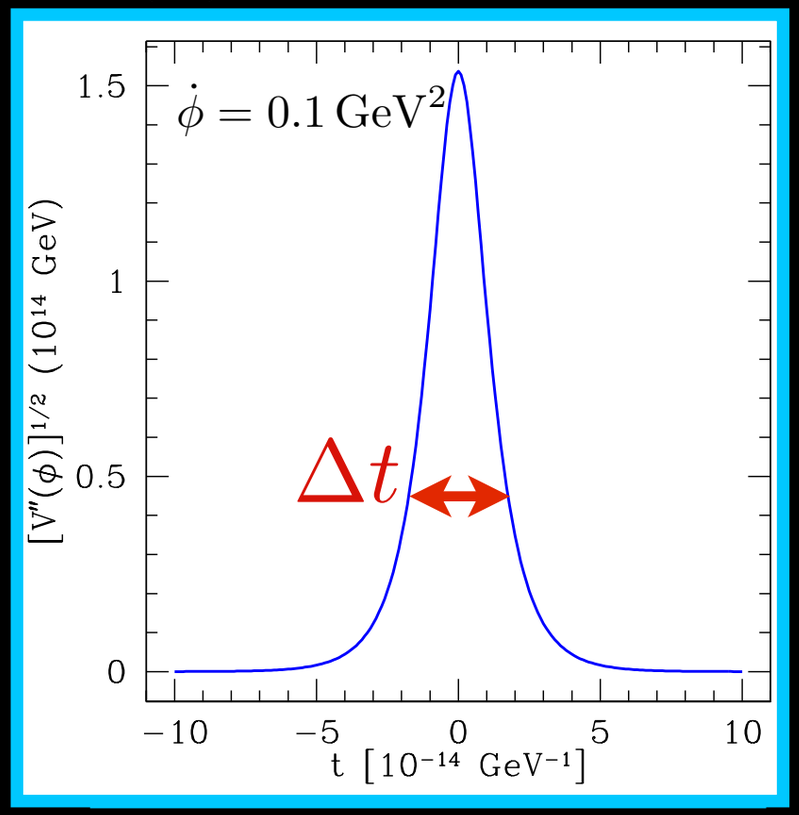
Quantum Particle Production

Rapid changes in $V''(\phi)$ excite perturbations!

$$\ddot{\phi}_k + \omega_k^2(t)\phi_k = 0 \qquad \omega_k^2 \equiv k^2 + V''(\bar{\phi})$$

Express in terms of Bogoliubov coefficients:

$$\phi_k(t) = \frac{\alpha_k(t)}{\sqrt{2\omega_k(t)}} e^{-i\int^t \omega_k(t')dt'} + \frac{\beta_k(t)}{\sqrt{2\omega_k(t)}} e^{+i\int^t \omega_k(t')dt'}$$



Occupation number: $n_k(t) = |\beta_k|^2$

We get particle production ($|\beta_k|^2 \gtrsim 1$) when

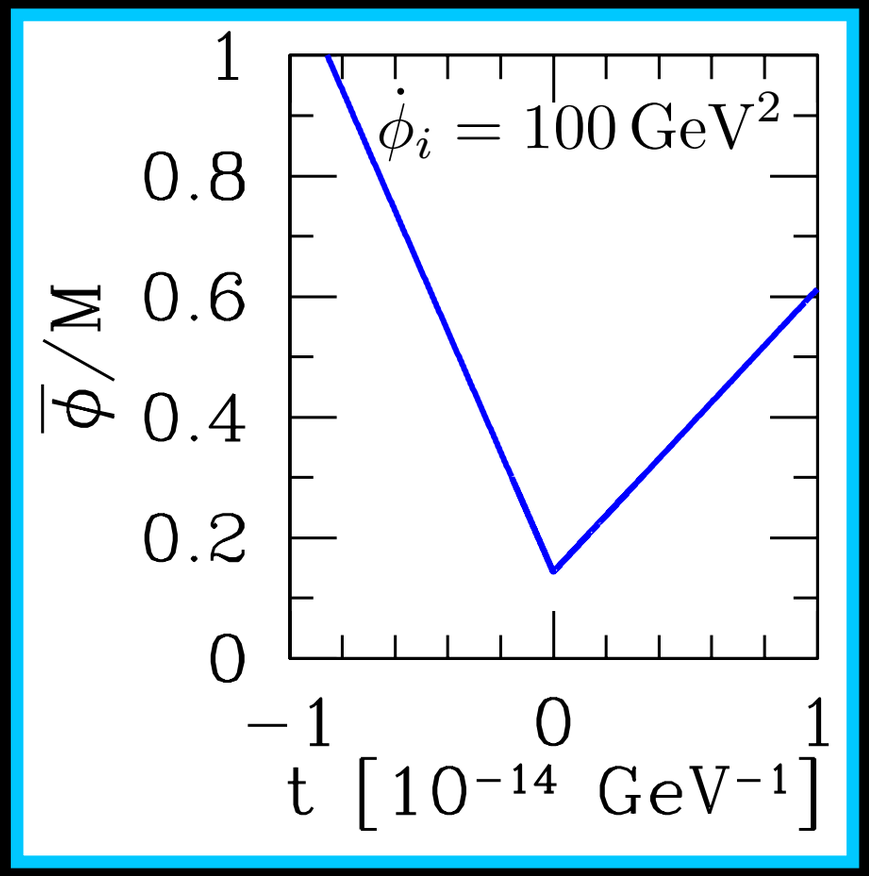
$$\frac{|\dot{\omega}_k|}{\omega_k^2} = \frac{|V'''(\phi)\dot{\phi}|}{2\omega_k^3} \gtrsim 1$$

