String Gas Cosmology and a Possible Explanation of the Blue Tilt in Gravitational Waves

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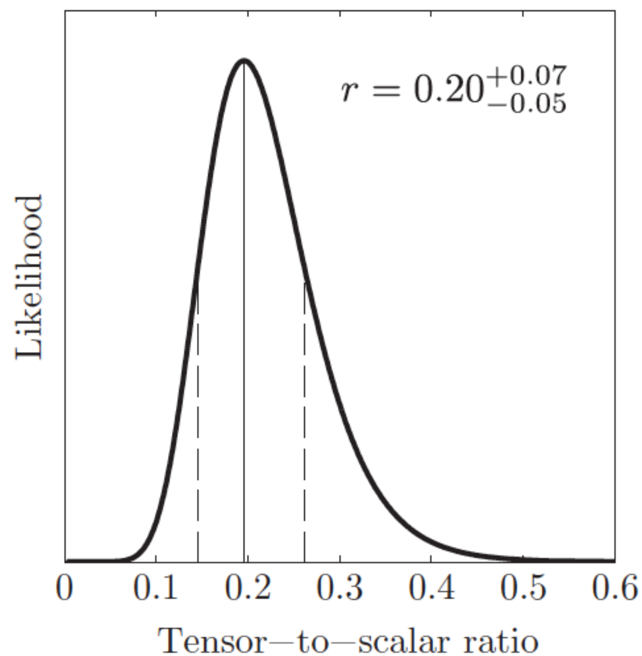
1) hep-th/0511140, 2) hep-th/0604127
3) hep-th/0606073, 4) hep-th/0608121
5) hep-th/0608186, 6)hep-th/0611193
7) hep-th/07121254, 8)hep-th/08104677
9)astro-ph/1403.4927

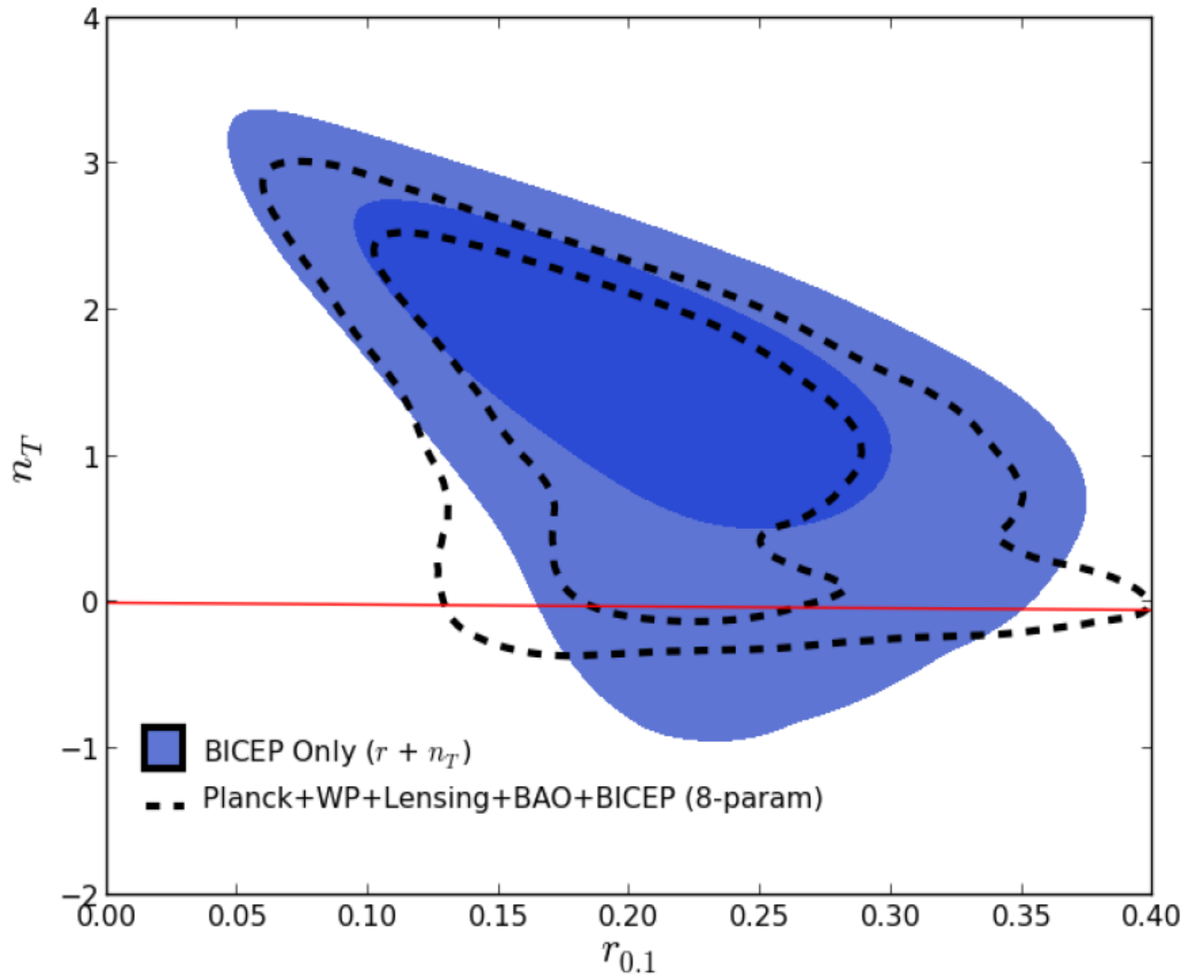
New Result, New Possibilities

Constraints on r=T/S



As We Heard it Today

Planck + BICEP Tensor Spectral Index



Courtesy of Will Kinney

String Cosmology

- ***Weaving cosmos with strings: AsemAn-0-ReesmAn bAftan (Persian Saying): A person who does not know what he is talking about; A philosopher; A Liar.***

The Motivation

- **Can string theory provide an alternative scenario to inflation?**

- **Can one study the structure formation in this scenario?**

Assumptions

- The universe is ***flat***!
- The topology of the universe is ***compact***, e.g., a torus.
- There are only ***d-***dimensions that are ***large*** and ***compact***.
- There are ***lots*** of entropy in the universe!

Our Initial Goal

- Here we study the generation of cosmological fluctuations during the early Hagedorn phase of string gas cosmology using the tools of string statistical mechanics.
- Since this early phase is quasi-static, the Hubble radius ($1/H(t)$) is very large (infinite in the limit of the exactly static case).
- In this phase, a gas of closed strings induces a scale-invariant spectrum of *scalar* and *tensor* metric fluctuations on all scales smaller than the Hubble radius.

$$\langle(\delta M)^2\rangle = \langle E^2\rangle - \langle E\rangle^2 = T^2 C_V \,.$$

Dilatonic String Cosmology

* The action:

$$\mathcal{A} = -\frac{1}{2\kappa_D^2}\int \sqrt{-G}\, d^D x\, e^{-2\phi}\left[{}^{(D)}R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right] + \int dt\; F(a,\beta)\,,$$

where $D = d + 1$ and F is the (one-loop) free energy which can be expressed in terms of the one-loop string partition function in a torus background of radii a and periodic Euclidean time of perimeter β.

