Nonlocal Gravity and Structure in the Universe

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Current cosmic acceleration:

a surprise, not explained by GR

- **Einstein equation (General Relativity)**

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

does not account for the observed acceleration of the Universe, so we need

New substance : Dark Energy or **New formulation: Modified Gravity**

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + \Delta T_{\mu\nu})$$

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- **Models in both camps can reproduce the observed expansion history.**
- **An emerging method to distinguish these models is to study**

Growth of structure for a fixed expansion history

We apply this technique to a particular modified gravity model, which in conclusion fails to pass this test, but we learned an important lesson regarding why it doesn't work.

A nonlocally modified gravity model

- **Deser and Woodard, PRL 99 (2007) 111301, 0706.2151**

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{-g}R\left[1 + f\left(\Box^{-1}R\right)\right] \quad \rightarrow \quad G_{\mu\nu} + \Delta G_{\mu\nu}(f) = 8\pi G T_{\mu\nu}$$

- **f can be fitted to produce the expansion history w/o Λ or DE. (See the next slide.)**

- **Advantages, main features, theoretical motivations, …**
- $\Box^{-1}R$ is dimensionless:
  no new mass parameter (typically required to be very small $10^{-33}eV$) is needed.
- $R \simeq 0$ during rad-dom & $\Box^{-1}R$ grows slowly (logarithmically) during mat-dom:
  the modification does not affect the expansion history until recently, exactly the type of modification we need for the current epoch of acceleration!
- $\Box^{-1}R$ is small in the Solar System: the model passes local tests of gravity
- For more general discussion on the issues of screening and the stability of the model: Deser and Woodard, JCAP 11 (2013) 036, 1307.6639
- Theoretical motivation for nonlocal terms might arise from a quantum theory: See for example, Polyakov, PLB 103, 207 (1981)

Fit ΛCDM expansion history w/o Λ

$$G_{\mu\nu} + \Delta G_{\mu\nu}(f) = 8\pi G T_{\mu\nu}$$

Specialize the modified field eqn to the FRW (homogeneous, isotropic, spatially flat) geometry and determine f so as to match with the ΛCDM expansion history, which is given as

$$H(t) = H_0 \sqrt{\Omega_\Lambda + \Omega_m / a^3 + \Omega_r / a^4} = \frac{d \ln a}{dt}$$

Note: once H_0 & Ω values are given, $H(t)$ is fixed.

- **Friedmann Eq.**

$$H^2(t) = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}(\rho_\Lambda + \rho_m + \rho_r)$$

To get exactly the same H(t) w/o ρ_Λ, need to make Newton's constant G growing with time:

$$H^2(t) = \frac{8\pi G}{3}\rho = \frac{8\pi G_{\text{eff}}(t)}{3}(\rho_m + \rho_r)$$

Problem:
Data says growth is a bit lower than what's expected in the ΛCDM model .

$$G_{\text{eff}}(t) > G$$

Can we make perturbations behave opposite way so as to suppress growth of structure?

Generically $f(\Box^{-1}R)$, $f(R)$, $\cdots$ any modification to GR

Perturbation Eqs. & growth of structure

To see the growth of structure, perturb the metric around the FRW background;

$$ds^2 = -(1+2\Psi(t,\vec{x}))dt^2 + a^2(t)(1+2\Phi(t,\vec{x}))dx^2$$

- **4 evolution Eqs. for 4 perturbations,** $\Psi\,,\ \Phi\,,\ \delta\,,\ \theta$

General Relativity	Nonlocal Gravity
$(\Phi+\Psi)=0$	$(\Phi+\Psi)=-(\Phi+\Psi)\left\{f(\bar{X})+\frac{1}{\bar{\Box}}\left[\bar{R}f'(\bar{X})\right]\right\}-\left\{f'(\bar{X})\frac{1}{\bar{\Box}}\delta R+\frac{1}{\bar{\Box}}\left[f'(\bar{X})\delta R\right]\right\}$
$\frac{k^2}{a^2}\Phi=4\pi G\bar{\rho}\delta$	$\frac{k^2}{a^2}\Phi+\frac{k^2}{a^2}\left[\Phi\left\{f(\bar{X})+\frac{1}{\bar{\Box}}\left[\bar{R}f'(\bar{X})\right]\right\}+\frac{1}{2}\left\{f'(\bar{X})\frac{1}{\bar{\Box}}\delta R+\frac{1}{\bar{\Box}}\left[f'(\bar{X})\delta R\right]\right\}\right]=4\pi G\bar{\rho}\delta$
$\dot{\delta}+H\theta=0\,,$	same
$H\dot{\theta}+\left(\dot{H}+2H^2\right)\theta-\frac{k^2}{a^2}\Psi=0$	same

