The Power Spectrum of Nambu-Goto Cosmic Strings

Andrei Lazanu and Paul Shellard

DAMTP, University of Cambridge

UNIVERSITY OF CAMBRIDGE

Outline

- Cosmic Strings overview
- Cosmic String Models
- The UETC Approach
- Nambu-Goto Simulations
- Convergence of UETCs in terms of the resolution of the grid
- Convergence of the power spectrum in terms of the number of eigenvectors used
- Combining the 3 simulations
- Cosmic strings power spectrum
- Constraints on the cosmic strings tension
- Degeneracies between cosmic strings and other cosmological parameters
- Summary

Cosmic Strings

- 1-dimensional topological defects
- Appear naturally as a result of symmetry-breaking processes in the early universe
- Give rise to observable cosmological consequences, such as line-like discontinuities in the CMB power spectrum
- Temperature power spectrum has 1 maximum, hence cannot be primary sources of anisotropy
- Contribution to the overall observed power spectrum of a few percent
- Active sources: they continuously seed perturbations throughout the history of the universe
- 2 approaches for simulating the evolution of cosmic strings

Cosmic String models

- The Abelian Higgs field theory model
  - Strings obtained as solutions to the relativistic generalization of the Ginzburg-Landau action
  - Simulations rely on extrapolation on many orders of magnitude, as their width cannot be resolved with current processing power

- The Nambu-Goto effective field action
  - Obtained as a first-order approximation from the Abelian-Higgs action by considering the string width to be small with respect to its length
  - A further simplification was made in the phenomenological **Unconnected segment model** (USM), where the strings are assumed to be formed from a number of uncorrelated randomly oriented straight string segments which have random velocities

The UETC approach

UETC = **U**n**E**qual **T**ime **C**orrelator

2-point correlation function of different components of the energy-momentum tensor

$$\sqrt{\tau\tau'}\langle\Theta_{\mu\nu}(\mathbf{k},\tau)\Theta_{\rho\sigma}(\mathbf{k},\tau')\rangle = c_{\mu\nu,\rho\sigma}(\mathbf{k}\tau,\mathbf{k}\tau')$$

At first-order in perturbation theory they store all information about the cosmic strings

Correlation of same type of components ➡ Positive definite-quantity ➡ diagonalizable

$$c(k\tau,k\tau') = \sum_i \lambda_i\, v^{(i)}(k\tau)v^{(i)T}(k\tau')$$

$$\lambda_1 > \lambda_2 > \cdots$$

Coherent eigenvectors ➡ can be fed individually into eigensolver ➡ Substitution of the energy-momentum tensor in terms of the eigenvectors

➡ $\Theta(k,\tau) \to \dfrac{v^{(i)}(k\tau)}{\sqrt{\tau}}$ ➡ Results summed up: $C_l^{\text{string}} = \sum_i \lambda_i C_l^{(i)}$

Cosmic strings modify usual perturbation equations

The perturbations they create are uncorrelated with primordial fluctuations

$C_l^{\text{total}} = C_l^{\text{inflation}} + C_l^{\text{strings}}$

Einstein equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

metric $g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu})$

Perturbations due to strings:

$$\delta T_0^0 = -\delta\rho + \Theta_0^0$$

$$\delta T_i^0 = (\rho + P) v_i + \Theta_i^0$$

$$\delta T_j^i = \delta P \delta_j^i + p\Sigma_j^i + \Theta_j^i$$

Using synchronous gauge perturbations at first order & splitting the equations into their scalar, vector and tensor components (in Fourier space), the evolution equations for the metric perturbations are obtained in terms of the matter perturbations and the cosmic strings.

General scalar-vector-tensor decomposition of a tensor:

$$T_{ij}(\mathbf{k}) = \frac{1}{3} T \delta_{ij} + \left(k_i k_j - \frac{1}{3}\delta_{ij} \right) T^S + \left(k_i T_j^V + k_j T_i^V \right) + T_{ij}^T$$

Perturbations for the equations

$$k\eta' = 4\pi G a^2 \sum_i \left(\rho_i + p_i\right) v_i - \frac{4\pi G}{k}\Theta^D$$

$$\ddot{h}^S + 2\frac{a'}{a}\dot{h}^S - 2k^2\eta = 16\pi G\left(a^2 p\Sigma^S + \Theta^S\right)$$

$$\ddot{h}^V + 2\frac{a'}{a}\dot{h}^V = 16\pi G\left(a^2 p\Sigma^V + \Theta^V\right)$$

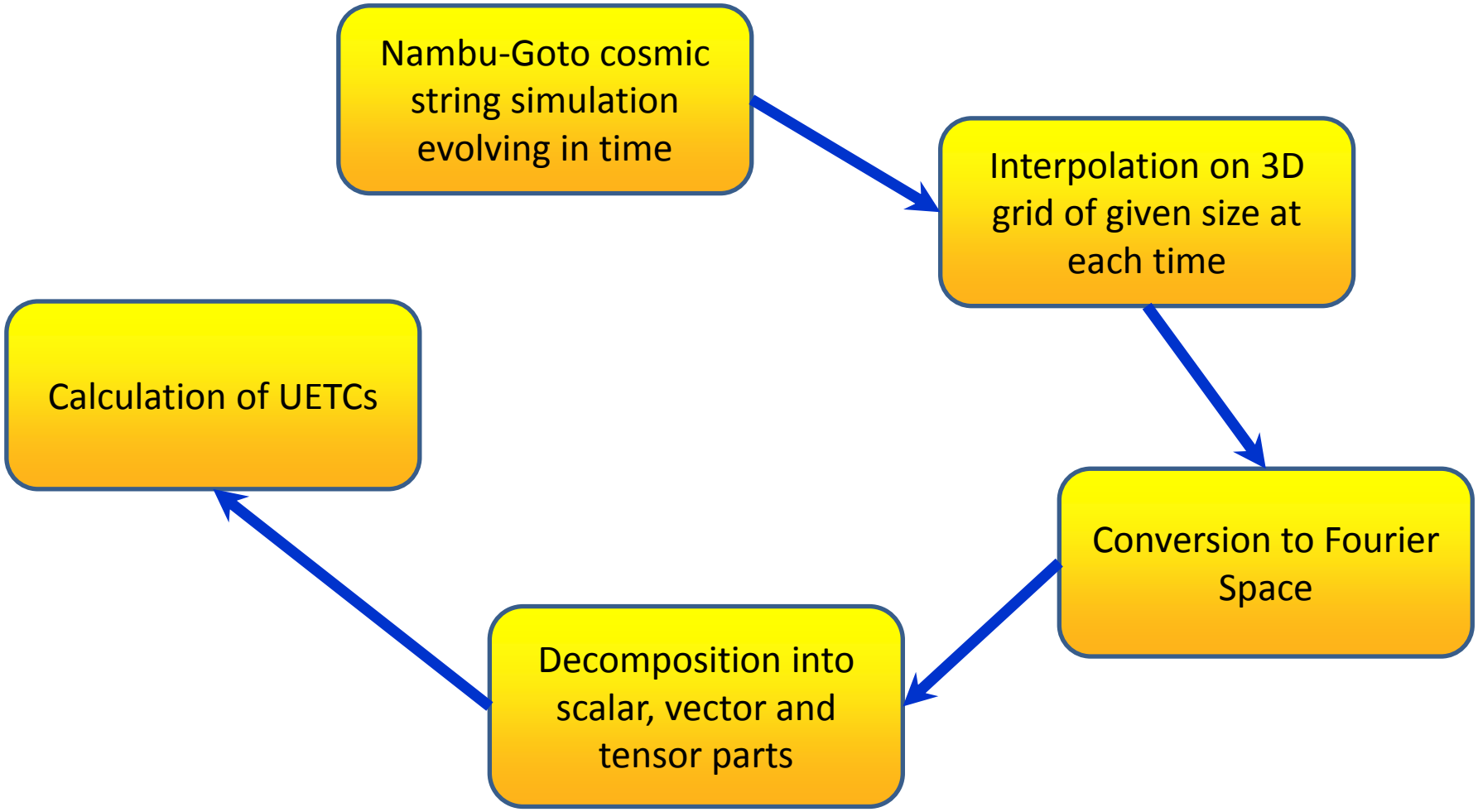
$$\ddot{h}^T + 2\frac{a'}{a}\dot{h}^T + k^2 h^T = 16\pi G\left(a^2 p\Sigma^T + \Theta^T\right)$$