* The Background:

$$ds^2 = -dt^2 + \sum_i^{(d)} a^2 d\theta_i^2\,,$$

Dilatonic String Cosmology: Field Equations

$$-(d)\dot{\mu}^2 + \dot{\varphi}^2 = e^{\varphi} E\,,$$

$$\ddot{\mu} - \dot{\varphi}\dot{\mu} = \frac{1}{2} e^{\varphi} P\,,$$

Note the invariance of the equations under

a(t) → 1/a(t)

$$\ddot{\varphi} - (d)\dot{\mu}^2 = \frac{1}{2} e^{\varphi} E\,,$$

where

$$\varphi \equiv 2\phi - (d)\mu\,, a(t) = e^{\mu(t)}\,,$$

$$V = (2\pi\sqrt{\alpha'})^9 a^{(d)} \equiv (2\pi\sqrt{\alpha'})^9 e^{(d)\mu}\,.$$

Equations in Terms of Original Dilaton

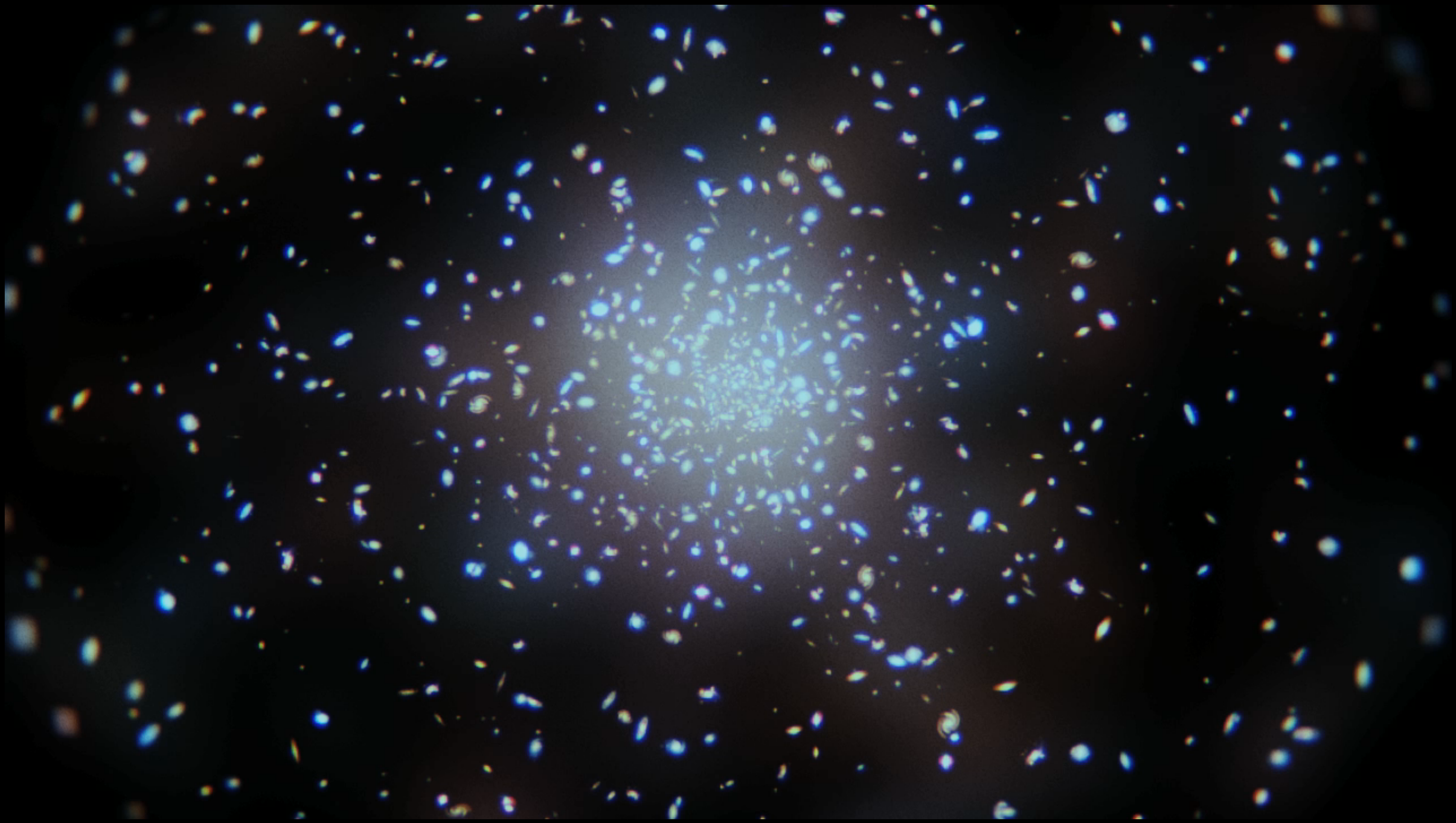
$$-(d)\dot{\mu}^2 + [\dot{\phi} - (d)\dot{\mu}]^2 = e^{\phi}\rho\,,$$

$$\ddot{\mu} - [\dot{\phi} - (d)\dot{\mu}]\dot{\mu} = \frac{1}{2}e^{\phi}p\,,$$

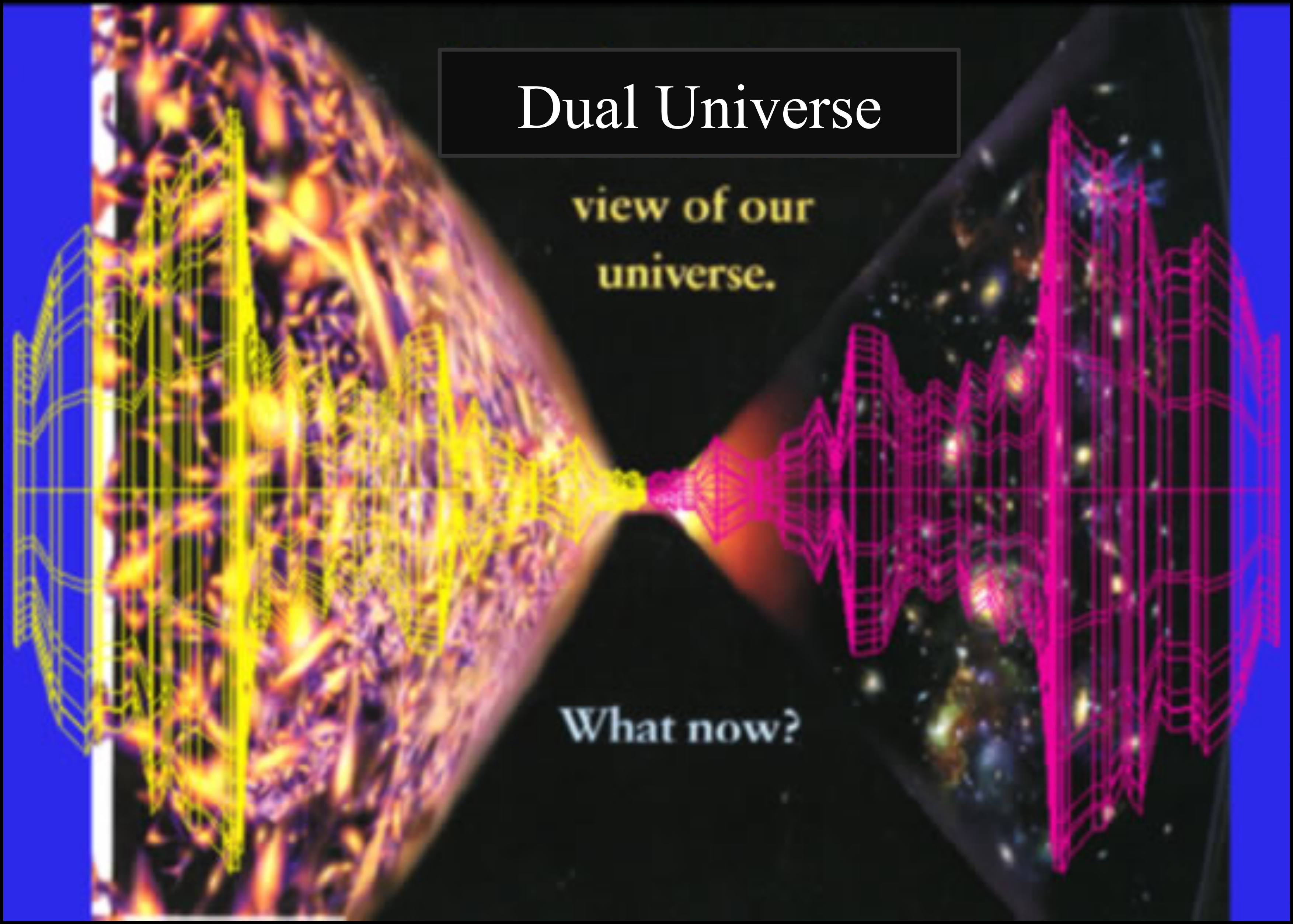
$$[\ddot{\phi} - (d)\ddot{\mu}] - (d)\dot{\mu}^2 = \frac{1}{2}e^{\phi}\rho\,.$$