Stress-energy conservation

$$\nabla^{\mu}\Delta G_{\mu\nu}=0$$

still holds in this nonlocal model

Blue < 0: time only,
At 0th order, only Blue matters.

Red > 0: time and space,
At 1st order, both blue and red matter.

$$\bar{X}\equiv\bar{\Box}^{-1}R$$

Parameterization of the deviations from GR

We solve the system of the 4 integro-differential eqs. for Ψ, Φ, δ, θ (numerically)

$$(\Phi+\Psi) = -(\Phi+\Psi)\left\{ f(\bar{X}) + \frac{1}{\bar{\Box}}\left[\bar{R} f'(\bar{X})\right]\right\} - \left\{ f'(\bar{X})\frac{1}{\bar{\Box}}\delta R + \frac{1}{\bar{\Box}}\left[f'(\bar{X})\delta R\right]\right\}$$

$$\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}\left[\Phi\left\{ f(\bar{X}) + \frac{1}{\bar{\Box}}\left[\bar{R} f'(\bar{X})\right]\right\} + \frac{1}{2}\left\{ f'(\bar{X})\frac{1}{\bar{\Box}}\delta R + \frac{1}{\bar{\Box}}\left[f'(\bar{X})\delta R\right]\right\}\right] \equiv \frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}E[\Phi,\Psi] = 4\pi G\bar{\rho}\delta$$

$$\dot{\delta} + H\theta = 0,$$

$$H\dot{\theta} + \left(\dot{H} + 2H^2\right)\theta - \frac{k^2}{a^2}\Psi = 0$$

and the following parameterizations are useful for analyzing the solutions:

$$\eta \equiv \frac{\Phi+\Psi}{\Phi}$$

$$\frac{G_{\rm eff}}{G} \equiv \frac{k^2\Phi}{4\pi G\bar{\rho}a^2\delta} = \frac{1}{1+\dfrac{E[\Phi,\Psi]}{\Phi}}$$

or

$$\Psi \equiv (1+\mu)\Psi_{\rm GR}$$

$$\Psi - \Phi \equiv (1+\Sigma)[\Psi_{\rm GR} - \Phi_{\rm GR}]$$

related by

$$\boxed{1+\mu = (1-\eta)\frac{G_{\rm eff}}{G}}$$

It turns out

$$\eta < 0 \quad \because \quad \Phi > 0,\ \Psi < 0,\ |\Psi| > |\Phi| \quad \Rightarrow \quad \boxed{1-\eta > 1}$$

$$\boxed{\frac{G_{\rm eff}}{G} < 1} \quad \because \quad E[\Phi,\Psi] > 0$$

i.e. Red > 0 is dominant over Blue < 0

$$\Box = -\partial_t^2 - 3H\partial_t + \frac{\nabla^2}{a^2}$$

Growth Equation

Combining the 4 evolution eqns. we have an eqn for δ:

General Relativity

$$\frac{d^2\delta}{da^2}+\left[\frac{d\ln(H)}{da}+\frac{3}{a}\right]\frac{d\delta}{da}-\frac{3}{2}\frac{\Omega_m}{h^2(a)a^5}\delta=0$$