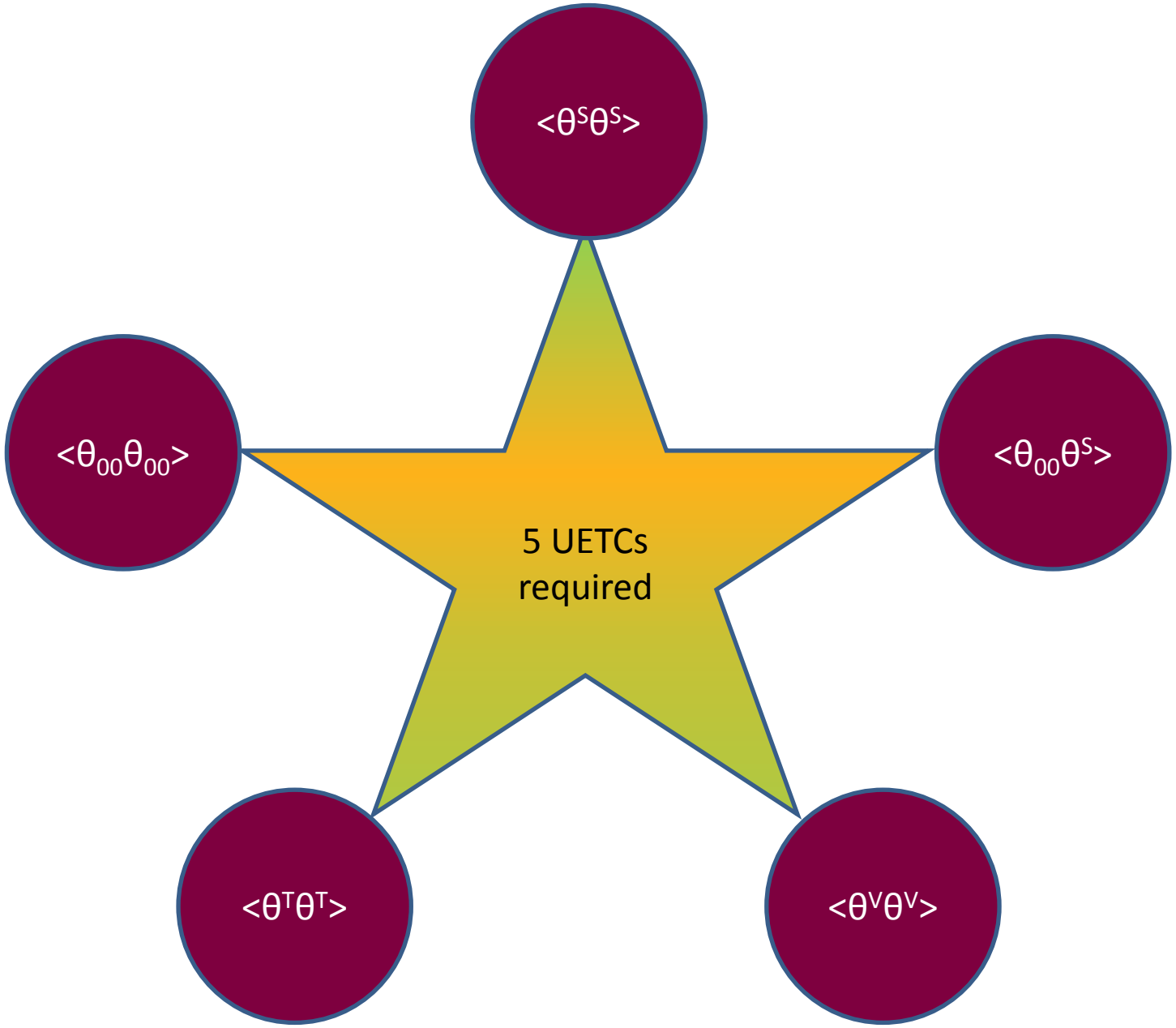
$$\dot{\Theta}^D = \Theta^D\left(-2H - \frac{k^2}{3H}\right) - \frac{k^2}{3}\left(2\Theta^S - \Theta_{00} - \frac{\dot{\Theta}_{00}}{H}\right)$$

$$\eta = -\frac{h - h^S}{6}$$

$$\Theta(k,\tau) \to \frac{v^{(i)}(k\tau)}{\sqrt{\tau}}$$

Nambu-Goto simulations




$<\theta^S\theta^S>$
$<\theta_{00}\theta_{00}>$
$<\theta_{00}\theta^S>$
5 UETCs required
$<\theta^T\theta^T>$
$<\theta^V\theta^V>$