$$\dot{E} + (d)\dot{\mu}P = 0 = \frac{\dot{S}}{\beta}$$

Dual Universe



Dual Universe



Hagedorn Era

$$\begin{aligned}
\varphi(t) &= \varphi_0 + \ln\left[\frac{2(2\pi\ell_s)^9}{(\kappa_{10}^2 E)}\frac{1}{t(t-t_0)}\right], \\
\mu(t) &= \mu_0 + \mu_1 \ln\left[\frac{t}{t-t_0}\right] \\
\mathcal{S} &\approx \beta_H E\,, \\
P &= \frac{\partial F}{\partial \ln a} = \frac{\partial}{\partial \ln a}(E - T_H \mathcal{S}) \approx 0
\end{aligned}$$

Late time behavior in Hagedorn era:

$$e^{\phi} \sim \frac{const.}{t^2}\ ,\ a(t) \sim const. \Rightarrow H \rightarrow 0$$

Radiation Dominated Era

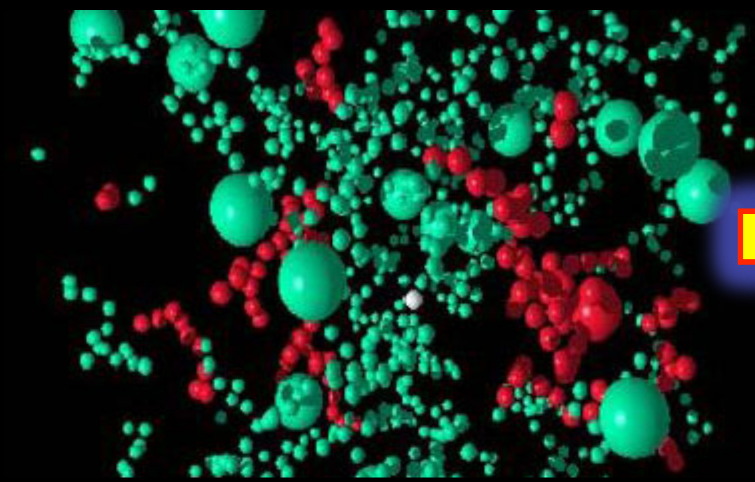
$$\mu(t) = \mu_0 + \left(\frac{2}{d+1}\right)\ln t\,,$$

$$\varphi(t) = \varphi_0 - \left(\frac{2d}{d+1}\right)\ln t\,,$$

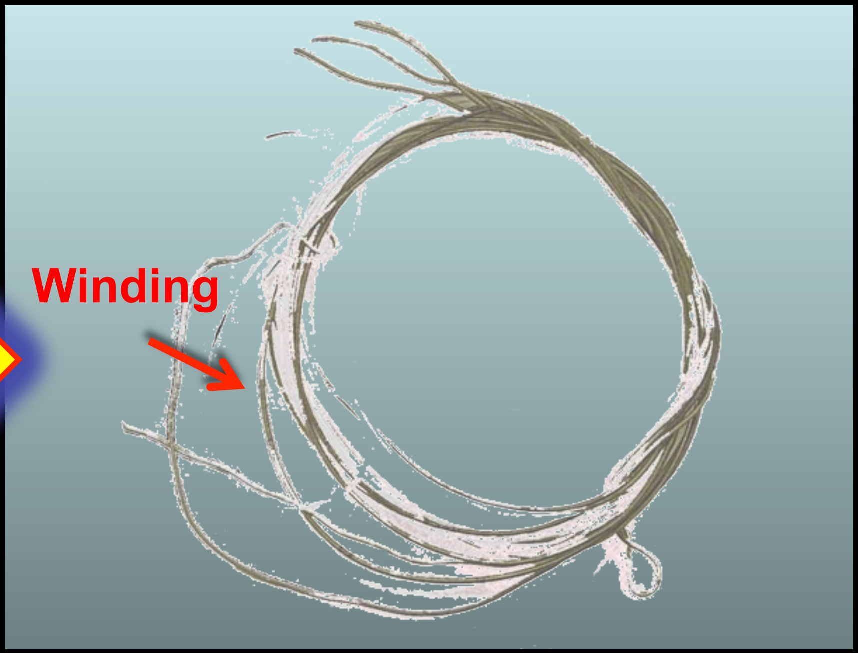
$$\phi = \frac{1}{2}[\varphi + (d)\mu] = \text{constant}$$

$$p = \frac{1}{d}\rho$$

Transition from Particles to Strings!



Particles

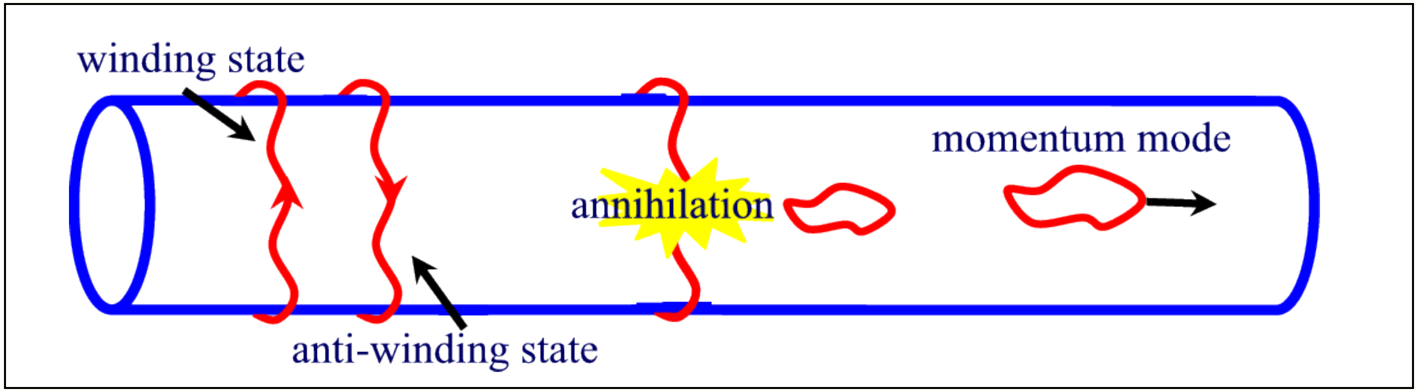


Strings

$$\varepsilon^2 = \frac{\mathbf{l}^2}{R^2} + \frac{\mathbf{w}^2 R^2}{\ell_s^4} + \frac{2}{\ell_s^2}\left[-2 + \sum_{I=1}^{d-1}\sum_{m=1}^{\infty} m\left(N_m^I + \tilde{N}_m^I\right)\right]$$

Why Three Large Dimensions?