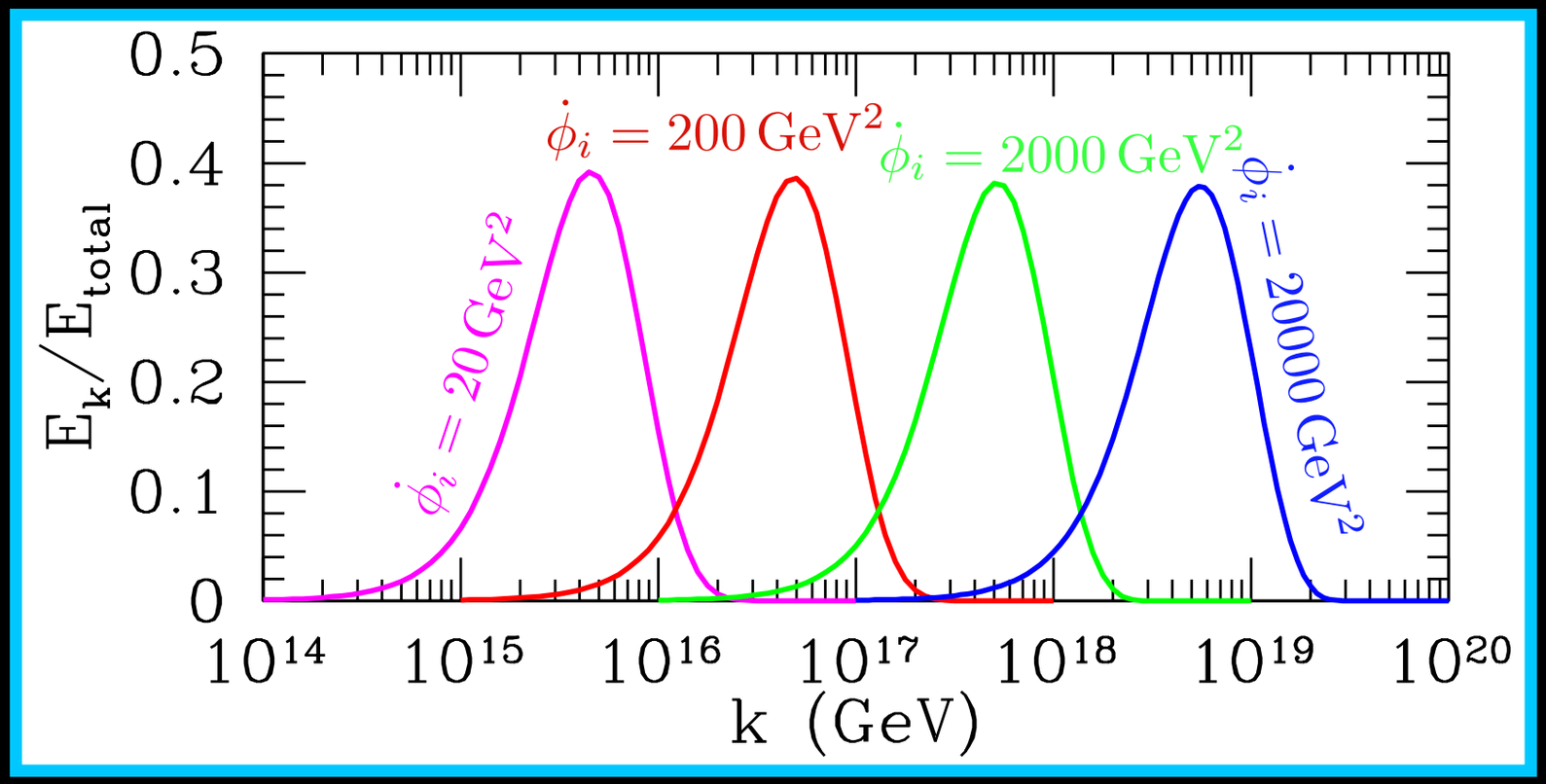
Modes with $k \gg \frac{1}{\Delta t}$ are not excited: $\frac{|\dot{\omega}_k|}{\omega_k^2} \ll 1$ for $k \gg \frac{1}{\Delta t}$

Numerical Results

Numerical results confirm these expectations.

- The chameleon bounces off its bare potential.
- Perturbations are generated during the bounce, taking energy away from the background evolution.
- The perturbation energy spectrum is peaked; most of the energy is in modes with $k_{\rm peak} \simeq (\Delta t)^{-1}$.