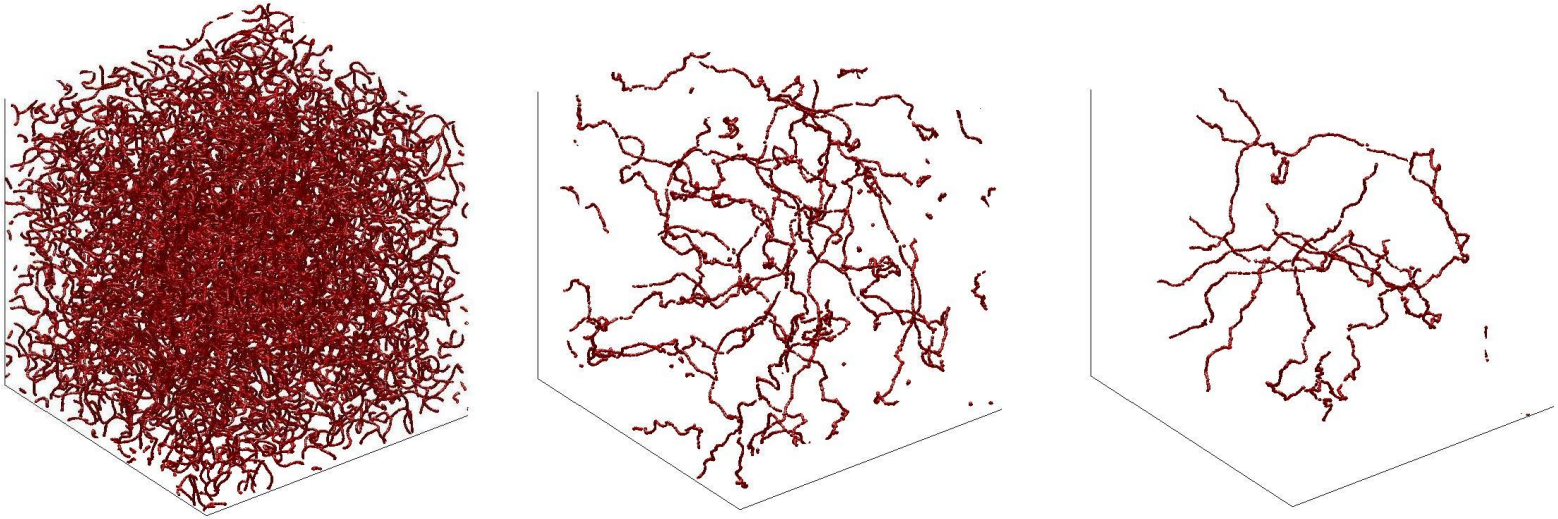
3 simulations

Radiation era: redshift 6348 to 700

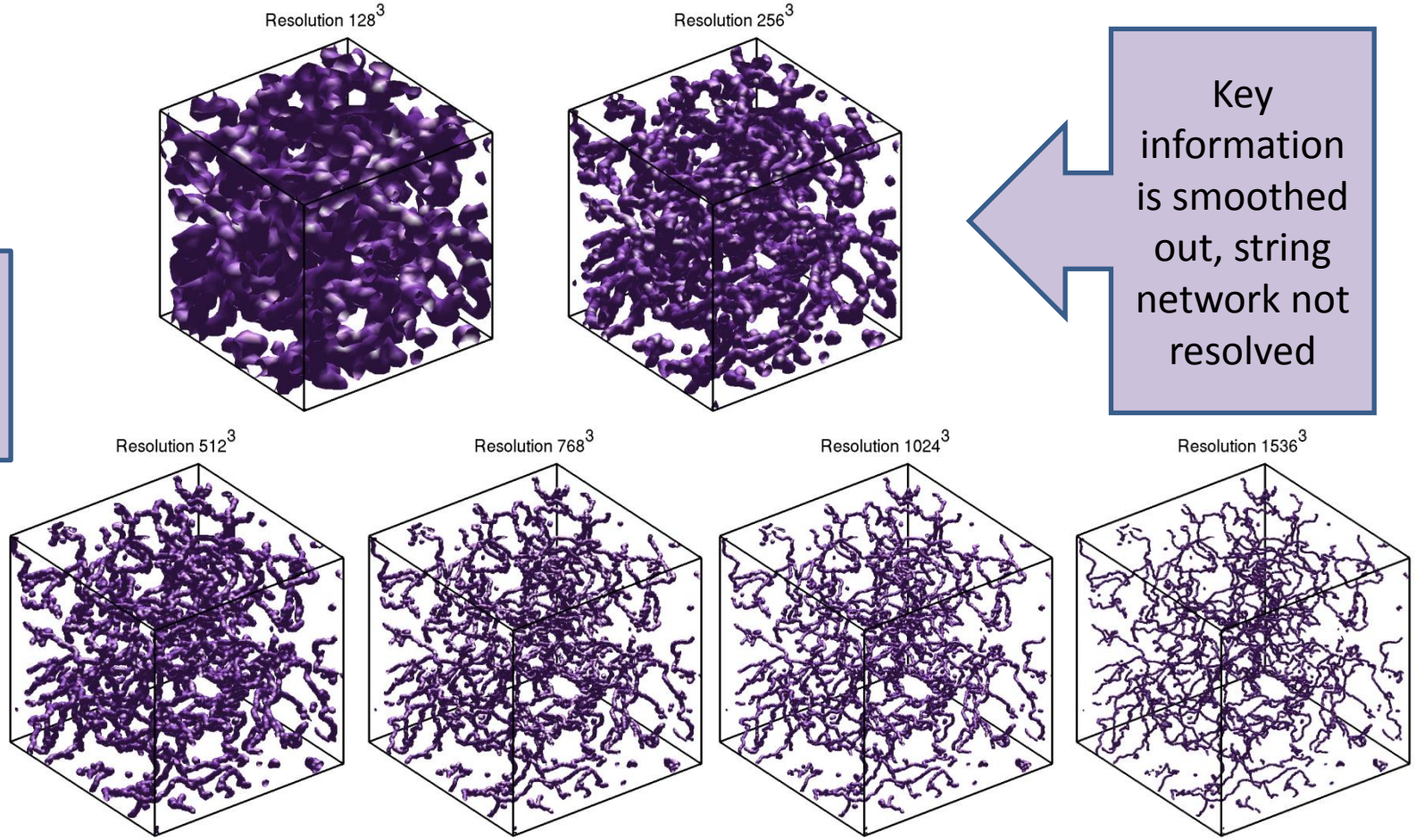
Matter era: redshift 945 to 37.5

Matter + Λ eras: redshift 55.4 to 0

Evolution of string network in the radiation era at beginning, middle and end of simulation



String network appearance in terms of resolution the grid it is interpolated on (radiation era simulation)

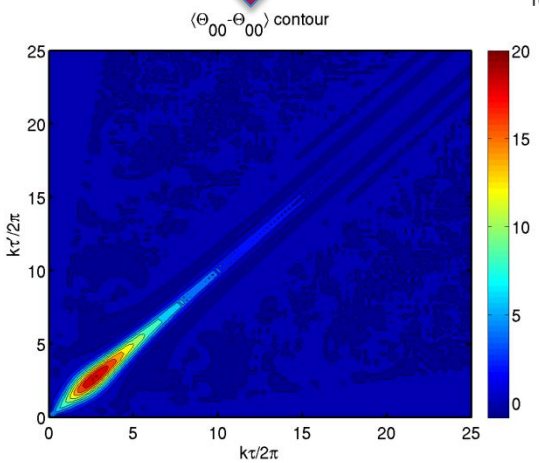


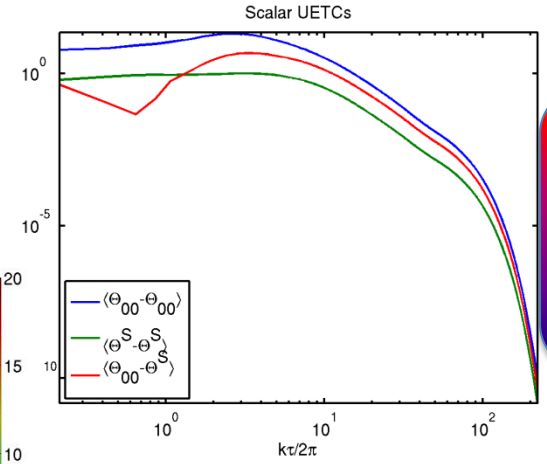
Key information is smoothed out, string network not resolved

Reliable predictions expected

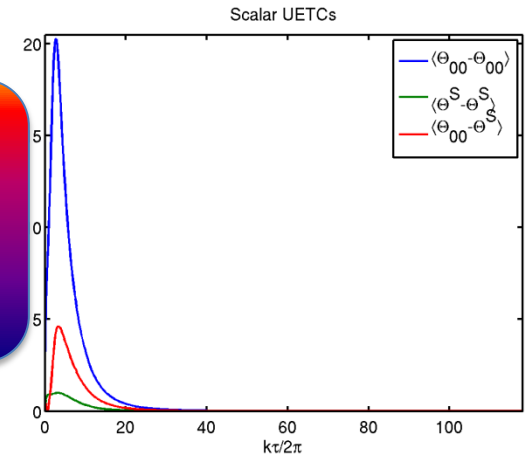
Chosen as balance between computational time and precision

