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DECONSTRUCTING DIMENSIONS AND MASSIVE GRAVITY

COSMO 2014

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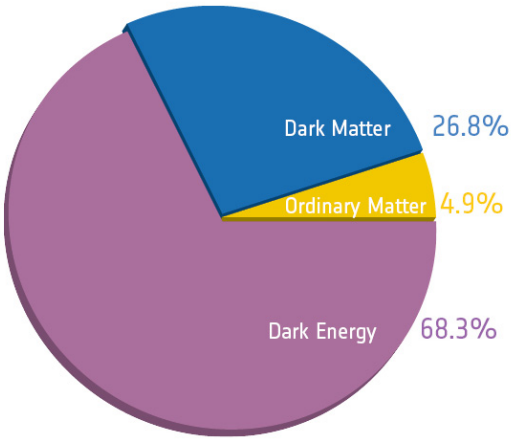
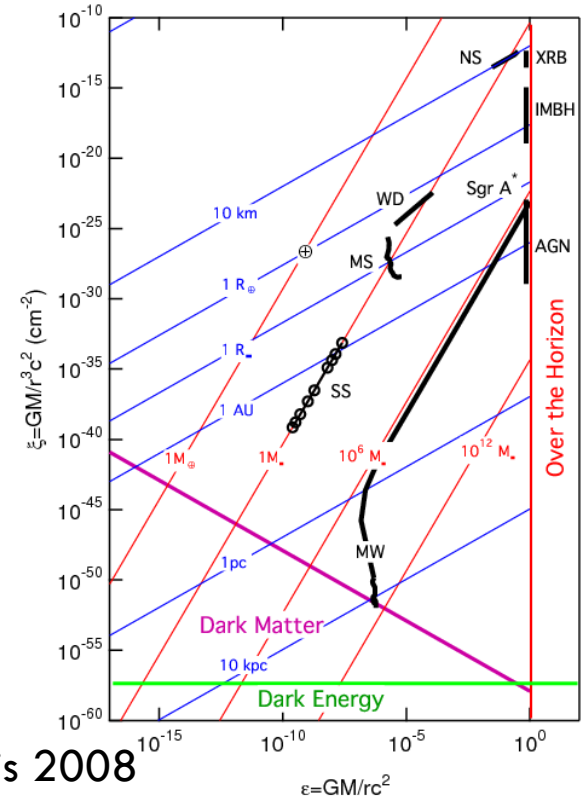
Why modify gravity (in the IR)?

Dark Matter and/or Dark Energy



Test Gravity on Large Scales



Psaltis 2008

Cosmological Constant Problem(s)

Λ

Massive Gravity

$$S = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4 x \, \left(R - \boxed{\frac{m^2}{4} \mathcal{U}(g, f)} \right)$$

Modify gravity by adding a mass term

External "reference metric" needed to make nontrivial potential. Breaks diff invariance. $f_{\mu\nu}$

Can address the Cosmological Constant Problem.

m can be small in a natural way.

$$\Lambda \to m$$

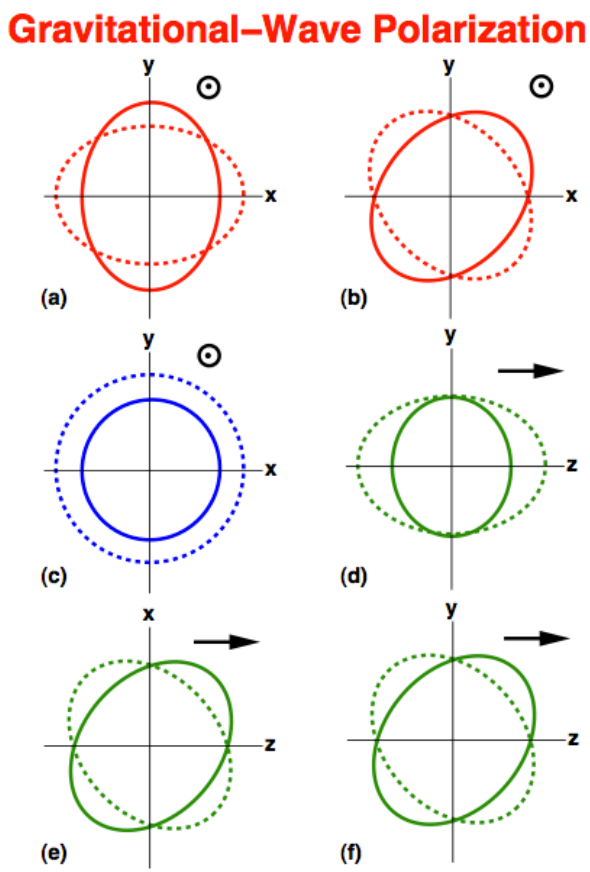
Degrees of Freedom of a Massive Graviton