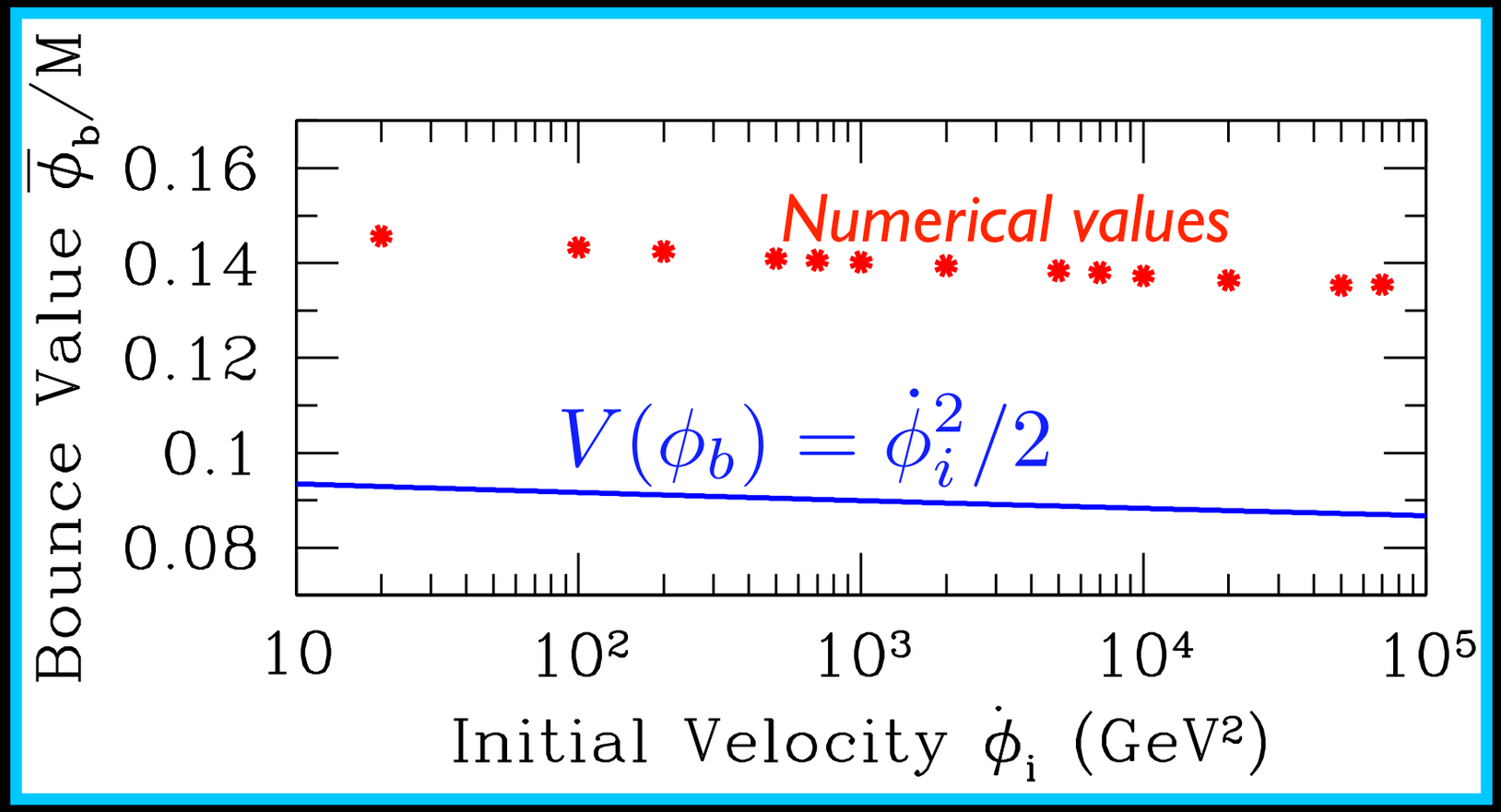


Numerical Surprise

The numerical results confirm our expectations...
except when they don't!

The chameleon turns around sooner than expected:

$$V(\phi_b) \ll \dot{\phi}_i^2/2$$



The Backreaction

Studying the backreaction of the perturbations on the chameleon background provides insight.

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) + \frac{1}{2}V'''(\bar{\phi})\langle\delta\phi^2\rangle = 0$$ *background equation with backreaction*

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) - \frac{V'''[\bar{\phi}(t)]}{16\pi^2}\int_0^t V'''[\bar{\phi}(t')]\dot{\bar{\phi}}(t')\mathrm{Ci}\left[2k_{\mathrm{IR}}(t-t')\right]dt' = 0$$

- the "dissipation" term is nonlocal; it has memory
- before the bounce, it acts like a friction term; it has the same sign as $\dot{\bar{\phi}}$ and it slows the chameleon down.
- but unlike friction, it does not decrease as the chameleon slows down. It is more like a potential, and it can turn the chameleon around!