$<\theta_{00}\theta_{00}>$ contour

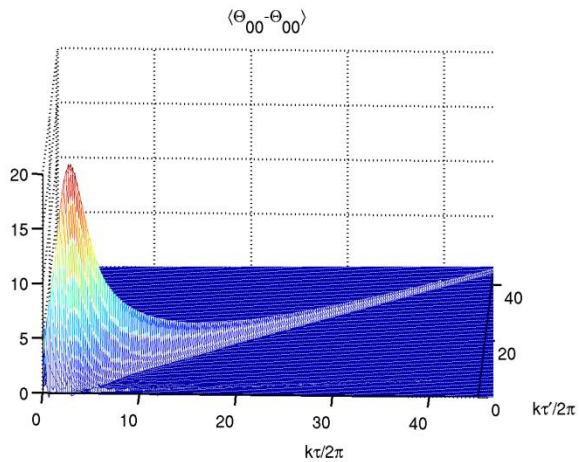


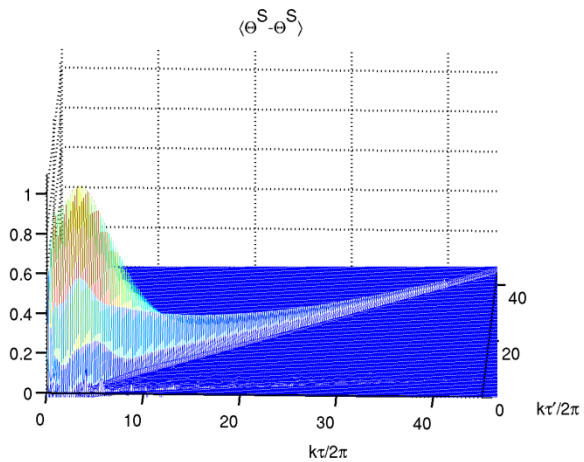


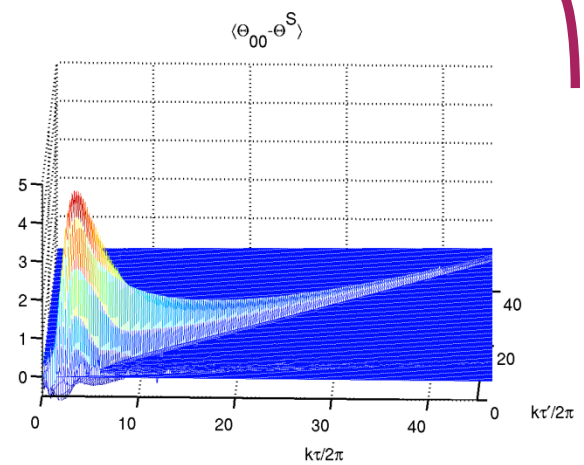
Diagonal views of scalar UETCs

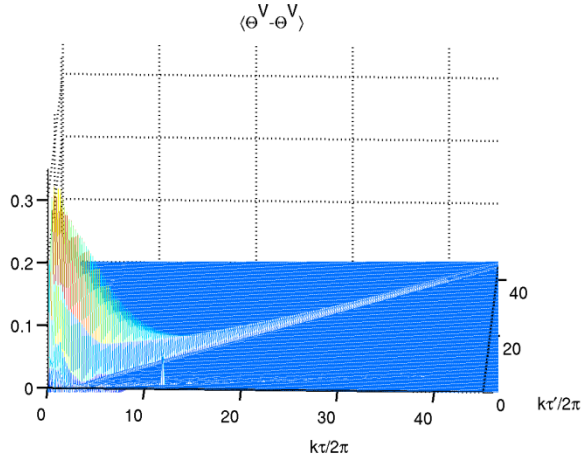


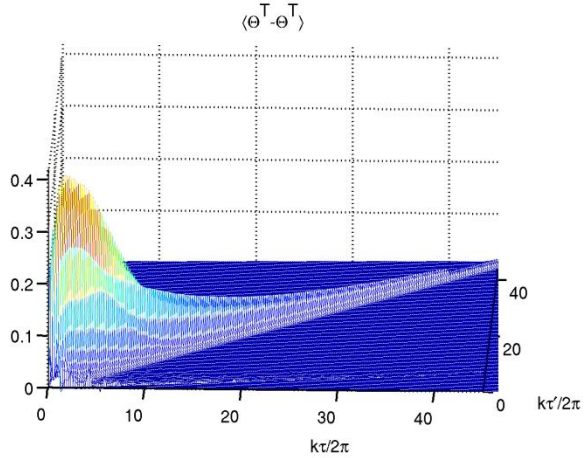
3D views of scalar UETCs









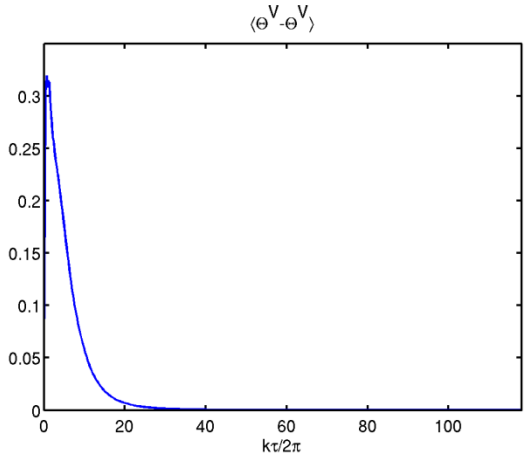


3D profile views

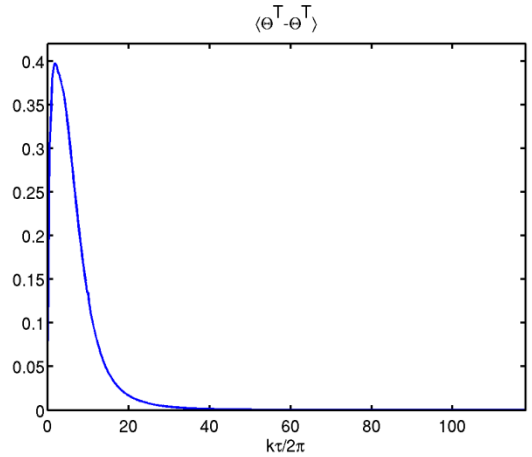
Vector and tensor UETCs

$<\theta^V\theta^V>$

$<\theta^T\theta^T>$

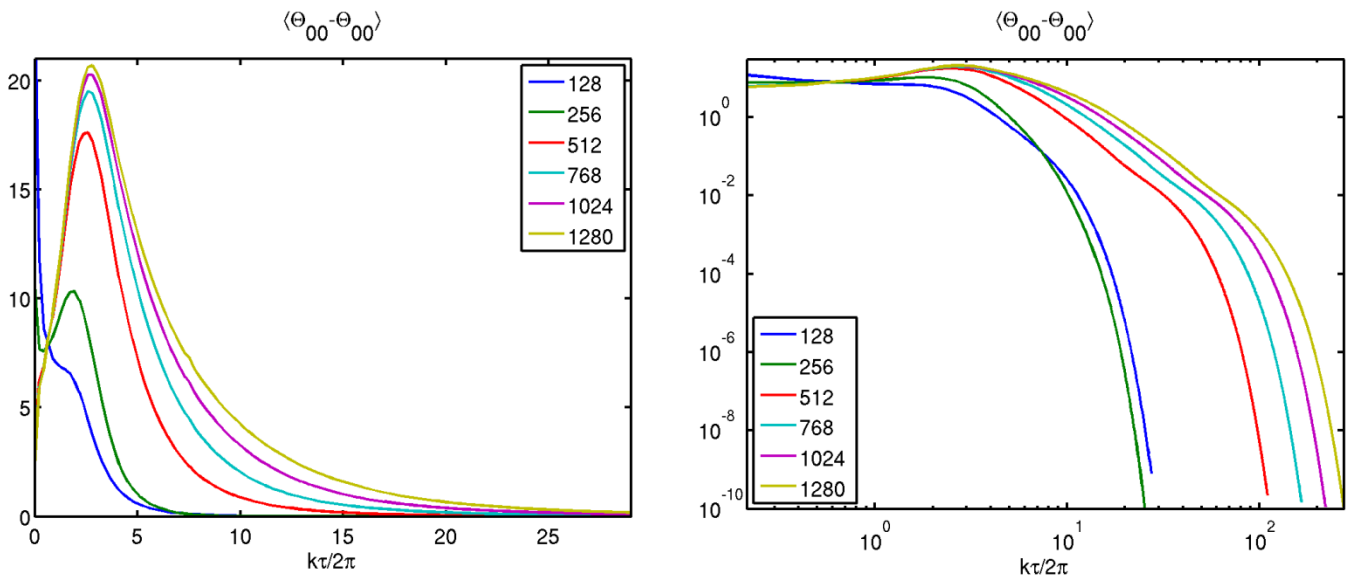