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i \mathrm{d}x^j$$

Tensors

Scalars

Vectors



Will 2006

Degrees of Freedom of a Massive Graviton

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Tensors

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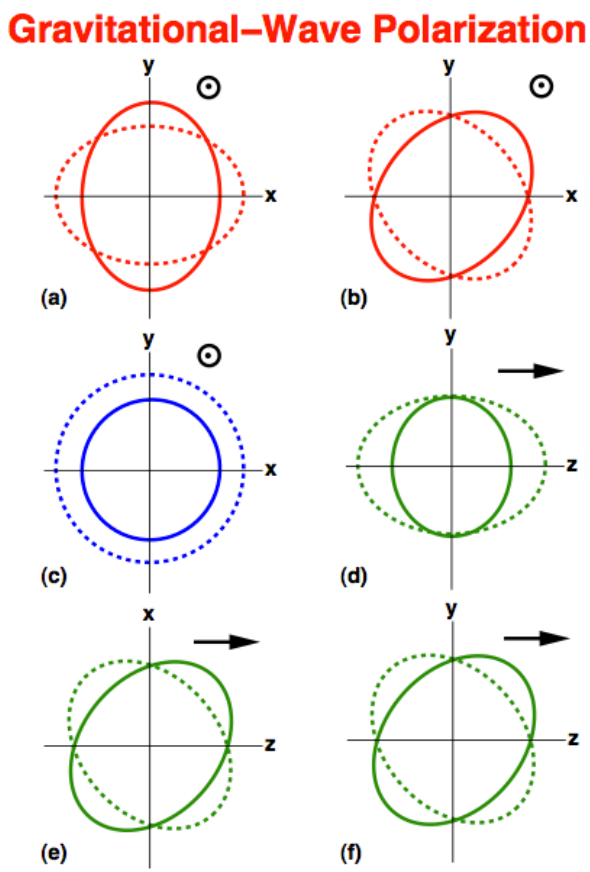


Will 2006

A healthy massive spin 2 particle has 5 polarizations

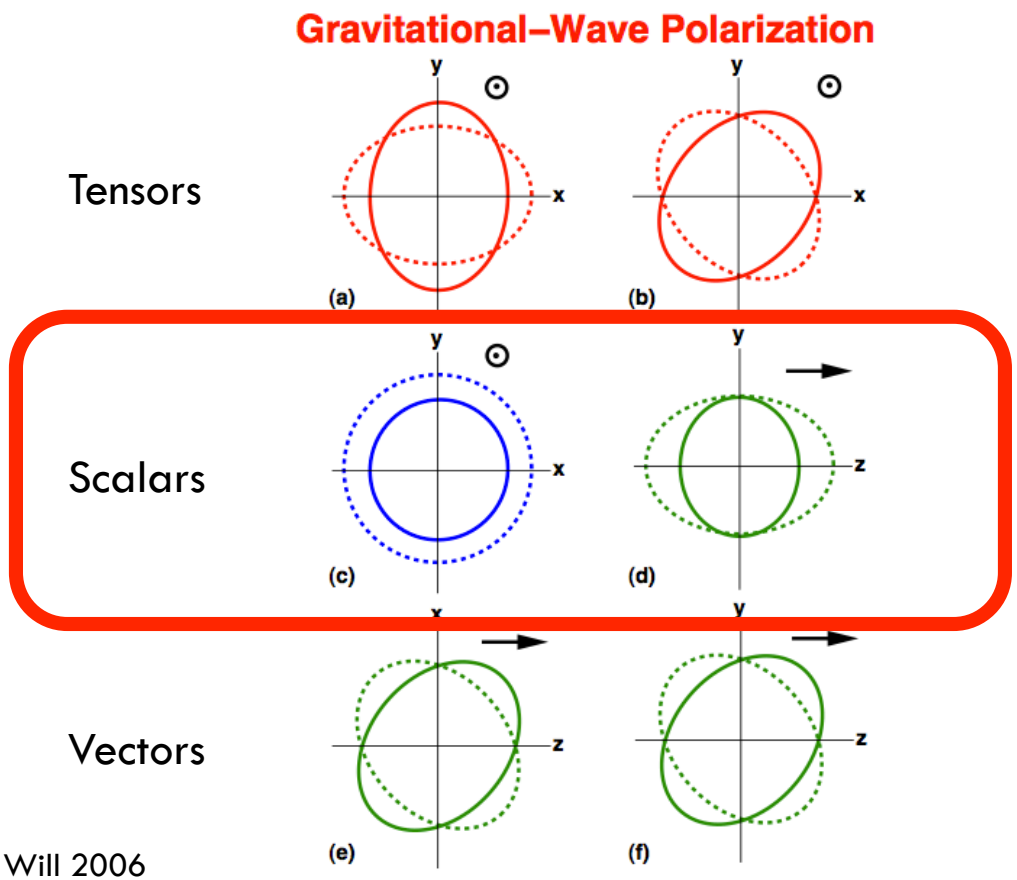
$$2s + 1 = 5$$

when

$$s = 2$$

Degrees of Freedom of a Massive Graviton

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i \mathrm{d}x^j$$



Will 2006

Two possible scalar modes:

-One is healthy, but must be screened via the Vainshtein Mechanism

Vainshtein, 1972

- The other is a ghost (negative kinetic energy) if it appears

Fierz and Pauli, 1939
Boulware and Deser 1972

dRGT Massive Gravity

de Rham et al, 2010

$$S = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left(R - \frac{m^2}{4} \mathcal{U}(g, f) \right)$$

$$\mathcal{U}(g) = \sum_{n=2}^{4} \alpha_n \mathcal{U}_n(g)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}^2][\mathcal{K}] + 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f}\right)^\mu_\nu$$

dRGT Massive Gravity

de Rham et al, 2010

$$S = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4x\sqrt{-g}\left(R - \frac{m^2}{4}\mathcal{U}(g,f)\right)$$

Very special structure chosen to avoid Boulware-Deser ghost

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]$$

Can we get a physical picture of why dRGT works?

de Rham, AM, Tolley, 1308.4136

Multi-gravity

Extension of massive gravity: Reference Metric becomes dynamical

$$S = \int \mathrm{d}^4x \frac{M_g^2}{2}\sqrt{-g}R[g] + \frac{M_f^2}{2}\sqrt{-f}R[f] - \frac{m^2 M_g^2}{4}\mathcal{U}(g,f)$$

Hassan and Rosen, 2011
Hinterbichler and Rosen, 2012

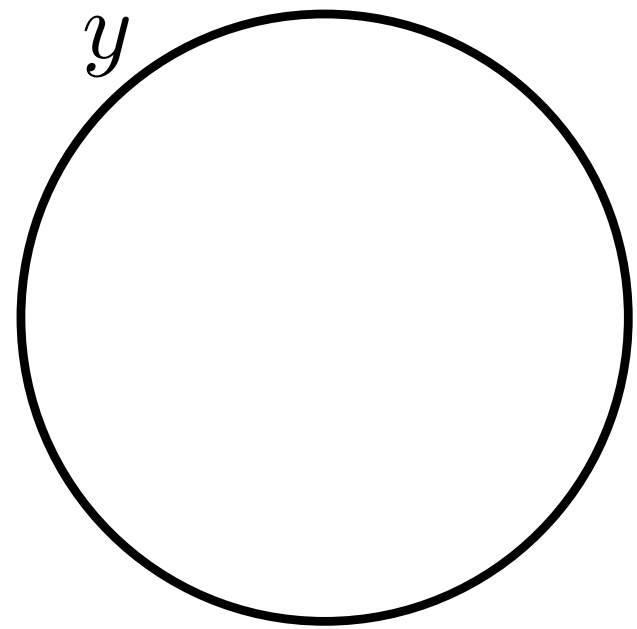
Can have arbitrary number of metrics

$$S = \int \mathrm{d}^4x \sum_I \frac{M_I^2}{2}\sqrt{-g_I}R[g_I] - \frac{m^2 M_1^2}{4}\mathcal{U}[g_I]$$

Kaluza-Klein Compactification

Consistent theories of massive gravitons arise in extra dimensional setups: Kaluza-Klein, DGP.

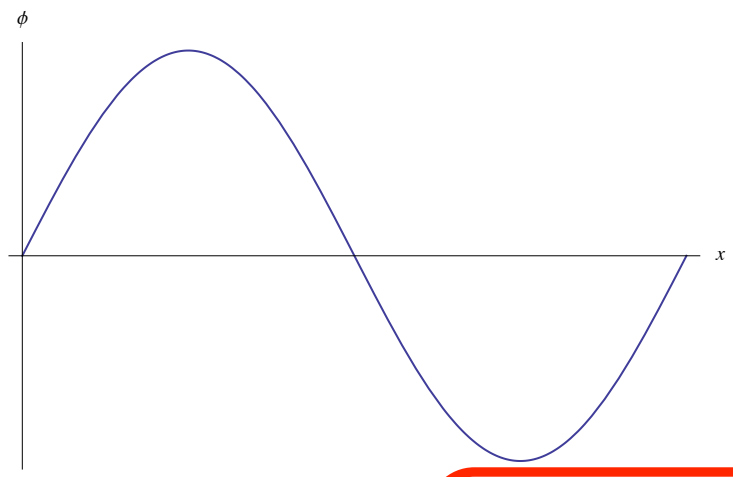
5D General Relativity has 5 propagating degrees of freedom.