$$V_D(\phi) = \frac{\kappa}{2}\left[V''\left(\bar{\phi}\right)\right]^2$$

$$0.02 \lesssim \kappa \lesssim 0.05$$

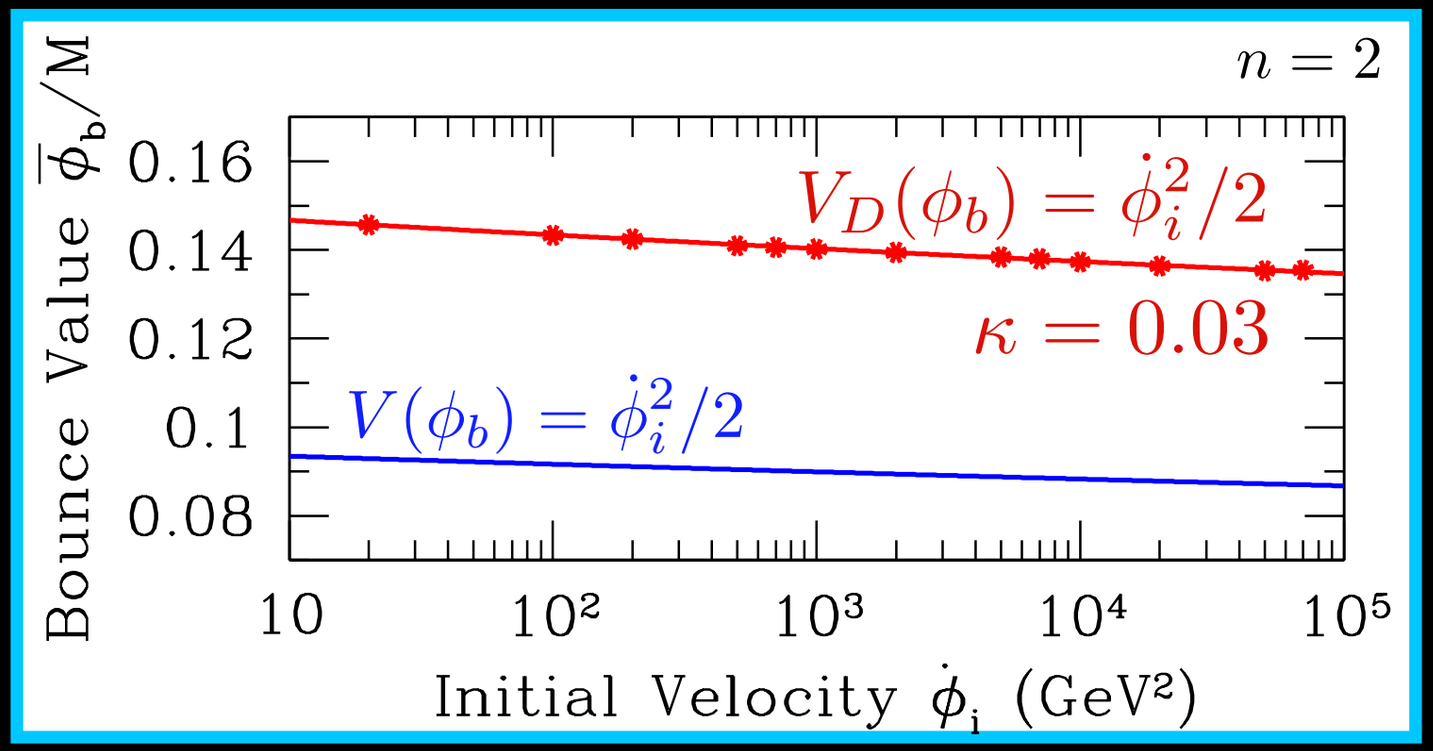
calibrate using numerical results

For $\phi \lesssim M, V_D(\phi)$ dominates over the chameleon's bare potential!

New Models for a New Potential

$V_D(\phi) = \frac{\kappa}{2}\left[V''\left(\bar{\phi}\right)\right]^2$ controls the chameleon's motion.