Nonlocal Gravity

$$\frac{d^2\delta}{da^2}+\left[\frac{d\ln(H)}{da}+\frac{3}{a}\right]\frac{d\delta}{da}-\frac{3}{2}(1+\mu)\frac{\Omega_m}{h^2(a)a^5}\delta=0$$

$$1+\mu=(1-\eta)\frac{G_{\text{eff}}}{G} > 1$$ **Growth gets enhanced**

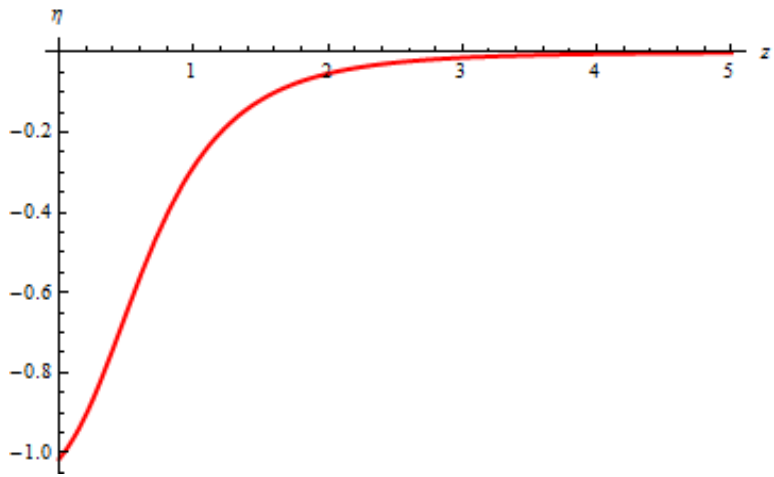
$$1+\mu=(1-\eta)\frac{G_{\text{eff}}}{G} < 1$$ **Growth gets suppressed**

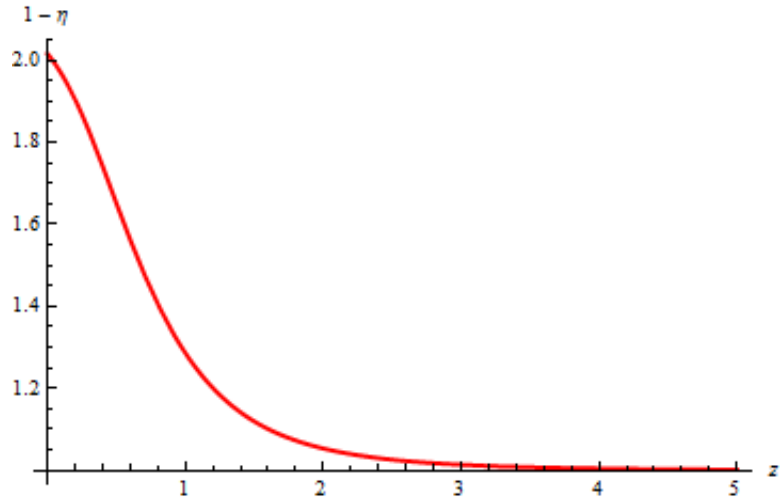
Who wins?

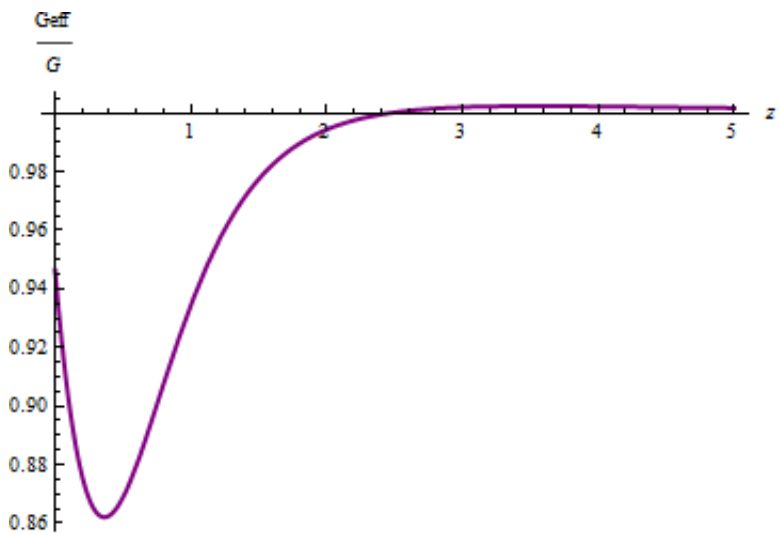
$$\frac{G_{\text{eff}}}{G} < 1$$ **or** $$1-\eta > 1$$ **(equivalently** $|\Psi|>|\Phi|$**)**