Coordinates: x^μ 4 large dimensions we observe

y 1 compact extra dimension

Kaluza-Klein Compactification



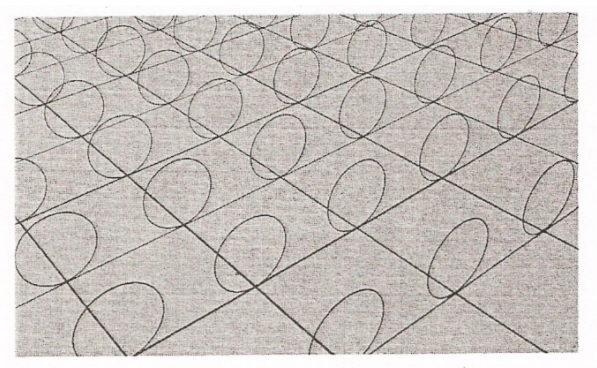
$$k_n = \frac{2\pi n}{L}$$

$$(\partial_y \phi)^2 \to -k_n^2 \phi_n^2$$

We interpret the wavenumber along the y direction as a mass in 4d

$$m_n = \frac{2\pi n}{L}$$

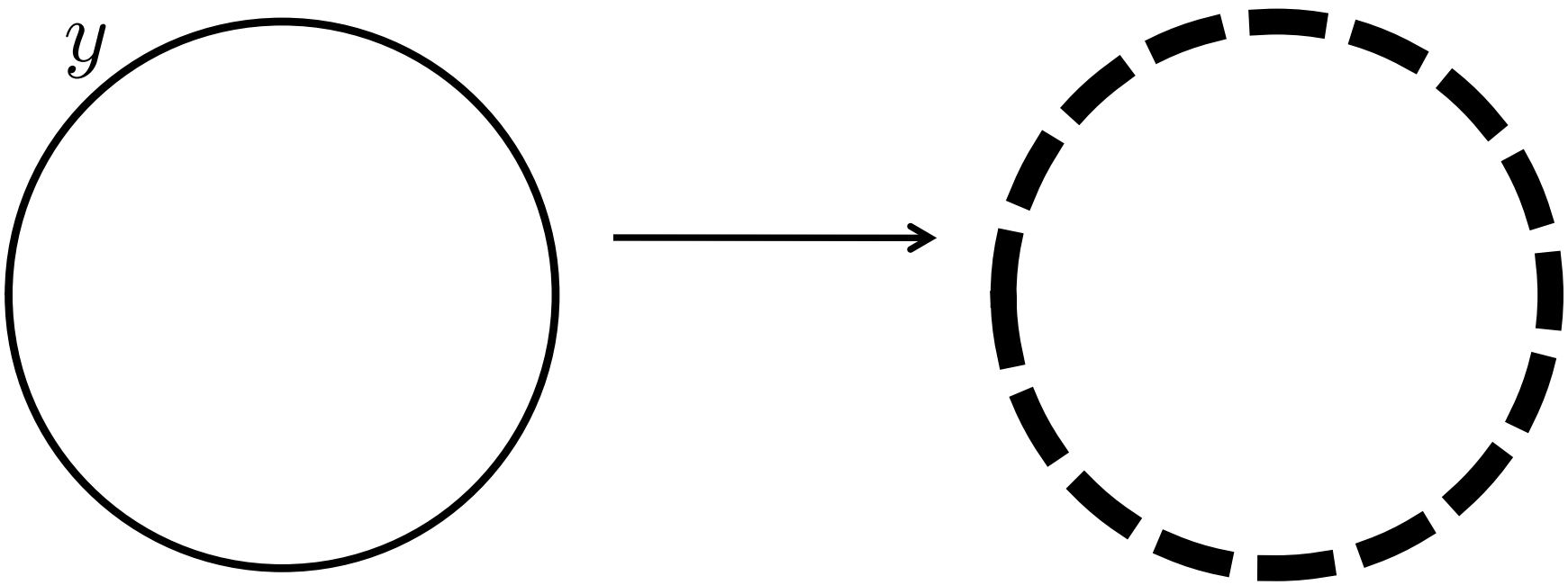
Infinite tower of massive modes



$$G^{(5)}_{AB} \longrightarrow \begin{cases} \vdots \\ m = 2\dfrac{2\pi}{L} & H^{(2)}_{\mu\nu} \\ m = \dfrac{2\pi}{L} & H^{(1)}_{\mu\nu} \\ m = 0 & g_{\mu\nu} \quad A_\mu \quad \phi \end{cases}$$

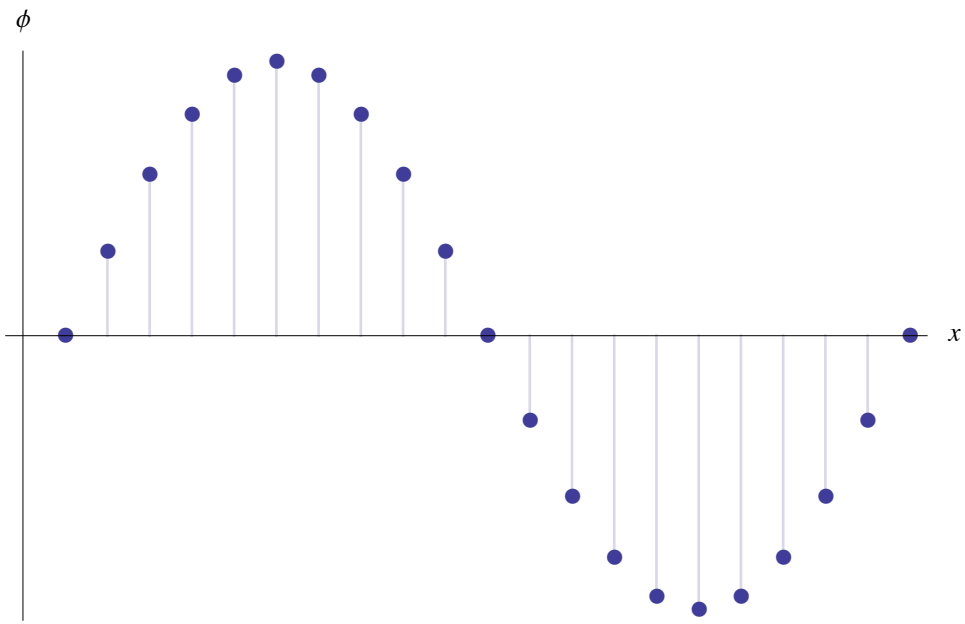
Consistent theory of an **infinite number** of massive gravitons