Brandenberger-Vafa Scenario



The mass spectrum of the string:

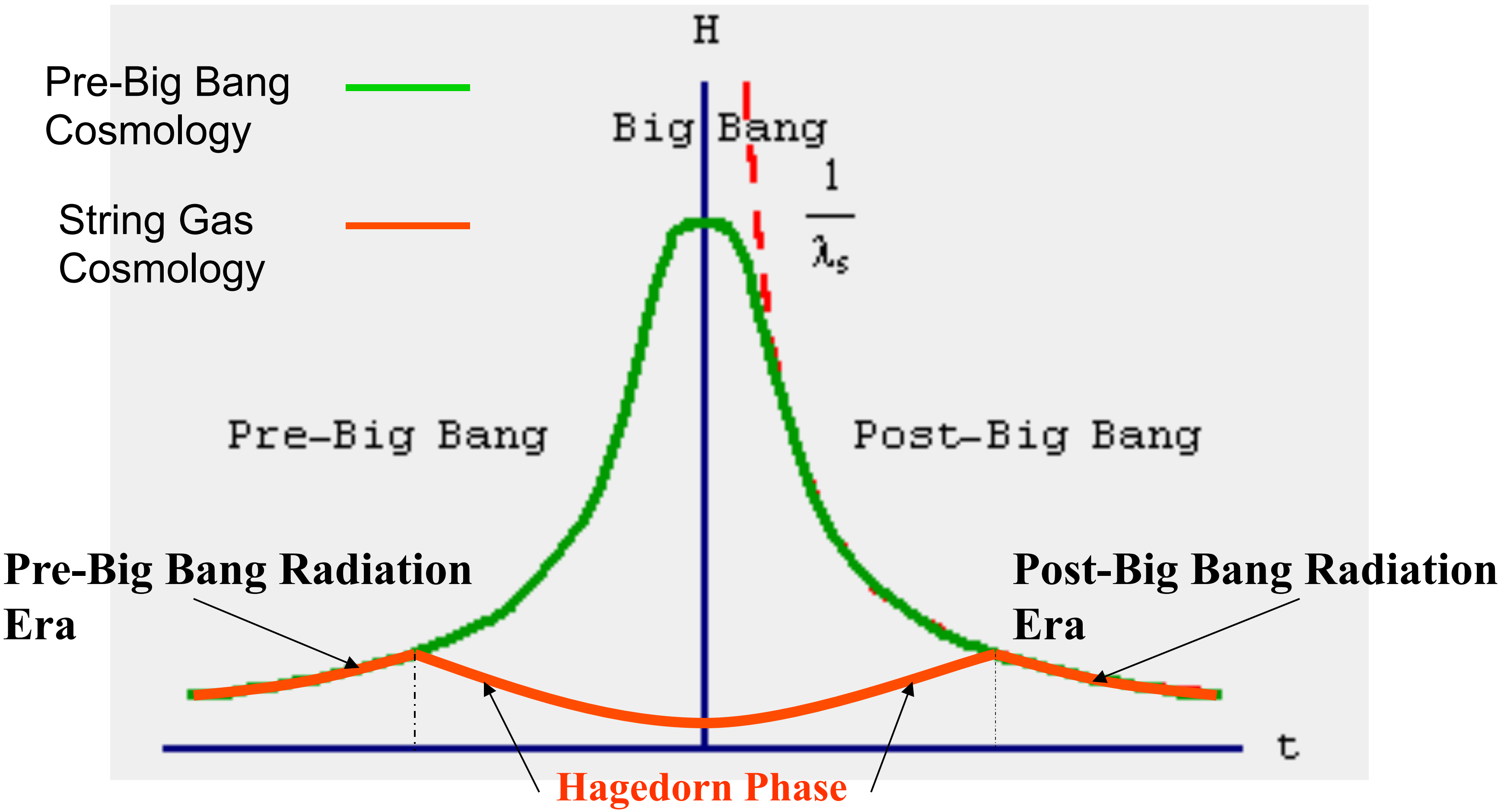
$$M^2 = \frac{n^2}{R^2} + w^2 R^2 + \text{oscillators}$$

Momentum Modes
(Prevent Contraction)

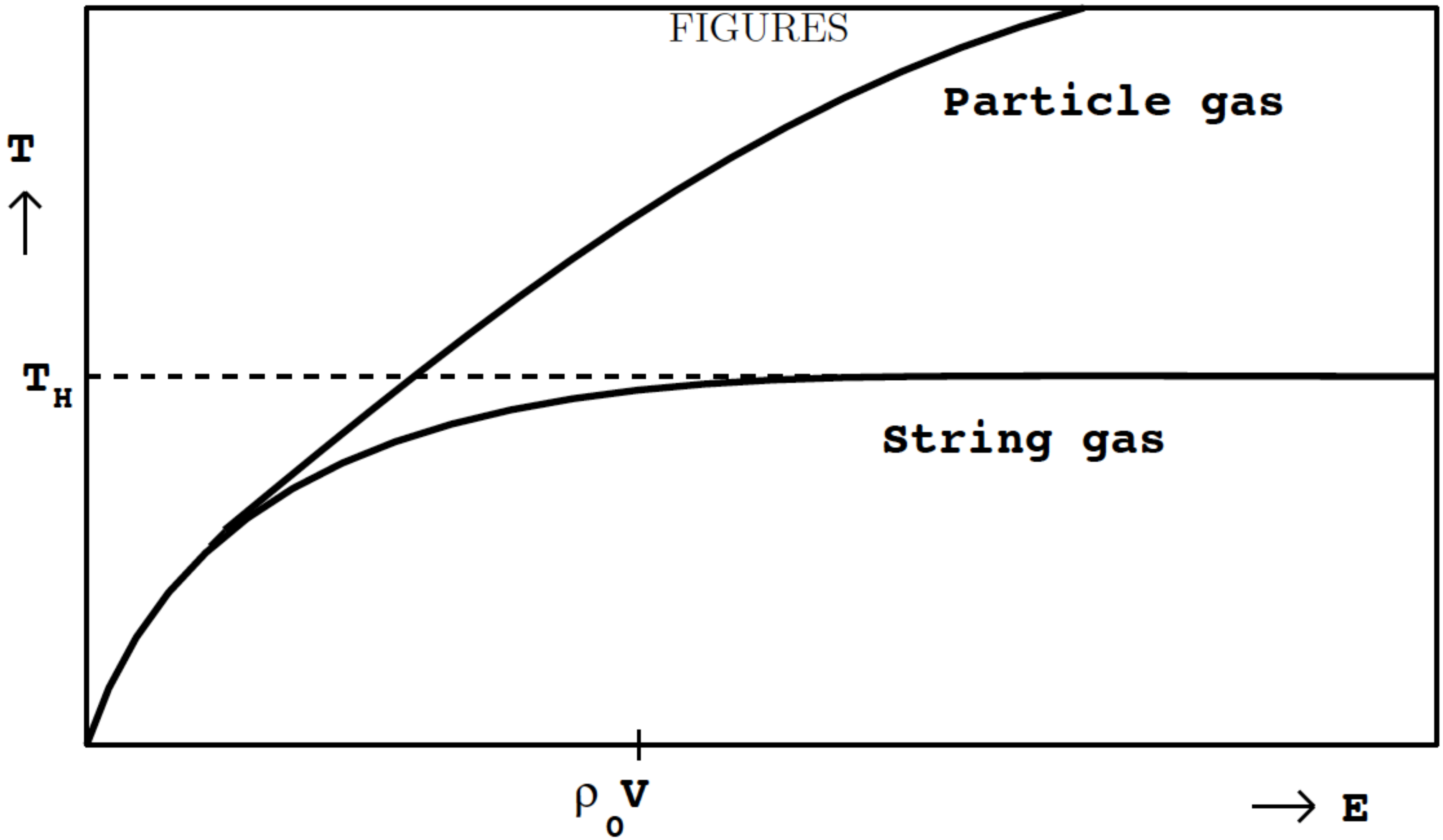
Winding Modes
(Prevent Expansion)

Strings intersect in at most 3 spatial dimensions!!!