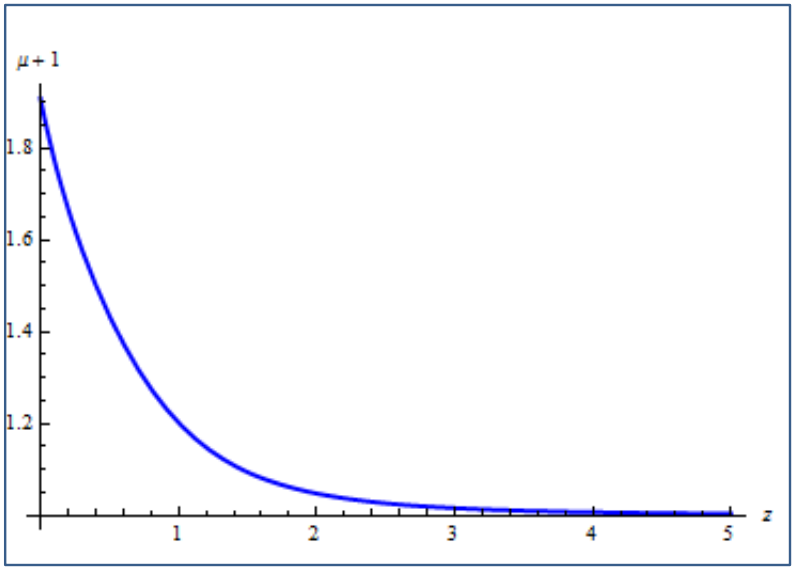
Partially successful: $\frac{G_{\text{eff}}}{G} < 1$ **but the effect of** η **overwhelms it.**

Figures for $1+\mu=(1-\eta)\dfrac{G_{\text{eff}}}{G} \quad > \quad 1$









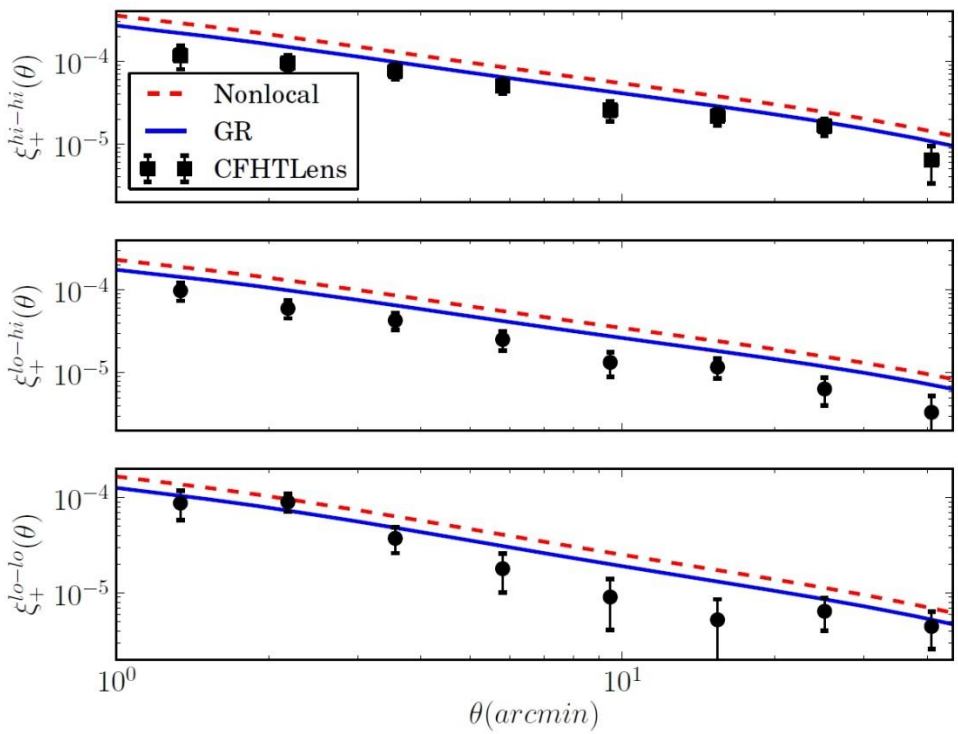
Weak Lensing

Fixing the redshift-distance relation and the initial amplitude of fluctuations, the power spectrum of the convergence of galaxies in two redshift bins is