Dimensional Deconstruction



Arkhani-Hamed et al, 2001, 2002

Dimensional Deconstruction: Intuition



Introduce new scale:
Lattice spacing m^{-1}

$$N = mL$$

Tower is truncated

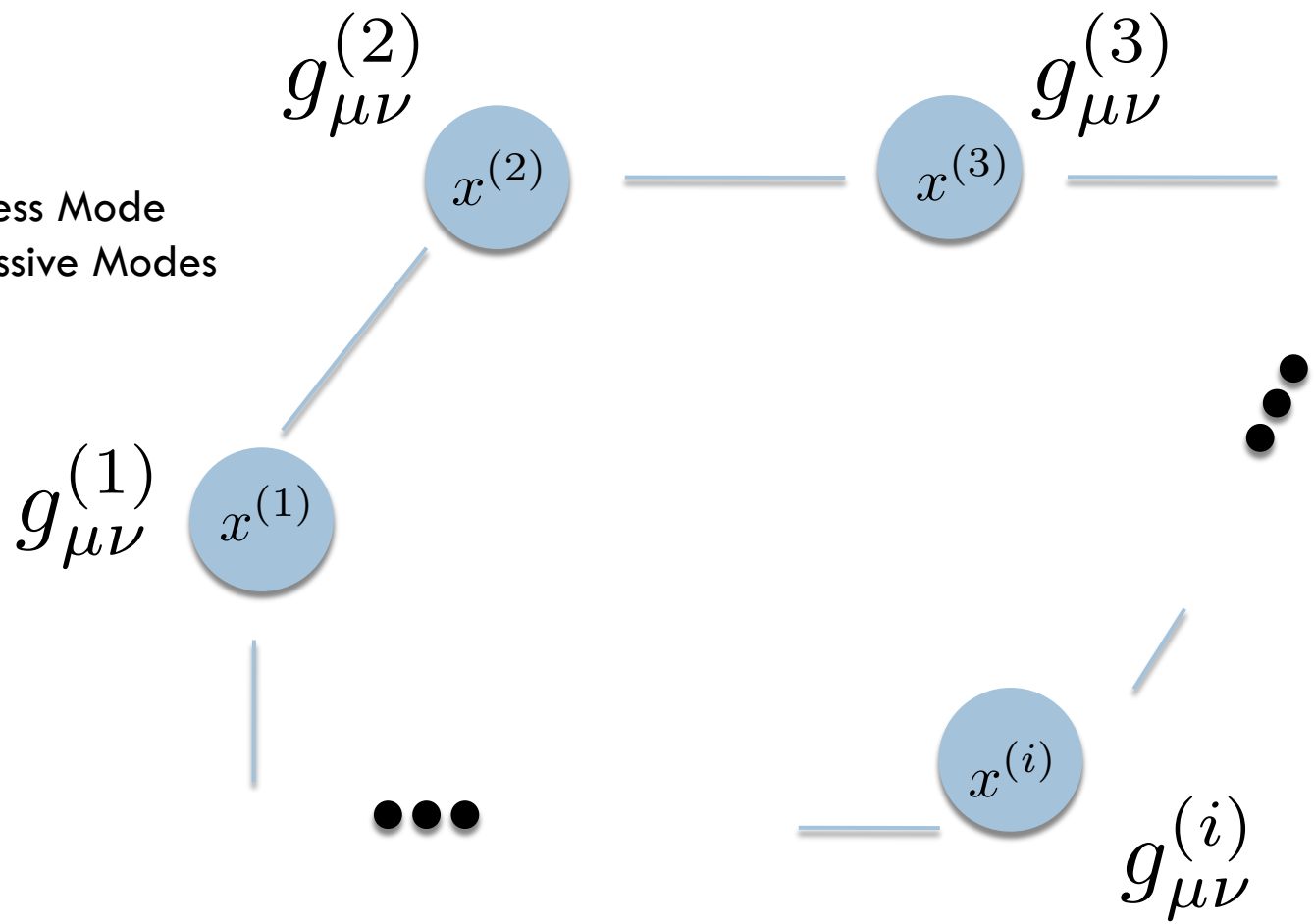
$$m_n = m \sin\left(\frac{n}{N}\right)$$

$$0 \leq n < N$$

Arkhani-Hamed et al, 2001, 2002

Deconstruction and Gravity

N Sites:
1 Massless Mode
N-1 Massive Modes



Can we do this while preserving the number of degrees of freedom?