Predict when the chameleon bounces: $V_D(\phi_b) = \dot{\phi}_i^2/2$



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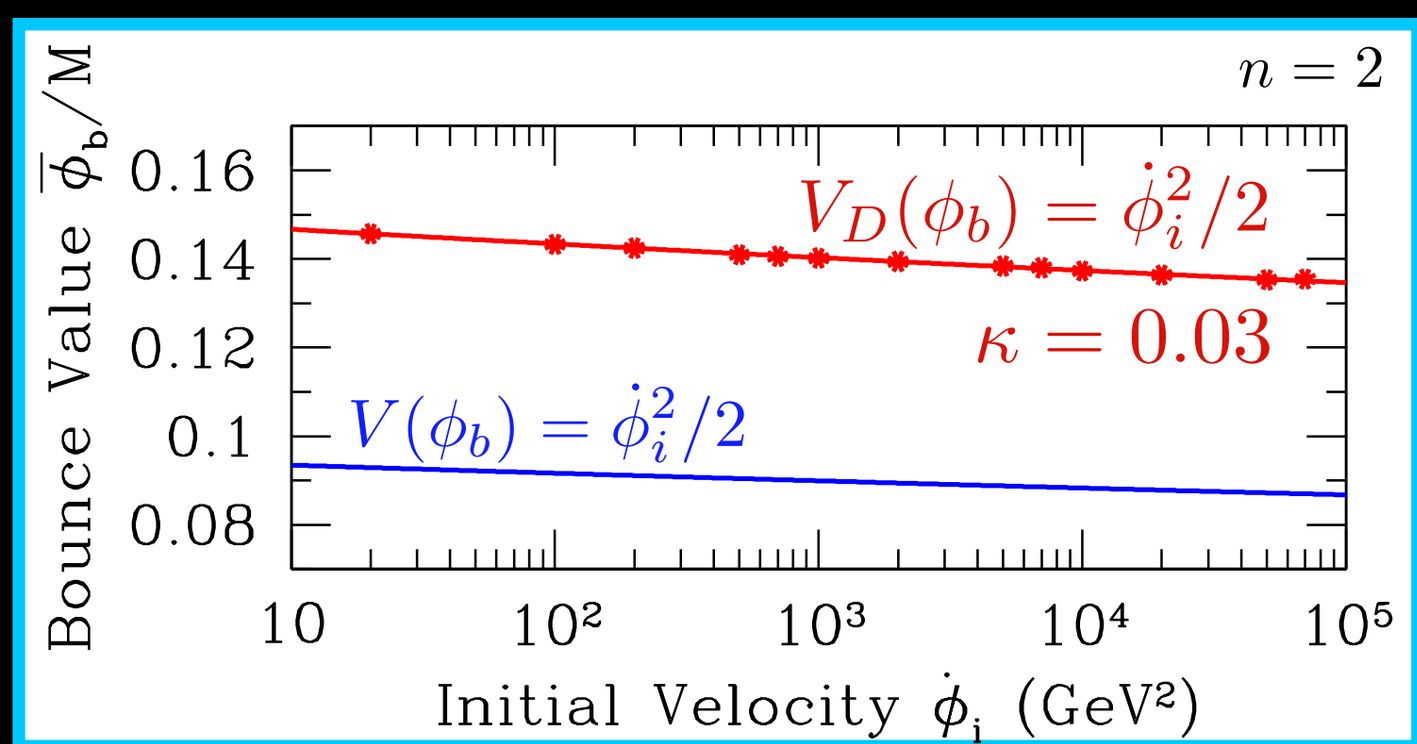
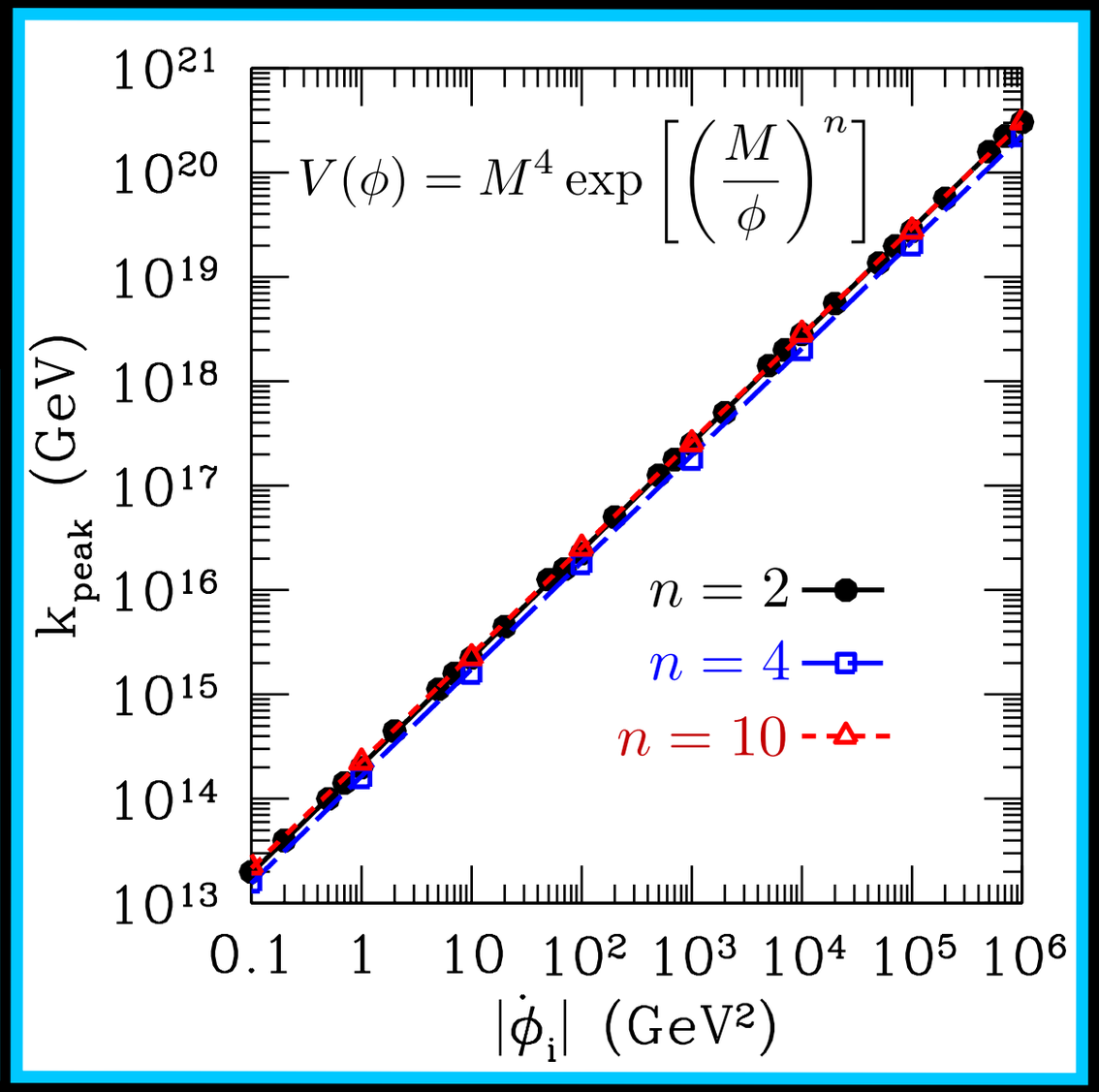
Predict the peak wavenumber in the perturbation energy spectrum:

$$k_{\rm peak} = \left[\Delta t \,|\, \Delta V'' = V''(\phi_b)\right]^{-1}$$

$$k_{\rm peak} \simeq \frac{n|\dot{\phi}_i|}{M}\left(\ln\left[\frac{\dot{\phi}_i^2}{n^4\kappa M^4}\right]\right)^{\frac{n+1}{n}}$$