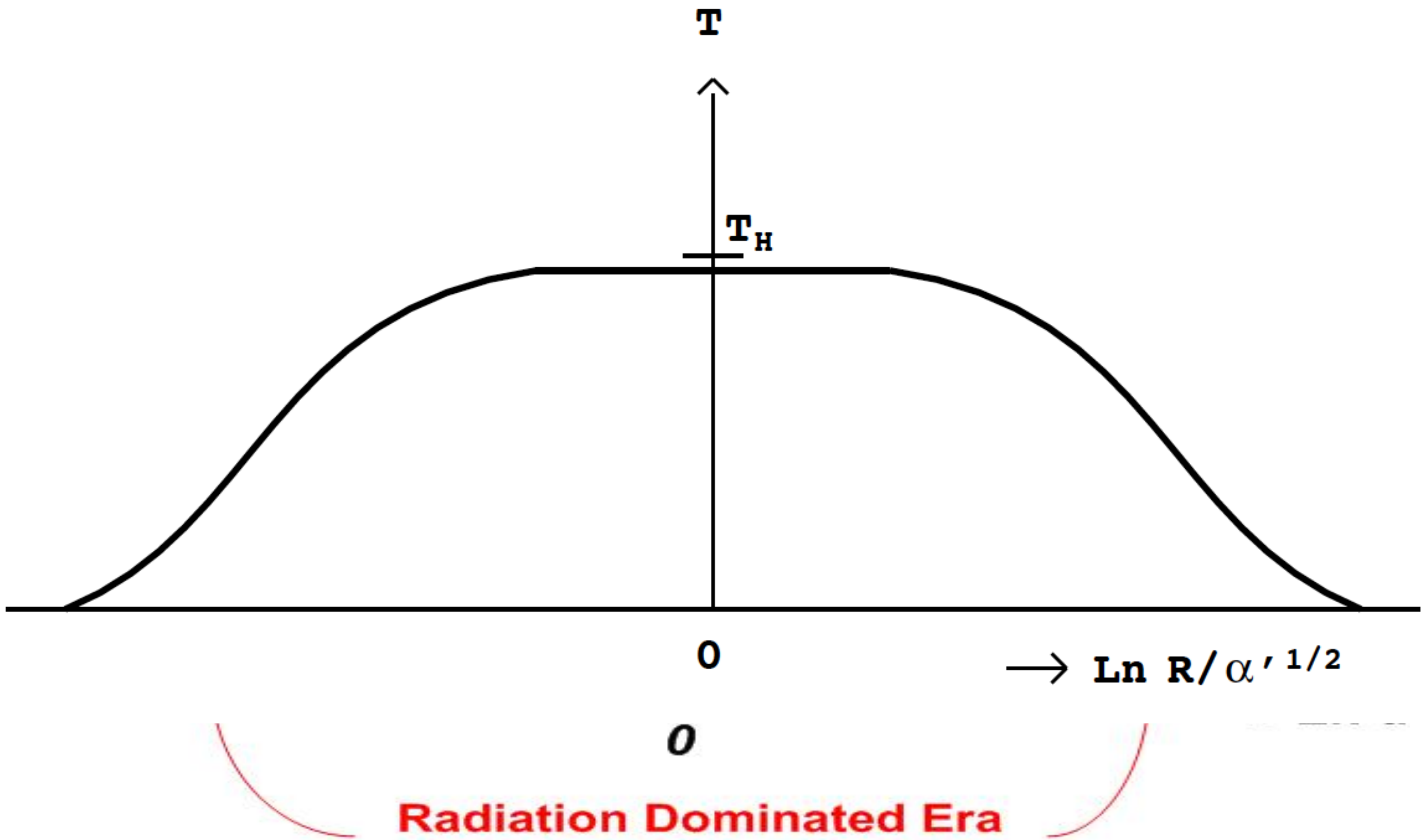
Classical String Cosmology



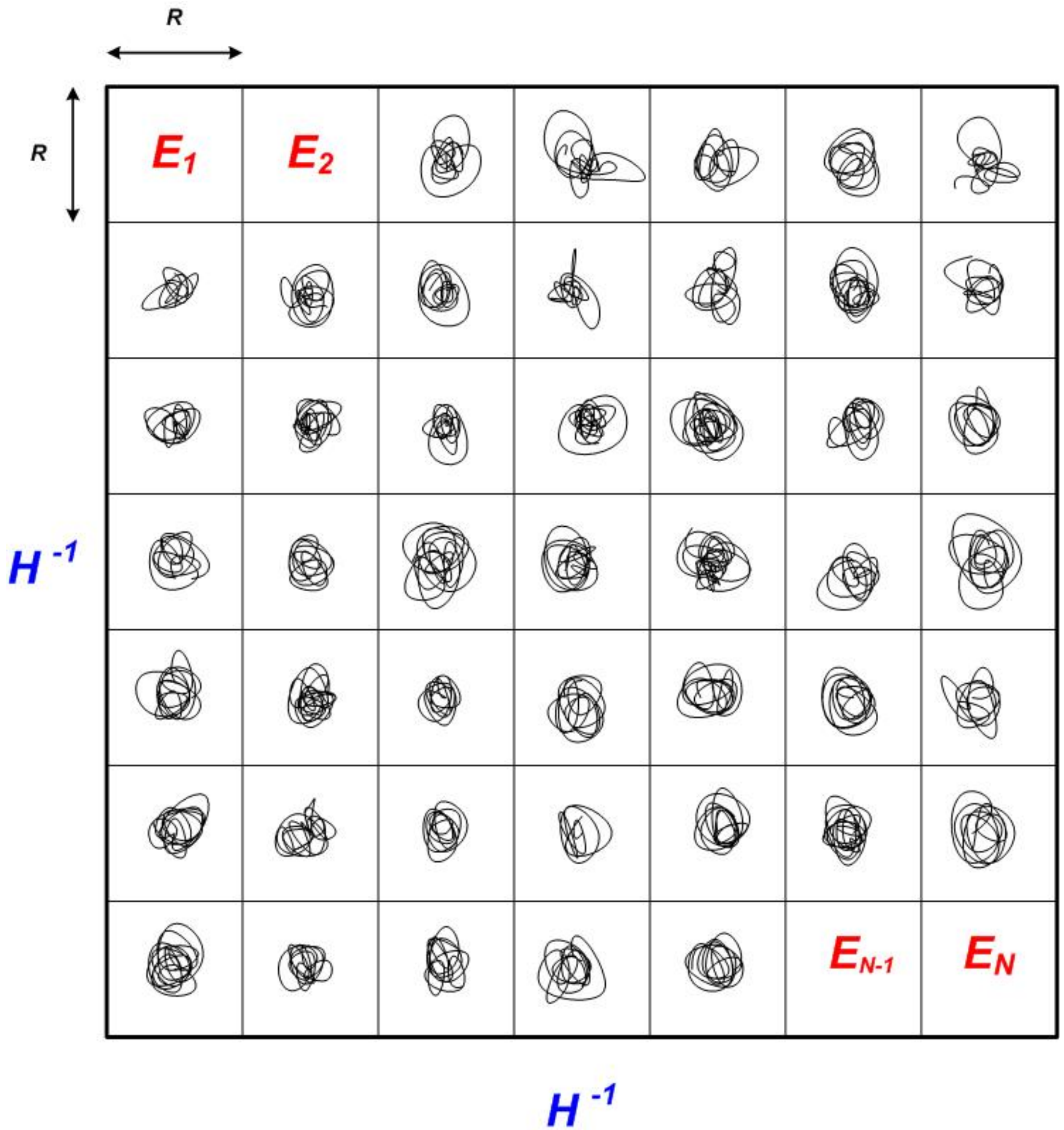
Behavior of Temperature



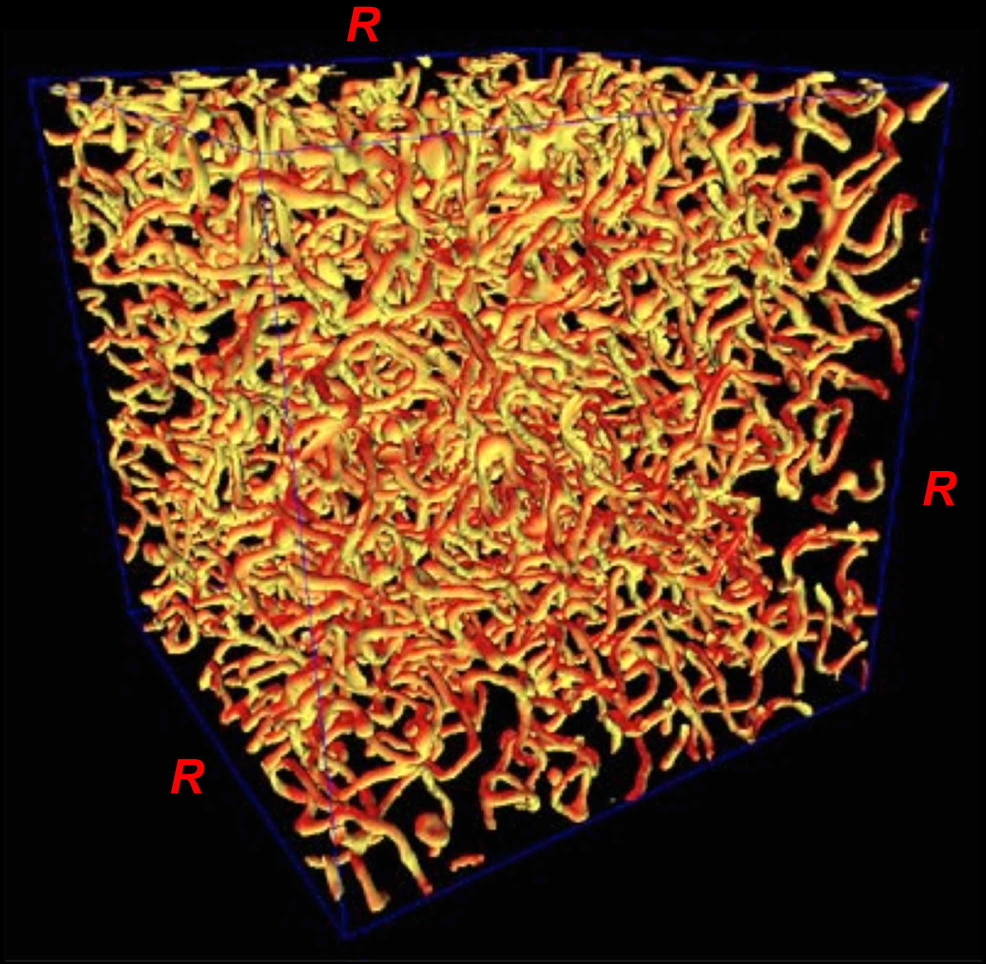
Behavior of Temperature



Kayhaanistan



Kayhaanak



Kayhaanak

Correlation Function

$$
\begin{aligned}
C^{\mu}{}_{\nu}{}^{\sigma}{}_{\lambda} &= \langle \delta T^{\mu}{}_{\nu} \delta T^{\sigma}{}_{\lambda} \rangle = \langle T^{\mu}{}_{\nu} T^{\sigma}{}_{\lambda} \rangle - \langle T^{\mu}{}_{\nu} \rangle \langle T^{\sigma}{}_{\lambda} \rangle \\
&= 2 \frac{G^{\mu\beta}}{\sqrt{-G}} \frac{\partial}{\partial G^{\beta\nu}} \left(\frac{G^{\sigma\delta}}{\sqrt{-G}} \frac{\partial \ln Z}{\partial G^{\delta\lambda}} \right) \\
&\quad + 2 \frac{G^{\sigma\beta}}{\sqrt{-G}} \frac{\partial}{\partial G^{\beta\lambda}} \left(\frac{G^{\alpha\delta}}{\sqrt{-G}} \frac{\partial \ln Z}{\partial G^{\delta\nu}} \right) ,
\end{aligned}
$$

with $\delta T^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \langle T^{\mu}{}_{\nu} \rangle$ and

$$
\boxed{\langle T^{\mu}{}_{\nu} \rangle = 2 \frac{G^{\mu\alpha}}{\sqrt{-G}} \frac{\partial \ln Z}{\partial G^{\alpha\nu}} ,}
$$

Energy Fluctuation