Discretizing

Integrals become sums

$$\int \mathrm{d}y f(x,y) \to \frac{1}{m}\sum_i f_i(x)$$

Derivatives need to be handled with care

$$\partial_y f(x,y) \to m\left(f_{i+1}(x) - f_i(x)\right)$$

Fix *y* diffs before discretizing

$$N_y$$

Linear Level

Straightforward at the linear level

Fix gauge $\quad h_{\mu y} = 0, \quad h_{yy} = 1$

Discretize $\quad \partial_y h_{\mu\nu}(x,y) \to m^2 \left(h_{\mu\nu}^{(i+1)}(x) - h_{\mu\nu}^{(i)}(x) \right)$

$$S_{GR}^{5d} = \int \mathrm{d}^4x \mathrm{d}y - \underbrace{\frac{1}{4} h_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} h_{\rho\sigma}}_{\text{Linearized GR}} - \frac{1}{8} \left([(\partial_y h)^2] - [\partial_y h]^2 \right)$$

$$\mathcal{U}_{F.P.}[h] = m^2 \left([h]^2 - [h^2] \right)$$

However, need a non-linear theory for Vainshtein mechanism

Implementing Deconstruction

$$S_{GR,5d} = M_5^3 \int \mathrm{d}^4x \; \mathrm{d}y \; \sqrt{-g} \; N_y \left(R + [K]^2 - [K^2] \right)$$

Implementing Deconstruction

$$S_{GR,5d} = M_5^3 \int \mathrm{d}^4x \; \mathrm{d}y \; \sqrt{-g} \; N_y \left(R + \boxed{[K]^2 - [K^2]} \right)$$

The potential will come from here after discretizing

Implementing Deconstruction

$$S_{GR,5d} = M_5^3 \int \mathrm{d}^4x \, \mathrm{d}y \, \sqrt{-g} \, N_y \left(R + [K]^2 - [K^2]\right)$$

The potential will come from here after discretizing

Fix the gauge

$$N_5 = 1 \qquad N_\mu = 0$$

$$K_{\mu\nu} = \frac{1}{2}\partial_y g_{\mu\nu}$$

Implementing Deconstruction

$$S_{GR,5d} = M_5^3 \int \mathrm{d}^4x \, \mathrm{d}y \, \sqrt{-g} \, N_y \left(R + [K]^2 - [K^2]\right)$$

The potential will come from here after discretizing

Fix the gauge

$$N_5 = 1 \qquad N_\mu = 0$$

$$K_{\mu\nu} = \frac{1}{2}\partial_y g_{\mu\nu}$$

Discretization prescription?

$$K_{\mu\nu} \to ?$$

Most procedures will introduce new DOFs

Discretizing the metric directly will introduce the BD ghost

$$\partial_y g_{\mu\nu} \to m\left(g^{(2)}_{\mu\nu} - g^{(1)}_{\mu\nu}\right)$$

The potential is a function of the Fierz-Pauli combination: this is known to lead to a BD ghost

Discretizing using the Vielbein

de Rham, AM, Tolley, 1308.4136

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

First write derivatives of the metric in terms of the vielbeins

$$\partial_y g \sim e \partial_y e$$

Then discretize the derivative

$$\partial_y e^a_\mu \to m \left(e^{(2),a}_\mu - e^{(1),a}_\mu \right)$$

$$K_{\mu\nu} \to \mathcal{K}_{\mu\nu}$$

We recover the dRGT structure

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f} \right)^\mu_\nu$$

Discretizing using the Vielbein

Many features of multi-gravity theories arise naturally in this construction

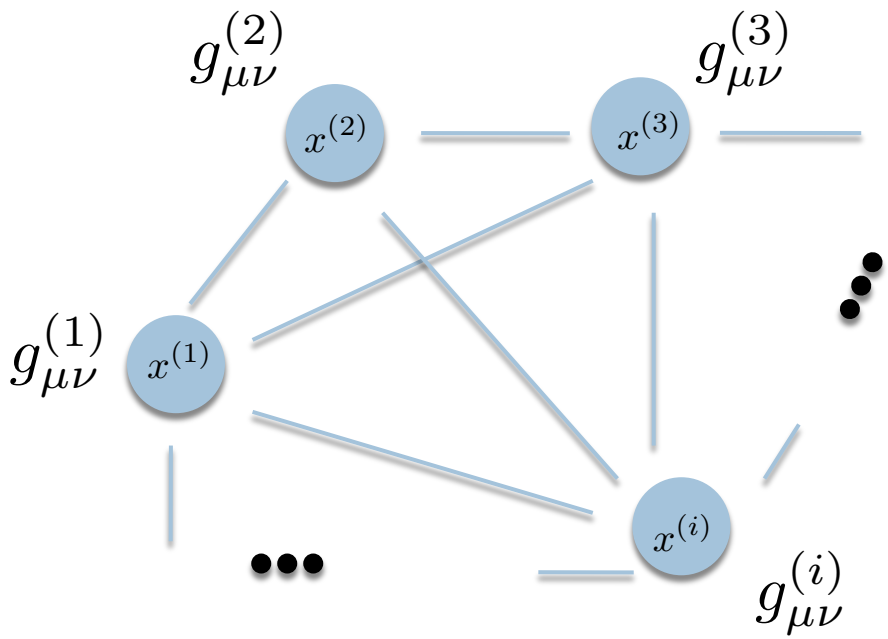
Symmetric Vielbein Condition

Strong Coupling Scale

Stuckleberg Fields

Discretizing using the Vielbein

Different discretizations of the derivative lead to different dRGT type interactions



