DECONSTRUCTING DIMENSIONS AND MASSIVE GRAVITY

Andrew Matas, Case Western Reserve University
Co-Authors: Claudia de Rham, Andrew Tolley

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Why modify gravity (in the IR)?

Dark Matter and/or Dark Energy

Test Gravity on Large Scales

Cosmological Constant Problem(s)

\( \Lambda \)

Psaltis 2008
Massive Gravity

\[ S = \frac{M_{Pl}^2}{2} \int d^4x \left( R - \frac{m^2}{4} U(g, f) \right) \]

Modify gravity by adding a mass term

External “reference metric” needed to make nontrivial potential. Breaks diff invariance.

Can address the Cosmological Constant Problem.

\( m \) can be small in a natural way.

\( \Lambda \rightarrow m \)
Degrees of Freedom of a Massive Graviton

\[ ds^2 = -dt^2 + (\delta_{ij} + h_{ij}) \, dx^i dx^j \]

Gravitational–Wave Polarization

- Tensors
- Scalars
- Vectors

Will 2006
Degrees of Freedom of a Massive Graviton

\[ ds^2 = -dt^2 + (\delta_{ij} + h_{ij}) \, dx^i dx^j \]

Tensors

Scalars

Vectors

Gravitational–Wave Polarization

A healthy massive spin 2 particle has 5 polarizations

\[ 2s + 1 = 5 \quad \text{when} \quad s = 2 \]

Will 2006
\[ ds^2 = -dt^2 + (\delta_{ij} + h_{ij}) \, dx^i dx^j \]

**Degrees of Freedom of a Massive Graviton**

Two possible scalar modes:

- One is healthy, but must be screened via the Vainshtein Mechanism

  Vainshtein, 1972

- The other is a ghost (negative kinetic energy) if it appears

  Fierz and Pauli, 1939
  Boulware and Deser 1972
dRGT Massive Gravity

de Rham et al, 2010

\[
S = \frac{M_{Pl}^2}{2} \int d^4 x \sqrt{-g} \left( R - \frac{m^2}{4} \mathcal{U}(g, f) \right)
\]

\[
\mathcal{U}(g) = \sum_{n=2} \alpha_n \mathcal{U}_n(g)
\]

\[
\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]
\]

\[
\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}^2][\mathcal{K}] + 2[\mathcal{K}^3]
\]

\[
\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]
\]

\[
\mathcal{K}_\nu^\mu = \delta_\nu^\mu - \left( \sqrt{g^{-1} f} \right)_\nu^\mu
\]
dRGT Massive Gravity

de Rham et al, 2010

\[ S = \frac{M_{Pl}^2}{2} \int d^4 x \sqrt{-g} \left( R - \frac{m^2}{4} U(g, f) \right) \]

Very special structure chosen to avoid Boulware-Deser ghost

\[
U_3 = [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 2[\mathcal{K}^4]
\]

\[
U_4 = [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]
\]

Can we get a physical picture of why dRGT works?

de Rham, AM, Tolley, 1308.4136
Multi-gravity

Extension of massive gravity: Reference Metric becomes dynamical

\[ S = \int d^4 x \left( \frac{M^2_g}{2} \sqrt{-g} R[g] + \frac{M^2_f}{2} \sqrt{-f} R[f] - \frac{m^2 M^2}{4} \mathcal{U}(g, f) \right) \]

Hassan and Rosen, 2011
Hinterbichler and Rosen, 2012

Can have arbitrary number of metrics

\[ S = \int d^4 x \sum_I \left( \frac{M^2_I}{2} \sqrt{-g_I} R[g_I] - \frac{m^2 M^2}{4} \mathcal{U}[g_I] \right) \]
Kaluza-Klein Compactification

Consistent theories of massive gravitons arise in extra dimensional setups: Kaluza-Klein, DGP.

5D General Relativity has 5 propagating degrees of freedom.