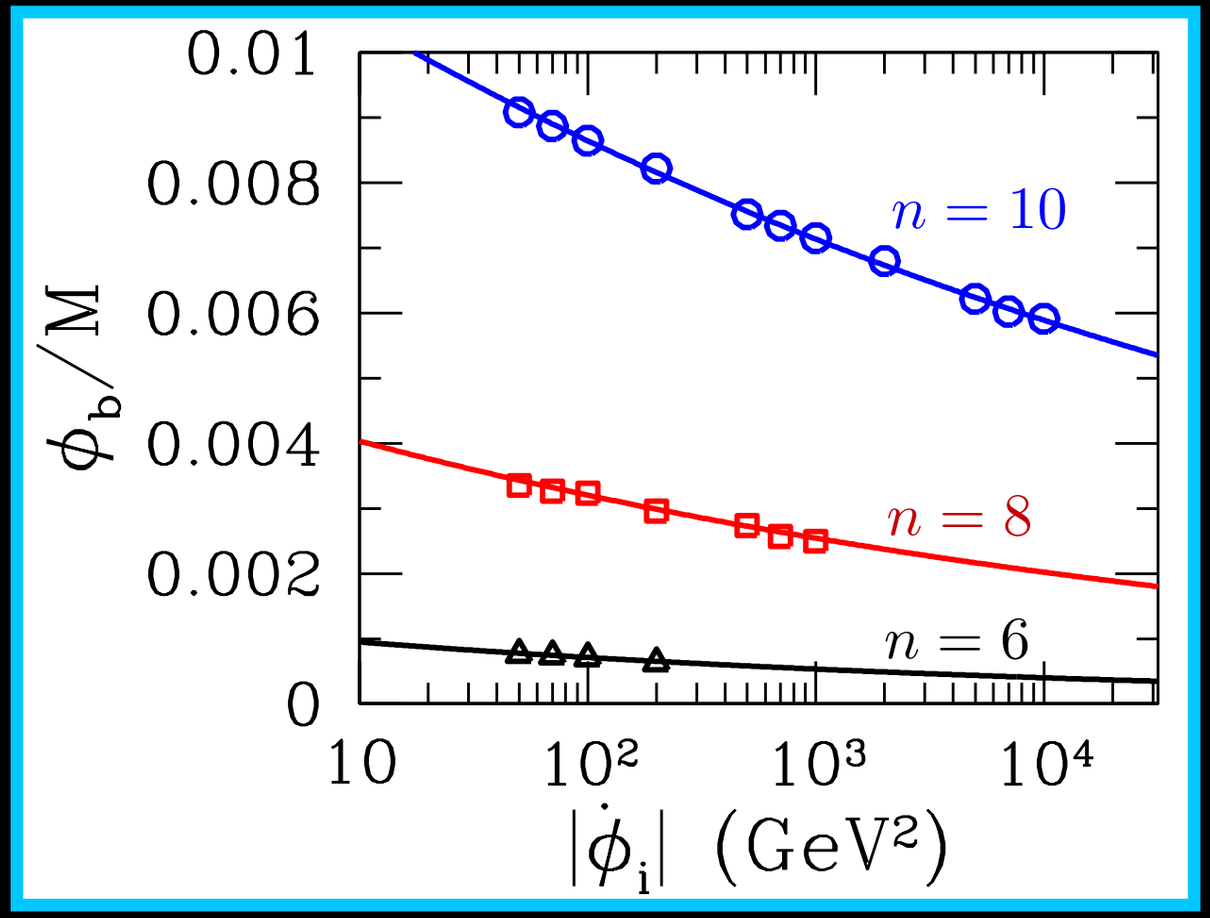


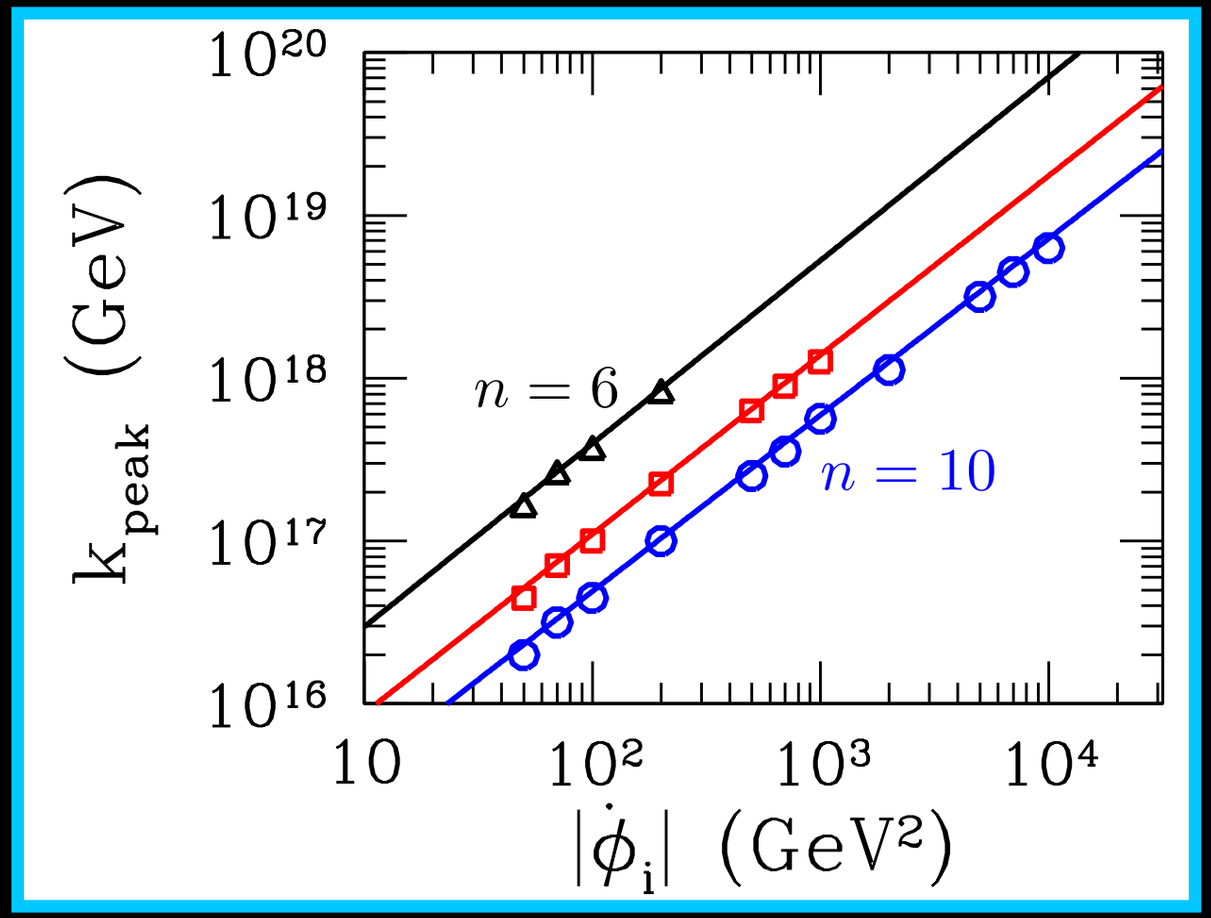
Power-Law Potentials

The model works equally well for power-law potentials:

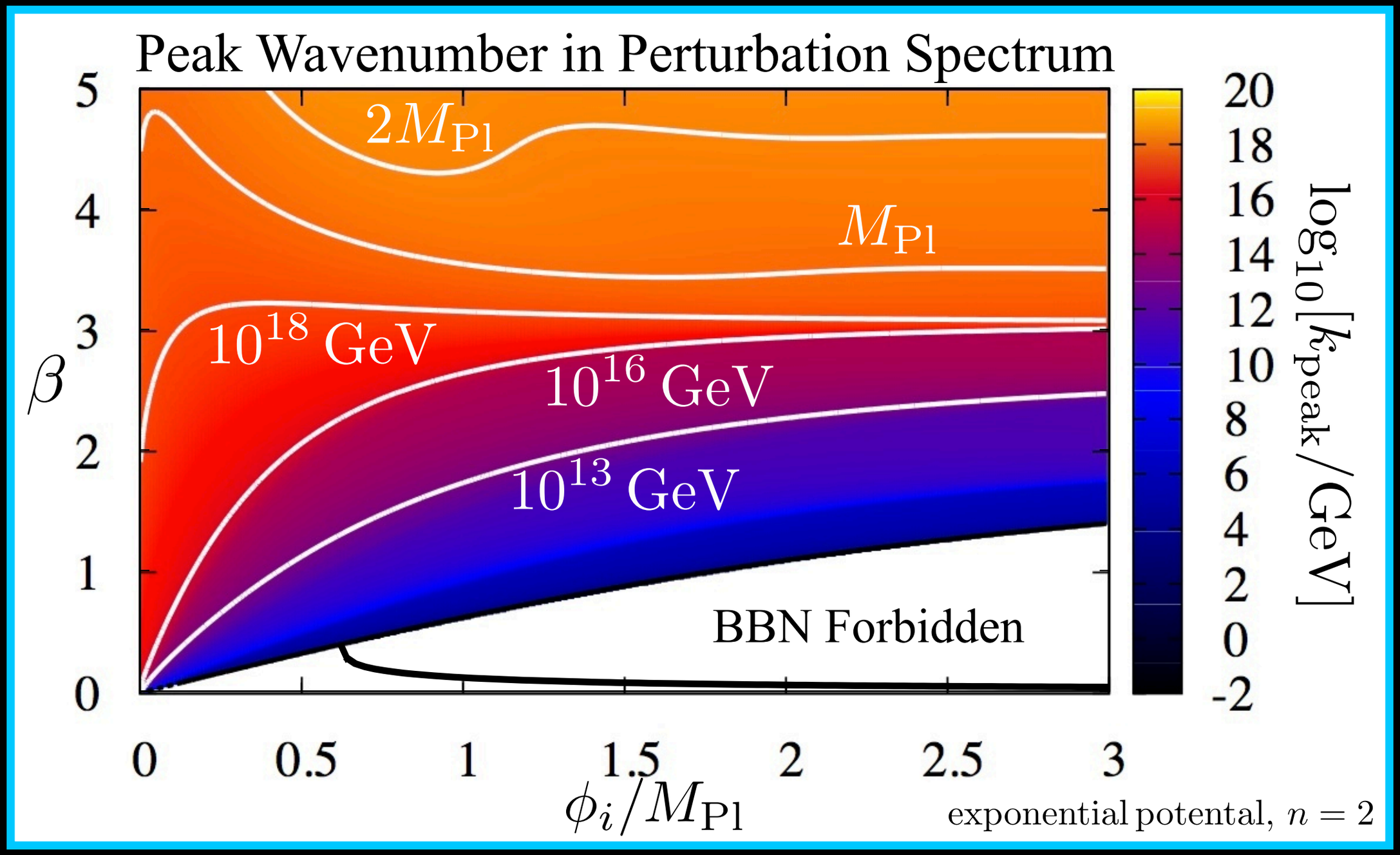
$$V(\phi) = M^4 \left(\frac{M}{\phi}\right)^n$$

Since ϕ bounces at a smaller value, the timescale for the rebound is shorter, which leads to even larger $k_{\rm peak}$!