Example:

$$\partial_y e^a_\mu(x,y) \to m\left(e^{(2),a}_\mu(x) - c e^{(1),a}_\mu(x)\right)$$

Scale of interactions

$$\Lambda_s \sim \left(\frac{M_5}{L}\right)^{1/2} \sim \left(M_4 m_1^2\right)^{1/3}$$

The scale that arises is the usual strong coupling scale for the lightest KK mode

Extensions

$$\partial_y \left[f(y) g(y) \right] = \partial_y f(y) g(y) + f(y) \partial_y g(y)$$

We can now apply this method in other cases

We find generically that other cases are more sensitive to discretization

Structures that are total derivatives in 5d (and thus safe) can become physical upon discretization

Gauss-Bonnet

We can apply Deconstruction to Gauss-Bonnet

$$S_{GB} = \alpha_{GB} \int \mathrm{d}^5 x \sqrt{-G} \left(\mathcal{R}^2_{\mu\nu\rho\sigma} - 4\mathcal{R}^2_{\mu\nu} + \mathcal{R}^2 \right)$$

5d 4d

$$\mathcal{R} \sim R + K^2$$

$$\mathcal{R}^2 \sim R^2 + \boxed{K^2 R} + K^4$$

New Kinetic Interaction?

Hinterbichler 2013

de Rham, AM, Tolley, 1311.6485

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5d 4d

$$\mathcal{R} \sim R + K^2$$

$$\mathcal{R}^2 \sim R^2 + \boxed{K^2 R} + K^4$$

The deconstructed theory has a ghost, can be seen in minisuperspace

New Kinetic Interaction?

Hinterbichler 2013

de Rham, AM, Tolley, 1311.6485

Gauss-Bonnet

We can apply Deconstruction to Gauss-Bonnet

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5d 4d

$$\mathcal{R} \sim R + K^2$$

$$\mathcal{R}^2 \sim R^2 + \boxed{K^2 R} + K^4$$

New Kinetic Interaction?

Hinterbichler 2013

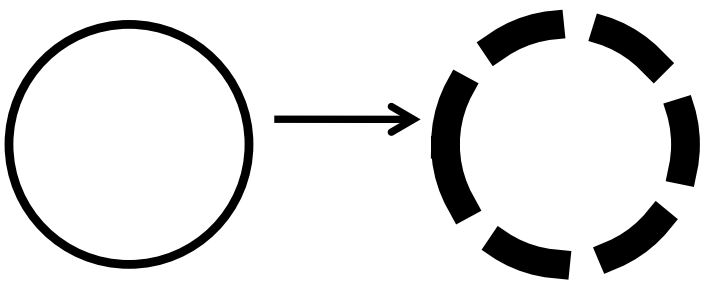
More general result: Only consistent kinetic term for Massive Gravity with flat reference metric in 4d is Einstein Hilbert

de Rham, AM, Tolley, 1311.6485

Charged Spin-2

Can try to keep the vector zero modes

$$\partial_\mu \tilde{h}_{n,\mu\nu} \to D_\mu \tilde{h}_{n,\mu\nu} = (\partial_\mu - iqnA_\mu)\,\tilde{h}_{n,\mu\nu}$$



$U(1)$ $\qquad$ Z_n

U(1) can be restored but this introduces new degrees of freedom

de Rham, AM, Ondo, Tolley, 1408.xxxx

Summary

Special structure of dRGT can be understood from a higher dimensional perspective

Higher derivative extensions are challenging because total derivative combinations can be broken

dRGT Massive Gravity: Vielbein

Special Structure avoids BD ghost

Hinterbichler and Rosen, 2012

$$\mathcal{U}(e,f) = \sum_{n=1}^{3} \beta_n \mathcal{U}_n(e,f)$$

$$\mathcal{U}_1(e,f) = \epsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge f^d$$

$$\mathcal{U}_2(e,f) = \epsilon_{abcd} e^a \wedge e^b \wedge f^c \wedge f^d$$

$$\mathcal{U}_3(e,f) = \epsilon_{abcd} e^a \wedge f^b \wedge f^c \wedge f^d$$

Vainshtein Mechanism

Non-linear interactions screen the helicity-0 mode

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{H_{\mu\nu}}{M_{\rm Pl}}$$

$$H_{\mu\nu} = h_{\mu\nu} + \frac{1}{m^2}\partial_\mu\partial_\nu\pi$$

Screening when $\partial^2\pi \gg m^2 M_{\rm Pl}$

Boulware-Deser ghost

Screening when $$\partial^2\pi \gg m^2 M_{\rm Pl}$$

$$\mathcal{U}(H) \to \mathcal{U}(\partial^2\pi)$$

Generically, 2nd derivative operators in the action will lead to EOMs that are higher order. This leads to a ghost, with mass