Coordinates:

$\begin{align*}
\left\{ x^\mu \right\} & : 4 \text{ large dimensions we observe} \\
y & : 1 \text{ compact extra dimension}
\end{align*}$
Kaluza-Klein Compactification

We interpret the wavenumber along the $y$ direction as a mass in 4d

$$k_n = \frac{2\pi n}{L}$$

$$(\partial_y \phi)^2 \rightarrow -k_n^2 \phi_n^2$$

$$m_n = \frac{2\pi n}{L}$$
Infinite tower of massive modes

Consistent theory of an infinite number of massive gravitons

\[ G_{AB}^{(5)} \]

\[ m = 2 \frac{2\pi}{L} \]

\[ m = \frac{2\pi}{L} \]

\[ m = 0 \]

\[ g_{\mu\nu} \]

\[ A_{\mu} \]

\[ \phi \]
Dimensional Deconstruction

$y$

Dimensional Deconstruction: Intuition

Introduce new scale:
Lattice spacing $m^{-1}$

$$N = mL$$

Tower is truncated

$$m_n = m \sin \left( \frac{n}{N} \right)$$

$0 \leq n < N$

Deconstruction and Gravity

$N$ Sites:
1 Massless Mode
$N-1$ Massive Modes

Can we do this while preserving the number of degrees of freedom?
Discretizing

Integrals become sums

\[ \int dy f(x, y) \rightarrow \frac{1}{m} \sum_i f_i(x) \]

Derivatives need to be handled with care

\[ \partial_y f(x, y) \rightarrow m \left( f_{i+1}(x) - f_i(x) \right) \]

Fix y diffs before discretizing

\[ N_y \]
Linear Level

Straightforward at the linear level

Fix gauge

\[ h_{\mu y} = 0, \quad h_{yy} = 1 \]

Discretize

\[ \partial_y h_{\mu \nu}(x, y) \rightarrow m^2 \left( h^{(i+1)}_{\mu \nu}(x) - h^{(i)}_{\mu \nu}(x) \right) \]

\[
S_{GR}^{5d} = \int d^4x dy - \frac{1}{4} h_{\mu \nu} \mathcal{E}^{\mu \nu \rho \sigma} h_{\rho \sigma} - \frac{1}{8} \left( [(\partial_y h)^2] - [\partial_y h]^2 \right)
\]

Linearized GR

\[ \mathcal{U}_{F.P.}[h] = m^2 \left( [h]^2 - [h^2] \right) \]

However, need a non-linear theory for Vainshtein mechanism
Implementing Deconstruction

\[ S_{GR,5d} = M_5^3 \int d^4x \, dy \, \sqrt{-g} \, N_y \left( R + [K]^2 - [K^2] \right) \]
Implementing Deconstruction

\[ S_{GR,5d} = M_5^3 \int d^4 x \ dy \ \sqrt{-g} \ N_y \ (R + [K]^2 - [K^2]) \]

The potential will come from here after discretizing.
Implementing Deconstruction

\[ S_{GR,5d} = M_5^3 \int d^4x \, dy \, \sqrt{-g} \, N_y (R + [K]^2 - [K^2]) \]

The potential will come from here after discretizing.

Fix the gauge

\[ N_5 = 1 \quad N_\mu = 0 \]

\[ K_{\mu\nu} = \frac{1}{2} \partial_y g_{\mu\nu} \]
Implementing Deconstruction

\[ S_{GR,5d} = M_5^3 \int d^4x \; dy \; \sqrt{-g} \; N_y \left( R + [K]^2 - [K^2] \right) \]

The potential will come from here after discretizing

Fix the gauge

\[ N_5 = 1 \quad N_\mu = 0 \]

\[ K_{\mu\nu} = \frac{1}{2} \partial_y g_{\mu\nu} \]

Discretization prescription?

\[ K_{\mu\nu} \rightarrow ? \]
Most procedures will introduce new DOFs

Discretizing the metric directly will introduce the BD ghost

\[ \partial_y g_{\mu\nu} \rightarrow m \left( g^{(2)}_{\mu\nu} - g^{(1)}_{\mu\nu} \right) \]