High-Energy Chameleons



$$k_{\rm peak} = 0.7\frac{\dot{\phi}}{M}\left(\frac{M}{\phi_b}\right)^3 \simeq 0.18\frac{\dot{\phi}}{M}\ln^{3/2}\left[\frac{2\dot{\phi}_i^2}{M^4}\right]$$

Summary: A Chameleon Catastrophe

AE, Barnaby, Burrage, Huang 1304.0009 (PRL) and 1310.5149 (PRD)

What happens when you kick a chameleon?

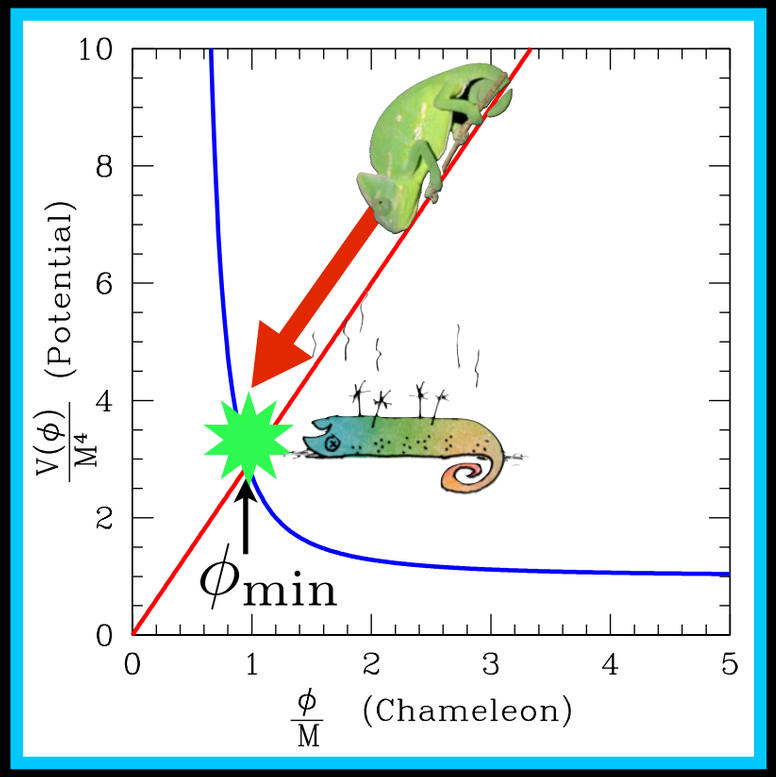
It hits its bare potential at a fatal velocity, and then it shatters into pieces!

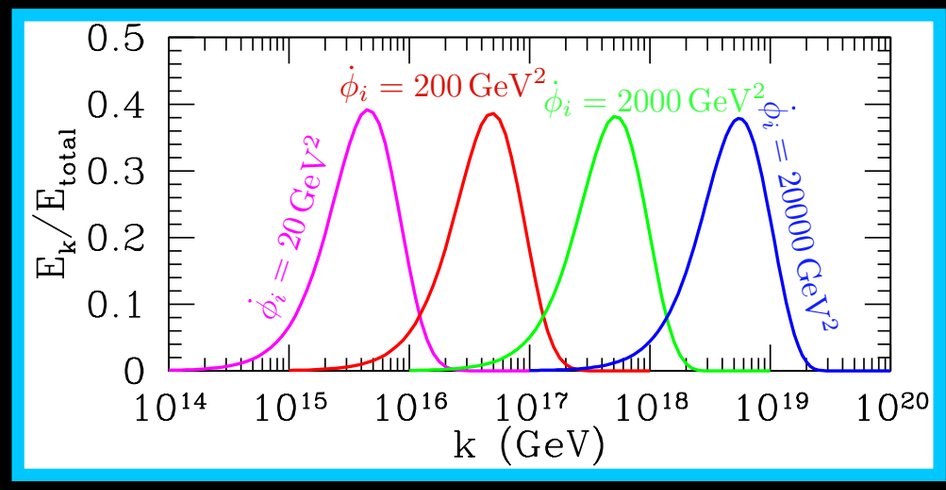
The chameleon's interaction with standard model particles hurtles it toward the minimum of its effective potential.

- Chameleons with $\beta > 1.8$ surf toward $\phi_{\rm min}$
- At impact, $\dot{\phi} \gtrsim {\rm GeV}^2$.

Because $\dot{\phi} \gg M^2$, the rebound is highly nonadiabatic, and perturbations are excited.

- Most (maybe all?) the chameleon's energy goes into perturbations.
- The perturbations have wavenumbers $k \gtrsim 10^{13}\,{\rm GeV}$
- The perturbations strongly interact with themselves and with matter: theoretical breakdown!
- Chameleons demonstrate how the presence of an extreme hierarchy of scales can challenge a theory's stability. Are there other examples?





Perturbation Equations

Perturbation Equations

Linear perturbations with first-order backreaction:

$$\ddot{\bar{\phi}} + V'(\bar{\phi}) + \frac{1}{2}V'''(\bar{\phi})\langle\delta\phi^2\rangle = 0$$

background equation with backreaction

$$\ddot{\phi}_k + \left[k^2 + V''(\bar{\phi})\right]\phi_k = 0$$

linearized perturbation equations

$$\langle\delta\phi^2\rangle = \int \frac{d^3k}{(2\pi)^3}\left(|\phi_k|^2 - \frac{1}{2\omega_k}\right)$$

$$\omega_k^2 \equiv k^2 + V''(\bar{\phi})$$