Now if we divide the universe inside the Hubble radius, H^{-1}, to small blocks of size $\ell_s \ll R \ll H^{-1}$, where R is almost independent of time during the Hagedorn phase. The partition function $Z = \exp(-\beta F)$, where $F = F(\beta\sqrt{-G_{00}}, R)$ is the string free energy with $\beta\sqrt{-G_{00}} = T^{-1}\sqrt{-G_{00}}$. Therefore $C^0{}_0{}^0{}_0$, becomes

$$\begin{aligned} C^0{}_0{}^0{}_0 &= \langle \delta\rho^2 \rangle = \langle \rho^2 \rangle - \langle \rho \rangle^2 \\ &= -\frac{1}{R^{2d}}\frac{\partial}{\partial\beta}\left(F + \beta\frac{\partial F}{\partial\beta}\right) = -\frac{1}{R^{2d}}\frac{\partial\langle E\rangle}{\partial\beta}, \\ &= \boxed{\frac{T^2}{R^{2d}}C_V} \end{aligned}$$

Specific Heat

The specific heat C_V can be obtained from the entropy of the system,

$$
\begin{aligned}
C_V &\equiv -\left[T^2\left(\frac{\partial S(E,R)}{\partial E}\right)_V\right]^{-1} \\
&= \frac{R^2}{\ell_s^3 T}\frac{1}{1-T/T_H},
\end{aligned}
$$

and thus

$$
\begin{aligned}
C^{0}{}_{0}{}^{0}{}_{0} &= \langle\delta\rho^2\rangle = \langle\rho^2\rangle - \langle\rho\rangle^2 \\
&= \frac{T}{\ell_s^3 R^{2(d-1)}}\frac{1}{1-T/T_H}
\end{aligned}
$$