The potential is a function of the Fierz-Pauli combination: this is known to lead to a BD ghost
Discretizing using the Vielbein

de Rham, AM, Tolley, 1308.4136

\[ g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab} \]

First write derivatives of the metric in terms of the vielbeins

\[ \partial_y g \sim e \partial_y e \]

Then discretize the derivative

\[ \partial_y e^a_\mu \rightarrow m \left( e^{(2),a}_\mu - e^{(1),a}_\mu \right) \]

We recover the dRGT structure

\[ K^\mu_{\mu\nu} \rightarrow \mathcal{K}_{\mu\nu} \]

\[ \mathcal{K}_\nu^\mu = \delta^\mu_\nu - \left( \sqrt{g^{-1}f} \right)^\mu_\nu \]
Discretizing using the Vielbein

Many features of multi-gravity theories arise naturally in this construction

Symmetric Vielbein Condition

Strong Coupling Scale

Stuckelberg Fields
Discretizing using the Vielbein

Different discretizations of the derivative lead to different dRGT type interactions

Example:
\[ \partial_y e^a_{\mu}(x, y) \rightarrow m \left( e^{(2), a}_{\mu}(x) - c e^{(1), a}_{\mu}(x) \right) \]
The scale that arises is the usual strong coupling scale for the lightest KK mode.
Extensions

\[ \partial_y [f(y)g(y)] = \partial_y f(y)g(y) + f(y)\partial_y g(y) \]

We can now apply this method in other cases

We find generically that other cases are more sensitive to discretization

Structures that are total derivatives in 5d (and thus safe) can become physical upon discretization
Gauss-Bonnet

We can apply Deconstruction to Gauss-Bonnet

\[ S_{GB} = \alpha_{GB} \int d^5x \sqrt{-G} \left( \mathcal{R}_{\mu\nu\rho\sigma}^2 - 4\mathcal{R}_{\mu\nu}^2 + \mathcal{R}^2 \right) \]

\[ \mathcal{R} \sim R + K^2 \]

\[ \mathcal{R}^2 \sim R^2 + [K^2 R + K^4] \]

New Kinetic Interaction?

Hinterbichler 2013

de Rham, AM, Tolley, 1311.6485
Gauss-Bonnet

We can apply Deconstruction to Gauss-Bonnet

\[ S_{GB} = \alpha_{GB} \int d^5x \sqrt{-G} \left( R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2 \right) \]

5d 4d
\[ R \sim R + K^2 \]

\[ R^2 \sim R^2 + \boxed{K^2 R} + K^4 \]

New Kinetic Interaction?

The deconstructed theory has a ghost, can be seen in minisuperspace

Hinterbichler 2013

de Rham, AM, Tolley, 1311.6485
Gauss-Bonnet

We can apply Deconstruction to Gauss-Bonnet

\[ S_{GB} = \alpha_{GB} \int d^5 x \sqrt{-G} \left( R^2_{\mu\nu\rho\sigma} - 4 R^2_{\mu\nu} + R^2 \right) \]

\[ 5d \quad 4d \]

\[ R \sim R + K^2 \]

\[ R^2 \sim R^2 + \boxed{K^2 R} + K^4 \]

New Kinetic Interaction?

More general result: Only consistent kinetic term for Massive Gravity with flat reference metric in 4d is Einstein Hilbert

Hinterbichler 2013

de Rham, AM, Tolley, 1311.6485
Charged Spin-2

Can try to keep the vector zero modes

\[ \partial_\mu \tilde{h}_{\mu,\nu} \rightarrow D_\mu \tilde{h}_{\mu,\nu} = (\partial_\mu - i q n A_\mu) \tilde{h}_{\mu,\nu} \]

\[ U(1) \]

\[ Z_n \]

de Rham, AM, Ondo, Tolley, 1408.xxxx

\[ U(1) \] can be restored but this introduces new degrees of freedom
Summary

Special structure of dRGT can be understood from a higher dimensional perspective.