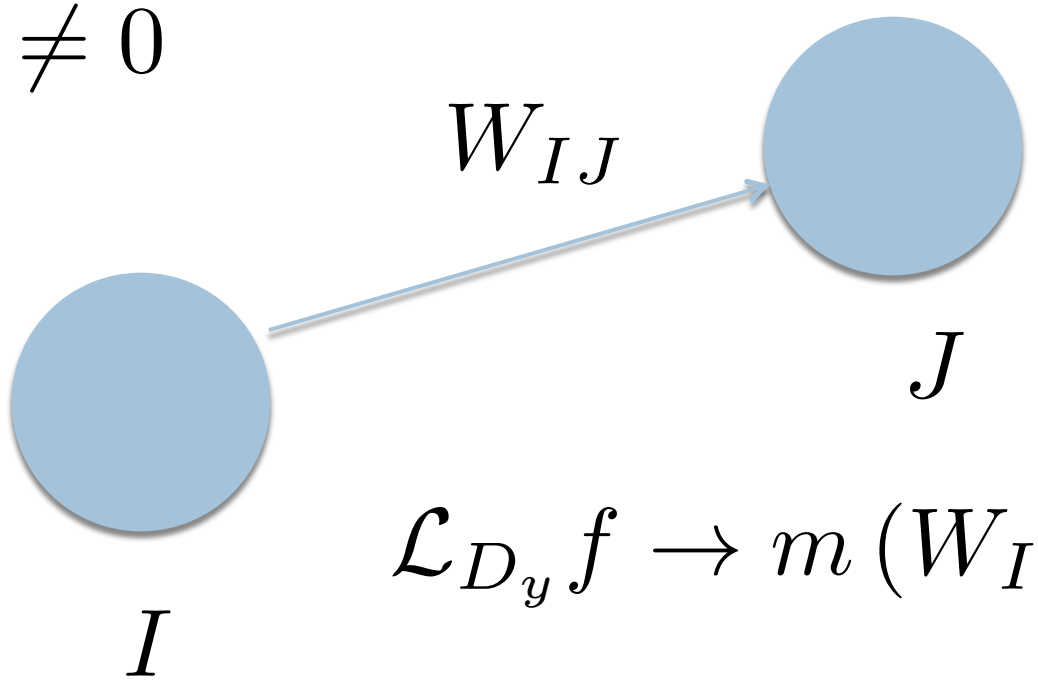
$$m^2_{\rm ghost} \sim \frac{M_{\rm Pl} m^4}{\partial^2\pi}$$

The mass of the ghost becomes small in the regime where the Vainshtein mechanism is active.

Wilson Lines

Can discretize in a gauge with the shifts, just get the Stuckleberg fields

$$N_\mu \neq 0$$



$$\mathcal{L}_{D_y} f \to m\left(W_{IJ} f_J - f_I\right)$$

Symmetric Vielbein Condition

Metric and Vielbein formalisms equivalent only when

$$e^{a,\mu}_{(1)} e^{b}_{(2),\mu} - e^{b,\mu}_{(1)} e^{a}_{(2),\mu} = 0$$

In Deconstruction this condition emerges in a natural way as a gauge choice

$$\Omega^{ab}_{y} = \frac{1}{2}\left(e^{a,\mu}\partial_y e^{b}_{\mu} - e^{\mu b}\partial_y e^{a}_{\mu}\right) = 0$$

Origin of Low Strong Coupling Scale

$$[K^2] - [K]^2 = \frac{m^2}{4}\left([(\partial_y h)^2] - [\partial_y h]^2\right) + O(M_{\rm Pl}^{-1})$$

Kinetic term for π

$$h_{\mu\nu} \to h_{\mu\nu} + 2\partial_\mu\partial_\nu\pi$$

$$(\partial_y^2\partial_\mu\pi)\partial^\mu\pi$$

Canonically Normalize

$$\pi \to \frac{\pi}{\partial_y}$$

Leads to low strong coupling scale since the y derivatives can be small

Scale of interactions

$\Lambda = (m^2 M_{\rm Pl})^{1/3}$

$$\mathcal{L}_{\rm GR,5d} \supset \int {\rm d}y \; \partial_y h \; \partial^2 \pi$$

Discretized Version

$$\sum_I h_{I+1} \frac{(\partial^2 \pi_I)^2}{\Lambda^3} \sim \frac{N}{\Lambda^3} \sum_{n_1,n_2,n_3} \tilde{h}_{n_1} \partial^2 \tilde{\pi}_{n_2} \partial^2 \tilde{\pi}_{n_3}$$

Diagonalize mass matrix (fourier transform)

Canonically Normalize

$$\tilde{h}_n \to \frac{1}{\sqrt{N}} \tilde{h}_n$$

$$\tilde{\pi}_n \to \frac{1}{\sqrt{N}} \frac{N}{n} \tilde{\pi}_n$$

Identify the strong coupling scale

$$\Lambda_s \sim \frac{1}{\sqrt{N}} \Lambda$$