Comparison

- Specific heat for open strings is positive:

$$C_V^{open} = \frac{2R^d}{\ell_s^{(d-1)}} \frac{T}{(1 - T/T_H)^3},$$

- Specific heat for non-compact dimensions is negative:

$$C_V^{NC} = -\frac{d+2}{2}\left(\frac{T_H}{T - T_H}\right)^2$$

- The specific heat of the ideal gas of point particles is positive

$$C_V^{particle} = (d+1)\left(\frac{d}{d+1}\right)^d R^d T^d$$

Metric Fluctuation

- On scales larger than the Hubble radius, gravity dominates the dynamics and metric fluctuations play the leading role. We will calculate here the spectrum of scalar metric fluctuations, fluctuation modes which couple to the matter sources. In the absence of anisotropic stress, there is only one physical degree of freedom, namely the relativistic generalization of the Newtonian gravitational potential. In longitudinal gauge, the metric then takes the form

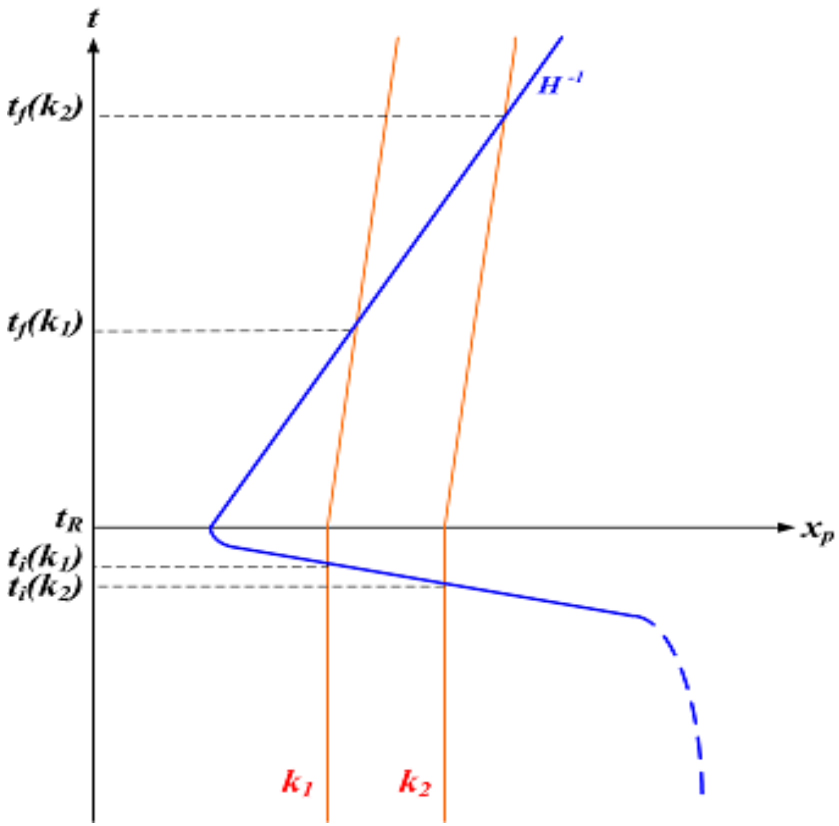
$$ds^2 = a^2(\eta)\left\{-(1+2\Phi)d\eta^2 + [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right\}$$

- On scales smaller than the Hubble radius, the gravitational potential Φ is determined by the matter fluctuations via the Einstein constraint equation (the relativistic generalization of the Poisson equation of Newtonian gravitational perturbation theory)

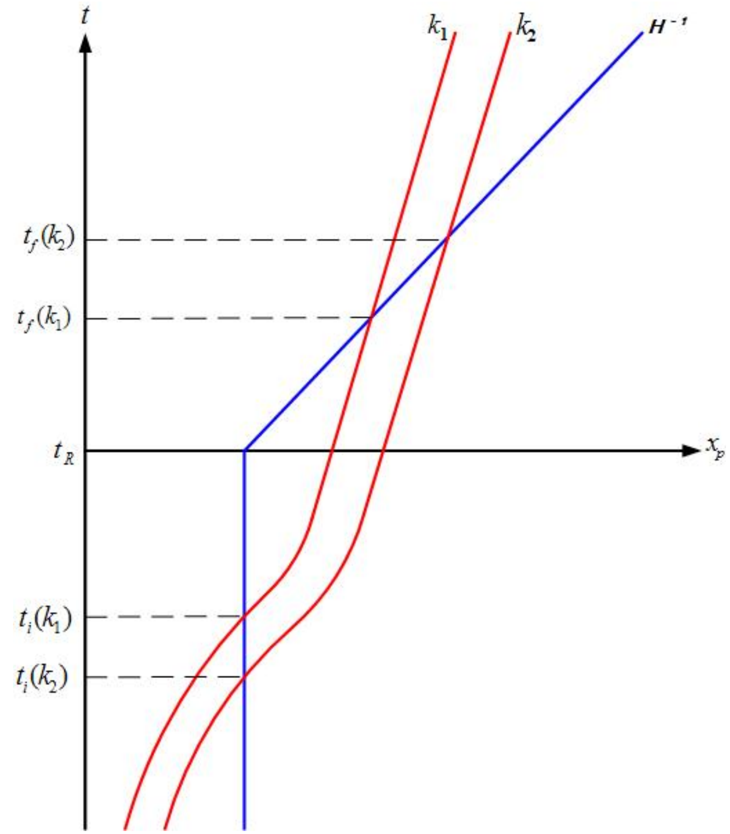
$$\nabla^2\Phi = \frac{2\pi^{d/2}G_{d+1}}{\Gamma[d/2]}\delta\rho\,.$$

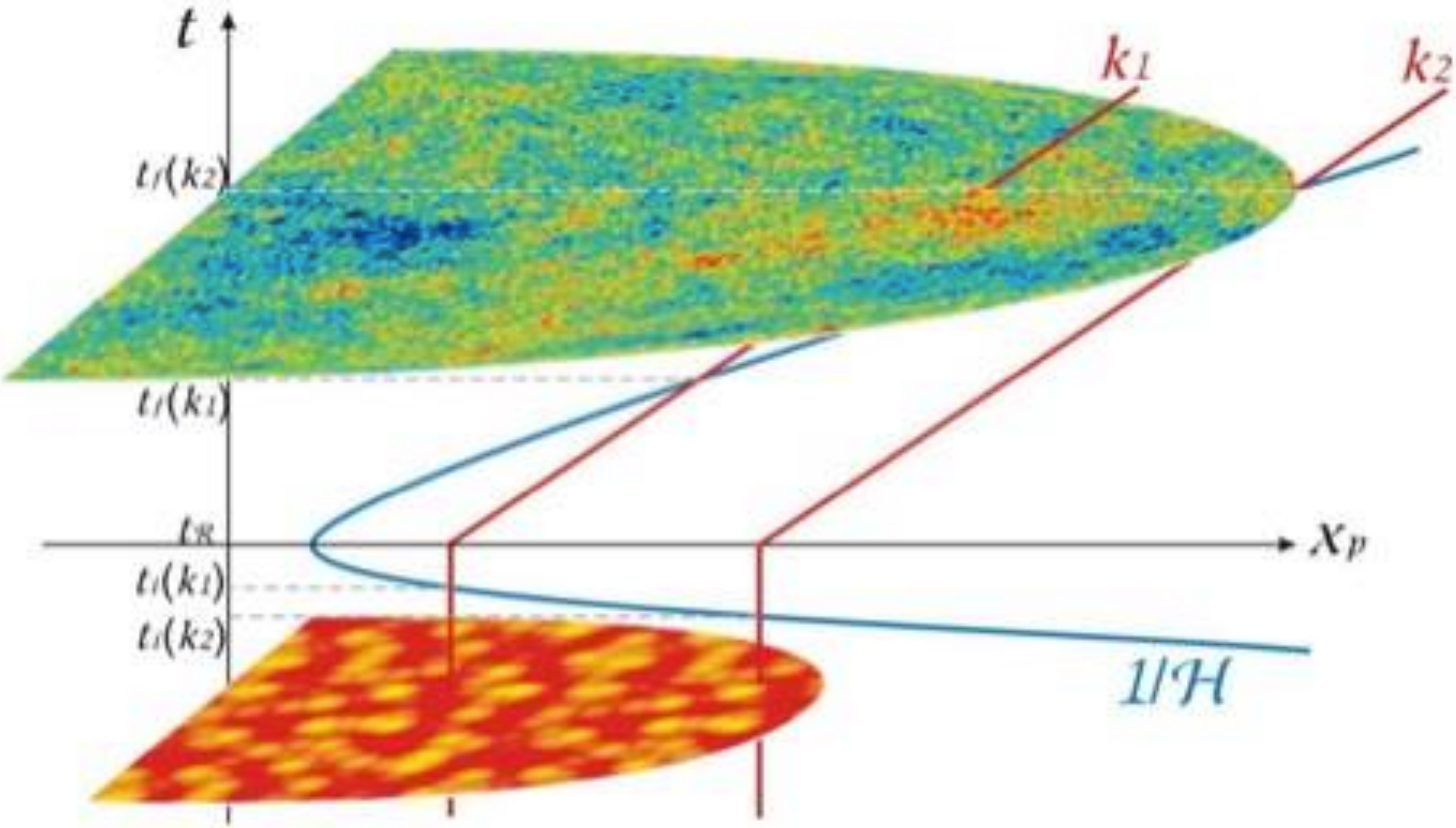
Spacetime diagram of the fluctuation modes