Higher derivative extensions are challenging because total derivative combinations can be broken.
dRGT Massive Gravity: Vielbein

Special Structure avoids BD ghost

\[ \mathcal{U}(e, f) = \sum_{n=1}^{3} \beta_n \mathcal{U}_n(e, f) \]

\[ \mathcal{U}_1(e, f) = \epsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge f^d \]

\[ \mathcal{U}_2(e, f) = \epsilon_{abcd} e^a \wedge e^b \wedge f^c \wedge f^d \]

\[ \mathcal{U}_3(e, f) = \epsilon_{abcd} e^a \wedge f^b \wedge f^c \wedge f^d \]

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Hinterbichler and Rosen, 2012
Vainshtein Mechanism

Non-linear interactions screen the helicity-0 mode

\[ g_{\mu\nu} = \eta_{\mu\nu} + \frac{H_{\mu\nu}}{M_{\text{Pl}}} \]

\[ H_{\mu\nu} = h_{\mu\nu} + \frac{1}{m^2} \partial_\mu \partial_\nu \pi \]

Screening when

\[ \partial^2 \pi \gg m^2 M_{\text{Pl}} \]
Boulware-Deser ghost

Screening when \[ \partial^2 \pi \gg m^2 M_{Pl} \]

\[ \mathcal{U}(H) \rightarrow \mathcal{U}(\partial^2 \pi) \]

Generically, 2\textsuperscript{nd} derivative operators in the action will lead to EOMs that are higher order. This leads to a ghost, with mass

\[ m_{\text{ghost}}^2 \sim \frac{M_{Pl} m^4}{\partial^2 \pi} \]

The mass of the ghost becomes small in the regime where the Vainshtein mechanism is active.
Wilson Lines

Can discretize in a gauge with the shifts, just get the Stuckelberg fields

\[ N_\mu \neq 0 \]

\[ \mathcal{L}_{D_y} f \rightarrow m (W_{IJ} f_J - f_I) \]
Symmetric Vielbein Condition

Metric and Vielbein formalisms equivalent only when

\[ e^{a,\mu}_{(1)} e^b_{(2),\mu} - e^b_{(1),\mu} e^{a}_{(2),\mu} = 0 \]

In Deconstruction this condition emerges in a natural way as a gauge choice

\[ \Omega^{ab}_{y} = \frac{1}{2} \left( e^{a,\mu}_{\mu} \partial_y e^b_{\mu} - e^{\mu b}_{\mu} \partial_y e^{a}_{\mu} \right) = 0 \]
Origin of Low Strong Coupling Scale

$$[K^2] - [K]^2 = \frac{m^2}{4} \left( \left[ (\partial_y h)^2 \right] - \left[ \partial_y h \right]^2 \right) + O(M_{Pl}^{-1})$$

Kinetic term for \( \pi \)

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + 2 \partial_{\mu} \partial_{\nu} \pi$$

Canonically Normalize

$$\pi \rightarrow \frac{\pi}{\partial_y}$$

Leads to low strong coupling scale since the \( y \) derivatives can be small
Scale of interactions

\[ \Lambda = \left( m^2 M_{P1} \right)^{1/3} \]

\[ \mathcal{L}_{GR,5d} \supset \int dy \, \partial_y h \ \partial^2 \pi \]

**Discretized Version**

\[ \sum_I h_{I+1} \frac{\left( \partial^2 \pi_I \right)^2}{\Lambda^3} \sim \frac{N}{\Lambda^3} \sum_{n_1,n_2,n_3} \tilde{h}_{n_1} \partial^2 \tilde{\pi}_{n_2} \partial^2 \tilde{\pi}_{n_3} \]

**Canonically Normalize**

\[ \tilde{h}_n \to \frac{1}{\sqrt{N}} \tilde{h}_n \]

\[ \tilde{\pi}_n \to \frac{1}{\sqrt{N}} \frac{N}{n} \tilde{\pi}_n \]

**Diagonalize mass matrix (fourier transform)**

**Identify the strong coupling scale**

\[ \Lambda_s \sim \frac{1}{\sqrt{N}} \Lambda \]