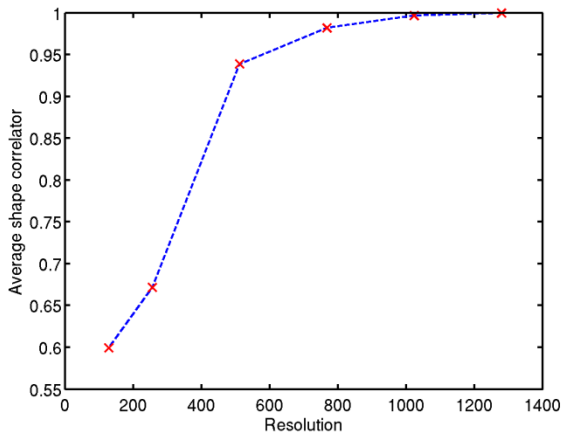
Equal time correlators



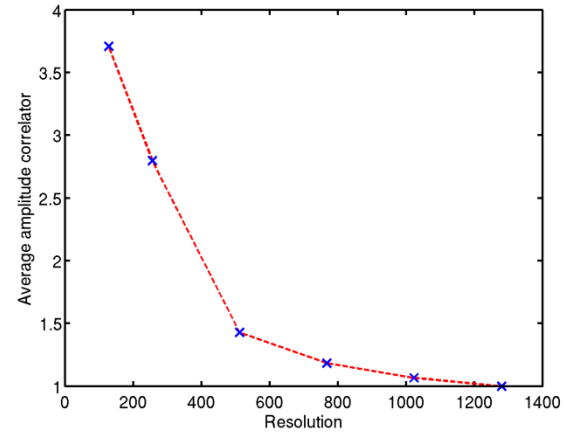
Convergence of UETCs in terms of the resolution of the grid

2D section of the $\langle\Theta_{00}\Theta_{00}\rangle$ UETC for the different resolutions





Comparison between different resolutions and 1280^3



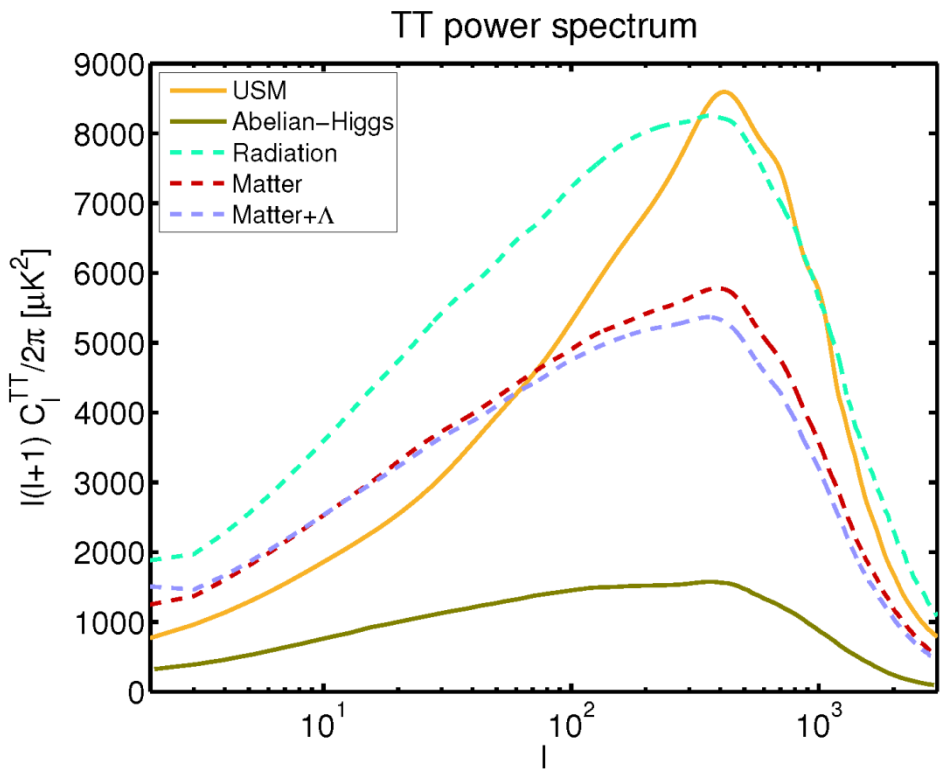
Shape correlator

Amplitude correlator

$$s_{A-B} = \frac{\sum_i \sum_j U^{sim1}(i,j) U^{sim2}(i,j)}{\sqrt{\sum_{i,j} (U^{sim1}(i,j))^2} \sqrt{\sum_{i,j} (U^{sim2}(i,j))^2}}$$

