Gravitational Waves and the Scale of Inflation

Mehrdad Mirbabayi

with L. Senatore, E. Silverstein, M. Zaldarriaga

Institute for Advanced Study

COSMO2014, Chicago

Aug 29, 2014

Does a measurement of r uniquely fix $H_{\rm inf}$?

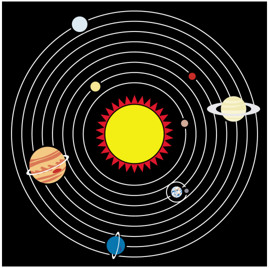
Does a measurement of r uniquely fix $H_{\rm inf}$?

Yes, if tensor modes are in vacuum during inflation:

$$\left\langle \gamma^{s}_{\mathbf{k}} \gamma^{s'}_{\mathbf{k}'} \right\rangle_{\rm vac} = (2\pi)^3 \delta^3(\mathbf{k}+\mathbf{k}') \delta^{ss'} \frac{1}{k^3} \frac{H_{\rm inf}^2}{M_{\rm pl}^2}$$

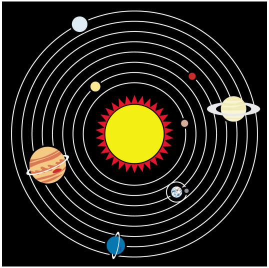
What if γ is not in vacuum?

Gravitational Waves in Solar System



Jupiter

Gravitational Waves in Solar System



Jupiter

Bremsstrahlung at center of Sun (Weinberg '65).

Examples of Tensor Emission During Inflation

1) Particle Production:

$$M_X^2 = M^2 \sin^2 \frac{\phi}{f}$$

Scattering of X particles emits gravity waves γ_X. Senatore et.al '11

2) Pseudo-scalar Inflaton:

$$\mathcal{L}_{\phi A} = \frac{\alpha}{f} \phi F \tilde{F}$$

Growing helical gauge field A excites the metric.

Sorbo '11, Barnaby et.al. '12, Mukohyama et.al. '14

Can γ_X be Larger than $\gamma_{\rm vac}$?

$\gamma_X > \gamma_{\rm vac}$?

1. Available energy density: $\frac{1}{2}\dot{\phi}^2 = M_{\rm pl}^2 H^2 \epsilon$
2. The energy in the auxiliary sector $\rho_X \ll M_{\rm pl}^2 H^2 \epsilon$
3. Estimate emission by $\partial^2\gamma \sim \rho_X / M_{\rm pl}^2$ at frequency $\omega \sim H$

$$\gamma_X \sim \frac{\rho_X}{M_{\rm pl}^2 H^2} \ll \epsilon$$

$\gamma_X > \gamma_{\rm vac}$?

1. Available energy density: $\frac{1}{2}\dot{\phi}^2 = M_{\rm pl}^2 H^2 \epsilon$
2. The energy in the auxiliary sector $\rho_X \ll M_{\rm pl}^2 H^2 \epsilon$
3. Estimate emission by $\partial^2 \gamma \sim \rho_X / M_{\rm pl}^2$ at frequency $\omega \sim H$

$$\gamma_X \sim \frac{\rho_X}{M_{\rm pl}^2 H^2} \ll \epsilon$$

There is a lot of room to outperform vacuum

$$\gamma_{\rm vac} \sim \frac{H}{M_{\rm pl}} \ll \gamma_X \ll \epsilon.$$

Punch Line

Scalars are emitted during energy transfer:

$$\frac{1}{2}\dot{\phi}^2 \to \rho_X \Longrightarrow \delta\phi_X$$

Punch Line

Scalars are emitted during energy transfer:

$$\frac{1}{2}\dot{\phi}^2 \to \rho_X \Longrightarrow \delta\phi_X$$

Large tensor emission **generically** leads to **Very Large** scalar emission

$$\gamma_X > \gamma_{\rm vac} \Longrightarrow \delta\phi_X \gg \delta\phi_{\rm vac}$$

$$r_{\rm max} \sim \epsilon^2$$

1. Exponential Expansion

Suppose emission is at a physical frequency $k/a \sim \omega_{\rm phys}$.