String Gas Cosmology



Inflationary Cosmology





t
k1
k2
tf(k2)
tf(k1)
tR
xp
ti(k1)
ti(k2)
1/H

Scalar Power Spectrum

$$\Delta_\Phi^2(k) \equiv \frac{4\pi^d G_{d+1}^2}{\Gamma^2[d/2]} k^{-4} \Delta_{\delta\rho_k}^2(k) \sim k^{n-1}$$

$$\boxed{\Delta_\Phi^2(k) \simeq \frac{\pi^{d/2} G_{d+1}^2}{2^{(d-3)}\Gamma^3[d/2]\ell_s^3} \frac{T}{(1 - T/T_H)} k^{2(d-3)}}$$

- Scale invariant in $d = 3$.
- Amplitude is suppressed by $(\ell_{pl}/\ell_s)^4$.
- Observational considerations sets

$$(\ell_{pl}/\ell_s) = g_s \sim 10^{-3} \ll 1$$

Red Tilt in Scalar Power Spectrum

- The temperature $T(k)$ decreases as k increases.
- T(k) is ***close*** to the Hagedorn temperature.
- The nominator of the right hand side of power spectrum dominates the final amplitude.
- The spectrum of scalar metric fluctuations has a ***red tilt*** (larger amplitude at larger wavelengths).

$$\boxed{1 - \frac{T(t_{exit}(k))}{T_H} \approx \left(\frac{k}{k_0}\right)^{\epsilon} \Rightarrow n_s - 1 \approx -\epsilon}$$

Producing Gravitational Waves

Tensor Modes

- Tensor modes produced by fluctuations of the wound strings around a compact space.
- Correlation function $C^i{}_j{}^i{}_j\ (i \neq j)$, namely the mean square fluctuation of $T^i{}_j\ (i \neq j)$ in a region of radius $R \sim k^{-1}$:

$$
\begin{aligned}
C^i{}_i{}^i{}_i &= \langle \delta T^i{}_i{}^2 \rangle = \langle T^i{}_i{}^2 \rangle - \langle T^i{}_i \rangle^2 = \frac{1}{\beta R^d} \frac{\partial}{\partial \ln R} \left(-\frac{1}{R^d} \frac{\partial F}{\partial \ln R} \right) = \frac{1}{\beta R^{(d-1)}} \frac{\partial p}{\partial R} \\
&\approx \frac{10}{3} \frac{(1 - T/T_H)}{\ell_s^3 R^{2(d-1)}} \left\{ \ln \left[\frac{\ell_s^3 T}{R^2 (1 - T/T_H)} \right] \left(1 - \frac{2}{5} \ln \left[\frac{\ell_s^3 T}{R^2 (1 - T/T_H)} \right] \right) \right\} \\
&\approx \frac{4}{3} \frac{T(1 - T/T_H)}{\ell_s^3 R^{2(d-1)}} \ln^2 \left[\frac{R^2}{\ell_s^3 T} (1 - T/T_H) \right],
\end{aligned}
$$

Power Spectrum of Tensor Modes

$$\boxed{\Delta_h^2 \sim \frac{1024\pi^2 G^2}{d}\frac{T(1-T/T_H)}{\ell_s^3} k^{2(d-3)} \ln^2 \frac{4\pi^2}{\ell_s^2 k^2 T}(1-T/T_H)}$$

- The power spectrum is scale invariant in $d = 3$ again.
- The key factor $(1 - T/T_H)$ now appears in the numerator and hence leads to a ***blue spectrum***.
- The tensor to scalar ratio

$$\boxed{r = \frac{\Delta_h^2}{\Delta_\Phi^2} \sim 21(1-T/T_H)^2 \ln^2\left[\frac{4\pi^2}{\ell_s^2 k^2 T}(1-T/T_H)\right]}$$

Blue Tilt in Tensor Power Spectrum

- Defining $\widehat{T}(k) \triangleq T(k)/T_H$, the tilt is

$$n_T = \frac{1 - 2\widehat{T}(k)}{1 - \widehat{\ }(k)} k \frac{d\widehat{T}(k)}{dk}$$

$$= -(n_s - 1)(2\widehat{T}(k) - 1)$$

- The spectrum of gravitational waves is determined by the *anisotropic pressure perturbations*.
- Since deeper in the Hagedorn phase, i.e. at *higher* $T(k)$, the pressure is *smaller*, the anisotropic pressure fluctuations should be smaller, as well.
- The amplitude of the gravitational wave spectrum will increase towards the *ultraviolet*, corresponding to a ***blue*** spectrum.

Some Estimation for the Blue Tilt

- Requiring COBE normalization for the power spectrum for the comoving curvature perturbation to requiring a tensor to scalar ratio of 0.2, fixes the string length to be given by

$$\ell_{pl} \approx 0.0016\ell_s$$

- The modes we observe exited when the temperature of the universe was $T \sim 0.99T_H$.
- The latter implying that the tensor tilt is essentially equal and ***opposite*** to the scalar tilt

$$\boxed{n_T \sim -(n_s - 1) \sim 0.03}$$

Conclusion I

- *Here, we have studied the generation and evolution of cosmological fluctuations in a model of string gas cosmology in which an early quasi-static Hagedorn phase is followed by the radiation-dominated phase of standard cosmology, without an intervening period of inflation.*

Conclusion II

- From theoretical aspects, string gas cosmology predicts *blue tensor spectra*. However,
- The tilt is *small*, at the same order-of-magnitude of scalar tilt.
- String gas cosmology predicts highly *Gaussian density* and tensor perturbations.
- Inflationary candidates with *blue* n_T produce considerable amount of *non-Gaussianities*.
- In case a *blue tensor* tilt *is* detected, non-Gaussianity should be the next test to distinguish between string gas and inflationary models.

Conclusion III

- *Although our cosmological scenario provides a new mechanism for generating a scale-invariant spectrum of cosmological perturbations, it does not solve all of the problems which inflation solves. In particular, it does not solve the flatness problem. Without assuming that the three large spatial dimensions are much larger than the string scale, we do not obtain a universe which is sufficiently large today.*