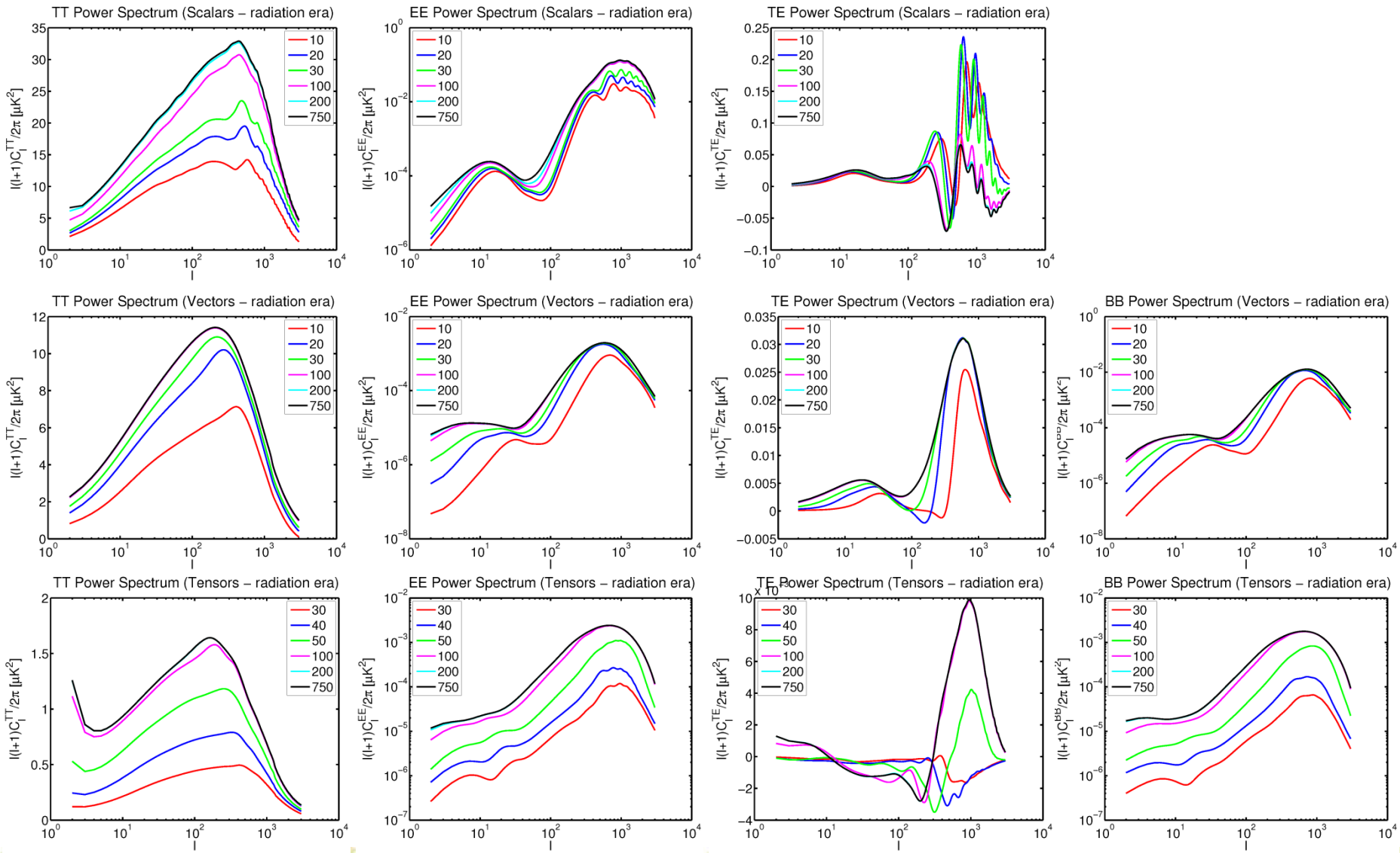
$$r_{A-B} = \frac{\sqrt{\sum_{i,j} (U^{sim1}(i,j))^2}}{\sqrt{\sum_{i,j} (U^{sim2}(i,j))^2}}$$

Cosmic String TT power spectrum



Predictions obtained from each of the 3 simulations compared to the USM and Abelian-Higgs results

Convergence of the power spectra in terms of the number of eigenvectors used (radiation era, $G\mu = 1.5\times10^{-7}$)



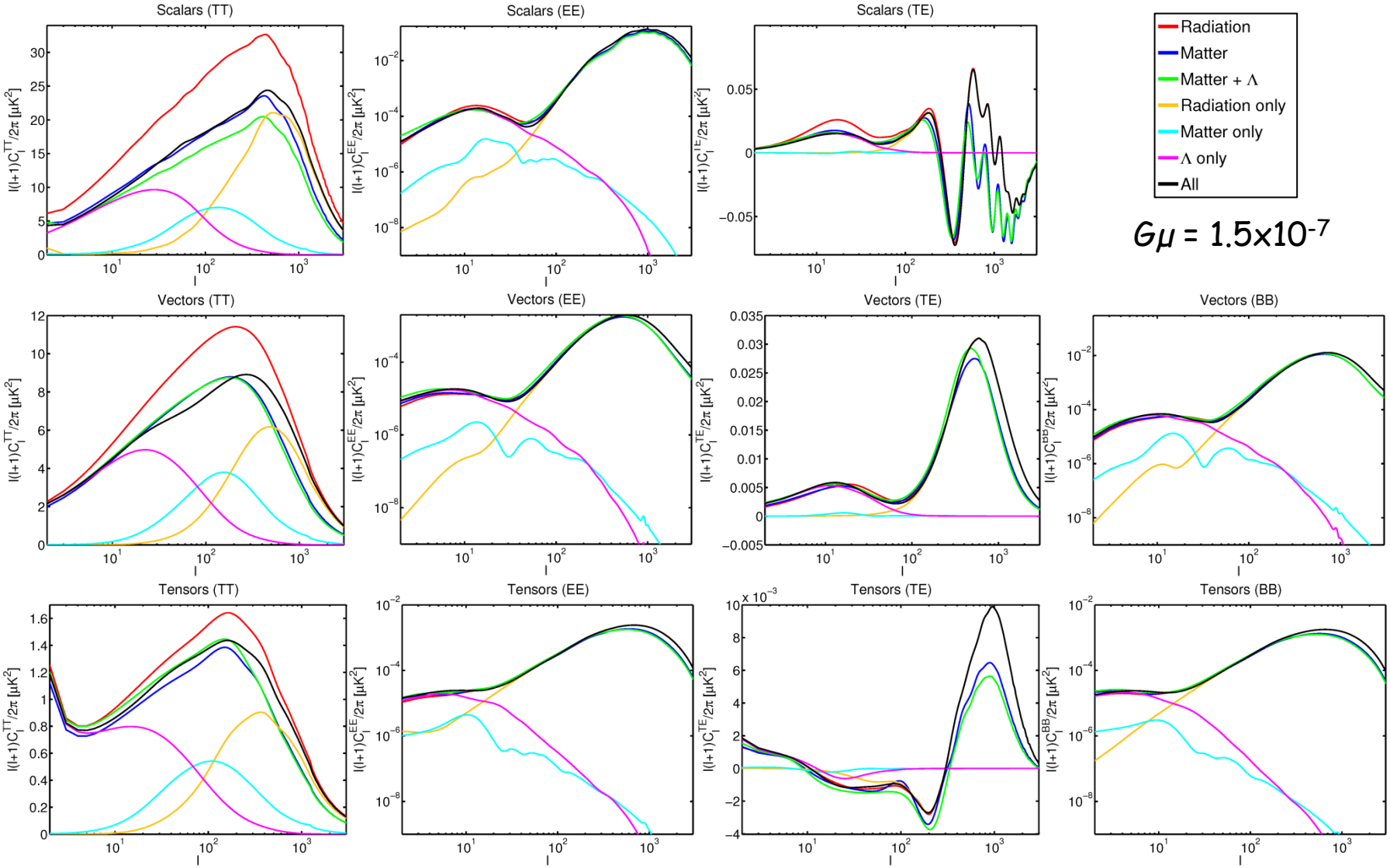
UNIVERSITY OF CAMBRIDGE

Combining the 3 simulations

- Each simulation is still considered separately
- For every simulation, each eigenvector is modified as follows:

$$\Theta(\mathbf{k},\tau) \rightarrow \frac{v(k\tau)}{\sqrt{\tau}} \rightarrow \begin{cases} \dfrac{v(k\tau)}{\sqrt{\tau}} & \text{if } \tau \,\epsilon \text{ time range used for UETC computation} \\ 0 & \text{else} \end{cases}$$