1.a. Then each mode can be excited in a period

$$\Delta t \sim H^{-1}$$

1. Exponential Expansion

Suppose emission is at a physical frequency $k/a \sim \omega_{\rm phys}$.

1.a. Then each mode can be excited in a period

$$\Delta t \sim H^{-1}$$

1.b. The waves redshift before horizon crossing $\gamma_{\rm freeze-out} = \frac{H}{\omega}\gamma_\omega$. It also takes more energy to excite γ_ω at higher ω. With fixed energy per Hubble volume:

$$N_\gamma \sim \frac{E_\gamma H^3}{\omega^4}.$$

1. Exponential Expansion

Suppose emission is at a physical frequency $k/a \sim \omega_{\rm phys}$.

1.a. Then each mode can be excited in a period

$$\Delta t \sim H^{-1}$$

1.b. The waves redshift before horizon crossing $\gamma_{\rm freeze-out} = \frac{H}{\omega}\gamma_\omega$. It also takes more energy to excite γ_ω at higher ω. With fixed energy per Hubble volume:

$$N_\gamma \sim \frac{E_\gamma H^3}{\omega^4}.$$

Most efficient emission is at $\omega \sim H$

2. Weak Gravity

Tensor emission is governed by linearized Einstein equation

$$G_{\mu\nu}^{\text{lin.}} \simeq \frac{1}{M_{\text{pl}}^2} T_{\mu\nu}$$

2. Weak Gravity

Tensor emission is governed by linearized Einstein equation

$$G_{\mu\nu}^{\rm lin.} \simeq \frac{1}{M_{\rm pl}^2} T_{\mu\nu}$$

2.a. N_X Incoherent Localized Events of mass M_X per Hubble volume :

$$\langle \gamma_X^2 \rangle \sim N_X \frac{M_X^2}{M_{\rm pl}^2} \frac{H^2}{M_{\rm pl}^2}$$

Comment1) This is an upper bound. Comment2) This can exceed $\langle \gamma^2 \rangle_{\rm vac}$.

2. Weak Gravity

Tensor emission is governed by linearized Einstein equation

$$G_{\mu\nu}^{\text{lin.}} \simeq \frac{1}{M_{\text{pl}}^2} T_{\mu\nu}$$

2.a. N_X Incoherent Localized Events of mass M_X per Hubble volume :

$$\langle \gamma_X^2 \rangle \sim N_X \frac{M_X^2}{M_{\text{pl}}^2} \frac{H^2}{M_{\text{pl}}^2}$$

Comment1) This is an upper bound. Comment2) This can exceed $\langle \gamma^2 \rangle_{\text{vac}}$.

2.b. Coherent emission by extended configurations (e.g. gauge field A):

$$N_X \longrightarrow \text{Number of Species}$$

$$M_X \longrightarrow \text{Energy / Hubble volume}$$

3. Scalar Emission from Energy Conservation

Scalar fluctuations $\delta\phi_X$ lead to: $\delta\rho_\phi = \dot{\phi}\delta\dot{\phi}_X$.

Energy conservation: $\int d^3x \delta\rho_\phi = M_X \Longrightarrow \delta\phi_X \propto \frac{M_X}{\dot{\phi}}$

3. Scalar Emission from Energy Conservation

Scalar fluctuations $\delta\phi_X$ lead to: $\delta\rho_\phi = \dot\phi\delta\dot\phi_X$.

Energy conservation: $\int d^3x\delta\rho_\phi = M_X \Longrightarrow \delta\phi_X \propto \frac{M_X}{\dot\phi}$

Converting to $\zeta_X^2 \sim N_X(H\delta\phi_X/\dot\phi)^2$ results in:

$$\langle\zeta_X^2\rangle \sim N_X \frac{M_X^2}{\epsilon M_{\rm pl}^2}\frac{H^2}{\epsilon M_{\rm pl}^2}$$

Comparison with γ_X gives $r_{\rm max} \sim \epsilon^2$.

3. Scalar Emission from Energy Conservation

Scalar fluctuations $\delta\phi_X$ lead to: $\delta\rho_\phi = \dot{\phi}\delta\dot{\phi}_X$.

Energy conservation: $\int d^3x \delta\rho_\phi = M_X \Longrightarrow \delta\phi_X \propto \frac{M_X}{\dot{\phi}}$

Converting to $\zeta_X^2 \sim N_X (H\delta\phi_X/\dot{\phi})^2$ results in:

$$\langle \zeta_X^2 \rangle \sim N_X \frac{M_X^2}{\epsilon M_{\rm pl}^2} \frac{H^2}{\epsilon M_{\rm pl}^2}$$

Comparison with γ_X gives $r_{\rm max} \sim \epsilon^2$.

In the concrete example of localized incoherent events

$$r_{\rm max} \simeq 0.3\epsilon^2$$

These scenarios can dominate vacuum tensor fluctuations and break the relation between r and $H_{\rm inf}$,

but then they dominate vacuum scalar fluctuations

and ϵ must be relatively large for observable values of r.

However, scalar and tensor tilts are less sensitive to ϵ:

$$n_s - 1 = -\frac{1}{2}\epsilon - \frac{5}{4}\frac{\dot{\epsilon}}{H\epsilon}$$

$$n_t = -\frac{1}{2}\epsilon - \frac{3}{4}\frac{\dot{\epsilon}}{H\epsilon}$$

Non-Gaussianity

If there are N_X incoherent emission events per H^{-4}:

$$\frac{\langle \zeta_X^3 \rangle}{\zeta_X^3} = f_{NL}\zeta_X \sim \frac{1}{N_X^{1/2}}$$

f_{NL} can be made small by increasing N_X.

Non-Gaussianity

If there are N_X incoherent emission events per H^{-4}:

$$\frac{\langle \zeta_X^3 \rangle}{\zeta_X^3} = f_{NL}\zeta_X \sim \frac{1}{N_X^{1/2}}$$

f_{NL} can be made small by increasing N_X. But there is an upper bound

$$\rho_X = N_X H^3 M_X \ll M_{\rm pl}^2 H^2 \epsilon$$

Combined with ζ_X gives

$$f_{NL} \gg 1$$

Away from the squeezed limit: $B(k_1, k_2, k_3) \propto \frac{1}{k_1^2 k_2^2 k_3^2}$

Conclusions

1. It is possible to have $\gamma_X \gg \gamma_{\rm vac}$.

2. Then $\zeta_X \gg \zeta_{\rm vac}$ such that $r \sim \epsilon^2$. Hence detectable r requires relatively large ϵ.

3. Tensor consistency condition $r = -2n_t$ is violated.

4. There is large non-Gaussianity $f_{NL} \gg 1$.

5. Generically, the same bound applies to multi-field models, but models in which scalar emission is suppressed can be built.

Thank you!

Exception– A two-field scenario

Consider a two filed inflationary model with both fields ϕ and ψ slow-rolling. Suppose the energy source for the auxiliary sector X is ψ:

$$M_X^2 = M^2 \sin^2 \frac{\psi}{f}.$$

Energy transfer from $\frac{1}{2}\dot{\psi}^2$ to X sector leads to $\delta\psi$ emission. If

$$\frac{\partial\zeta}{\partial\psi} \ll \frac{H\dot{\psi}}{\dot{\phi}^2 + \dot{\psi}^2}$$

then the contribution to scalar spectrum can be made small. This seems non-generic, but can be realized for instance if the re-heating surface is determined by ϕ:

$$V(\phi, \psi) = \theta(\phi - \phi_0) U(\phi, \psi).$$