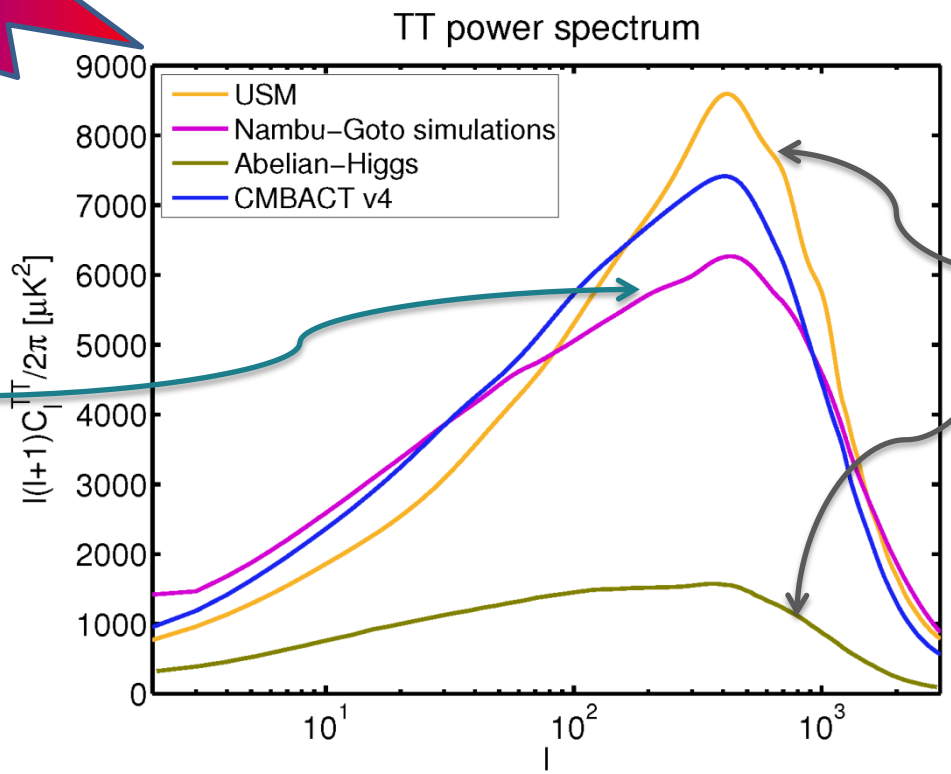
- C_ls are calculated for each of this modified eigenvectors and eventually are results are summed up
- Resulting power spectrum is smooth (see next slide)
- Procedure is approximate, but errors are small; however, it cannot be generalised safely to many smaller simulations, as errors will be increased



$G\mu = 1.5\times10^{-7}$

Final power spectrum

Result obtained from the simultations



Planck Collaboration, Paper XXV arXiv:1303.5085

Comparison between the cosmic string power spectra obtained the Nambu-Goto simulations, the standard USM and Abelian-Higgs methods and the result from the revised CMBACT code (default parameters); $G\mu/c^2 = 2.04\times10^{-6}$ in all cases.

Constraints on cosmic strings

Constraints on the maximum values of the string parameters obtained with COSMOMC using the best fit power spectrum at 95% confidence level with the 6 standard parameters ($\Omega_b h^2$, $\Omega_c h^2$, τ, θ, A_s, n_s) and strings , through parameter f_{10} (which represents the fractional power spectrum due to strings at l=10).

2 scenarios

Planck + WMAP polarization

Planck + WMAP polarization + BICEP2

Additional parameters added:

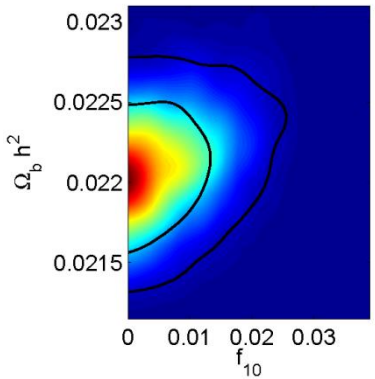
- ✓ N_{eff}
- ✓ running of the spectral index
- ✓ tensor modes (r)
- ✓ sterile neutrinos

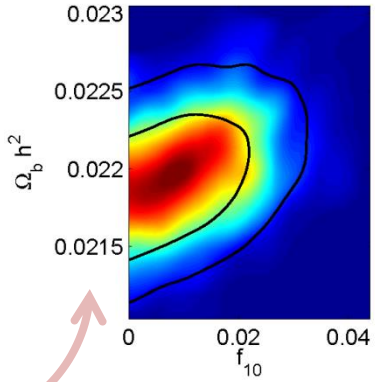
Additional likelihoods added:

- ✓ SPT/ACT
- ✓ Baryon acoustic oscillations (BAO)

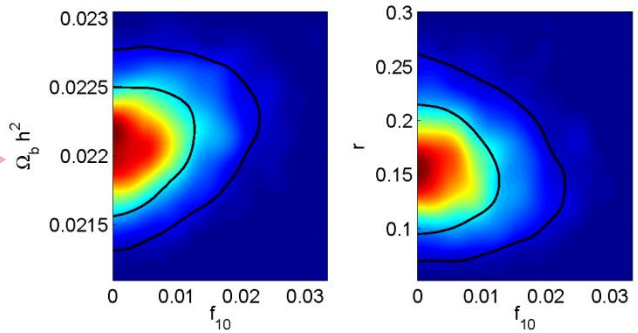
Constraints at 2σ level

Standard ΛCDM 6-parameter model	
Planck + WP	Planck + WP + BICEP2
$G\mu/c^2 < 1.49 \times 10^{-7}$	$G\mu/c^2 < 1.74 \times 10^{-7}$





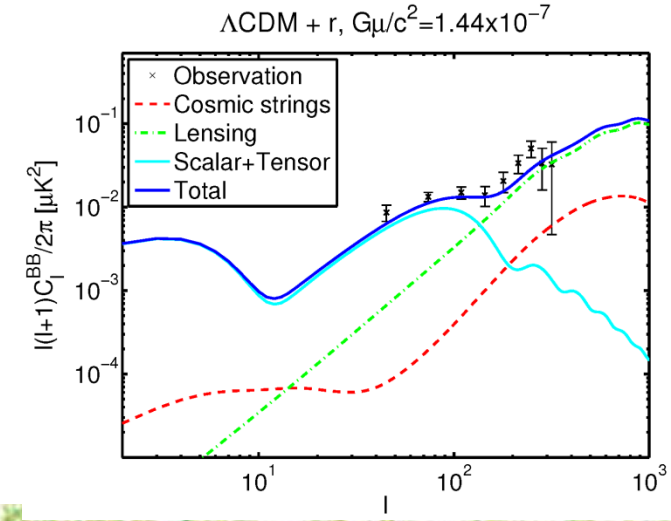
For BICEP2 need tensor modes to fit BB power spectrum



degeneracy between strings and baryons due to BB signal

degeneracy disappears after introducing tensor modes, but string contribution is reduced

$G\mu/c^2 < 1.44 \times 10^{-7}$



Degeneracies beteen cosmic strings and other cosmological parameters

Most interesting scenarios are the ones involving N_{eff} as degeneracies are very important

Cosmic string contribution is increased

Value of N_{eff} and H_0 are very big

Added additional likelihoods to try to solve the problem: HighL (SPT/ACT), BAO

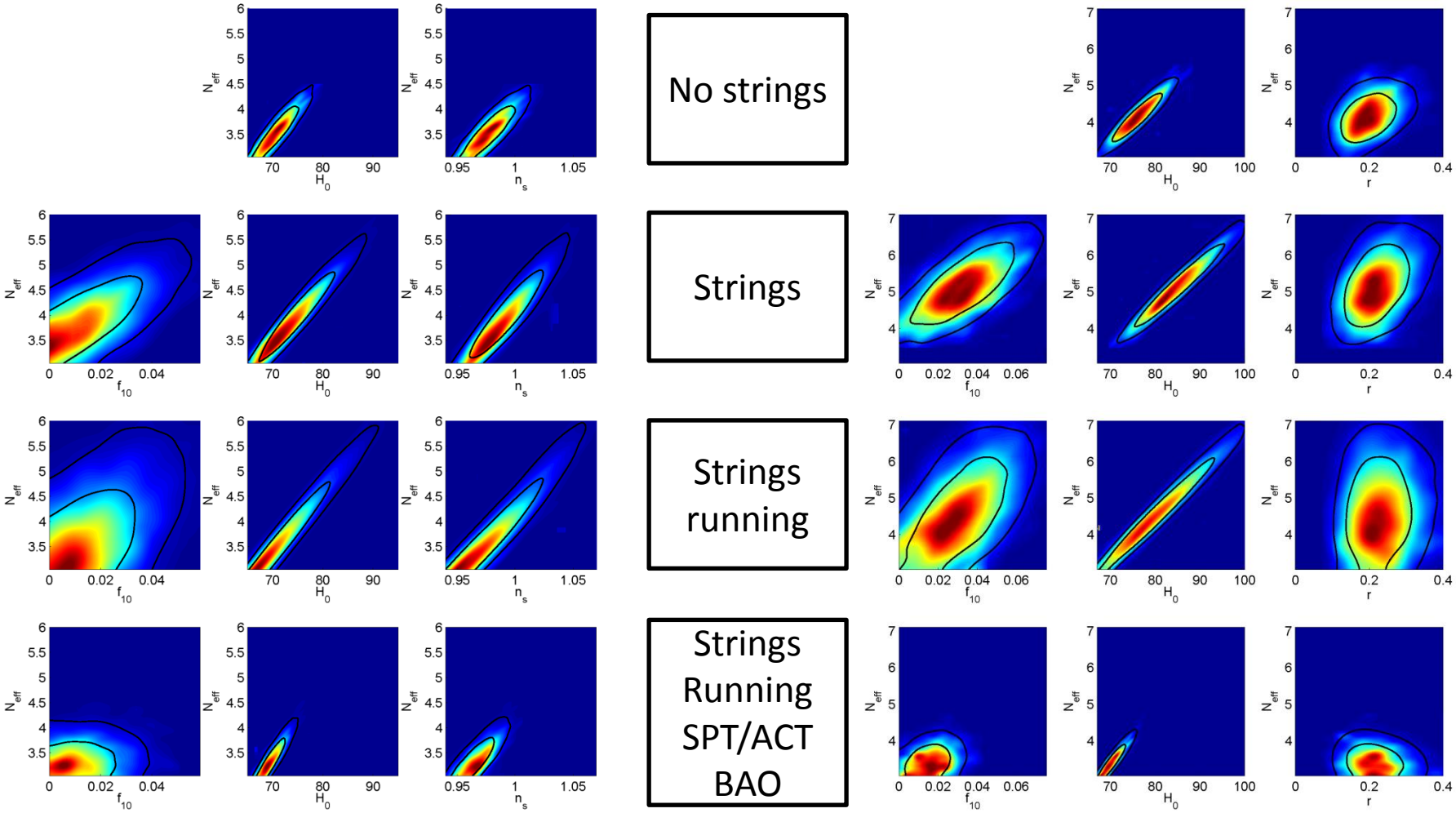
Problem solved by BAO

Next slide: illustration of this with N_{eff} and running

- $N_{eff} > 4$
- $H_0 > 75$ (Planck case)
- $H_0 > 85$ (Planck + BICEP2 case)

Planck + WMAP polarization + N_{eff}

Planck + WMAP polarization + BICEP2 + r + N_{eff}



Parameters used	Constraint on $G\mu/c^2$ (2σ)	
ΛCDM + strings +	**Planck**	**Planck + HighL + BAO**
-	1.49×10^{-7}	1.25×10^{-7}
n_{run}	1.88×10^{-7}	
r	1.42×10^{-7}	1.19×10^{-7}
n_{run}, r	1.99×10^{-7}	
N_{eff}	2.28×10^{-7}	1.58×10^{-7}
N_{eff}, r	2.49×10^{-7}	1.56×10^{-7}
N_{eff}, n_{run}	2.28×10^{-7}	1.95×10^{-7}
N_{eff}, $m_\nu^{sterile}$	2.36×10^{-7}	
N_{eff}, r, $m_\nu^{sterile}$	2.57×10^{-7}	

Planck, WMAP polarization

Planck, WMAP polarization + BICEP2

Parameters used	Constraint on $G\mu/c^2$ (2σ)	
ΛCDM + strings +	**Planck + BICEP2**	**Planck + BICEP2 + HighL + BAO**
-	1.74×10^{-7}	1.44×10^{-7}
n_{run}	2.25×10^{-7}	
r	1.44×10^{-7}	1.20×10^{-7}
n_{run}, r	2.07×10^{-7}	1.88×10^{-7}
N_{eff}, r	2.72×10^{-7}	1.70×10^{-7}
N_{eff}, r, n_{run}	2.65×10^{-7}	1.88×10^{-7}
N_{eff}, $m_\nu^{sterile}$	2.99×10^{-7}	
N_{eff}, r, $m_\nu^{sterile}$	2.85×10^{-7}	

Summary

- ✓ Unequal-time correlators (UETCs) obtained from 3 high-resolution Nambu-Goto simulations
- ✓ Cosmic strings power spectrum obtained from the UETCs from each simulation
- ✓ Overall cosmic string power spectrum computed by combining the simulations
- ✓ Temperature power spectrum situated between the standard Abelian-Higgs and USM results
- ✓ Constraints on $G\mu/c^2$ obtained with COSMOMC in different inflationary scenarios using Planck and BICEP2 likelihoods (and WMAP polarization), with different non-minimal parameters: tensor modes, running, additional degrees of freedom, sterile neutrinos

Acknowledgements

I would like to thank the following organizations for financial support for this conference:

Department of Applied Mathematics and Theoretical Physics (DAMTP)

UNIVERSITY OF CAMBRIDGE

Gonville & Caius College, Cambridge