$$C_l^{ij} = \left(\frac{3\Omega_m H_0^2}{2}\right)^2 \int_0^\infty d\chi \frac{g_i(\chi) g_j(\chi)}{a^2(\chi)} P(l/\chi;\chi)\left[1+\Sigma(\chi)\right]^2$$

where $g_i(\chi) \equiv \int_\chi^\infty d\chi' \frac{dn_i}{d\chi'}\left(1-\frac{\chi}{\chi'}\right)$ **the weighting function in each redshift bin**

$dn_i / d\chi$ **the redshift distribution of source galaxies in bin *i***



ξ in three different redshift bins as measured in CFHTLenS (black points with error bars).

Top and bottom panels: correlation function in the high and low redshift bins resp.
Middle panel: the cross spectrum.

Both GR and nonlocal models have the redshift-distance relation corresponding to Planck parameters.

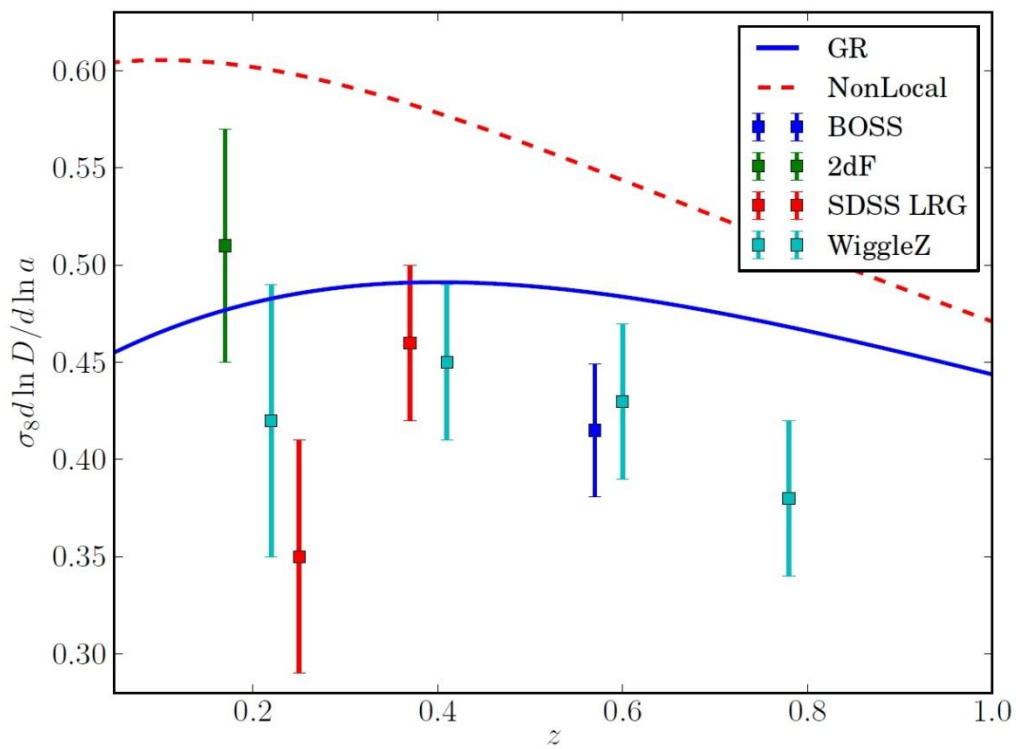
5.9-σ preference for GR over the nonlocal model

Redshift Space Distortions

Redshift space distortions probe the product of

the growth rate $\beta \equiv d\ln D / d\ln a$ and $\sigma_8(z)$ a measure of the clustering amplitude.

Here the *growth function* $D(a)$ is the solution to the growth eq. with initial condition $D(a) = a$.



Measurement of $\beta\sigma_8$

This is directly measured in spectroscopic surveys capable of probing redshift space distortions.

Data points come from
BOSS,
2dF,
SDSS LRG,
WiggleZ.

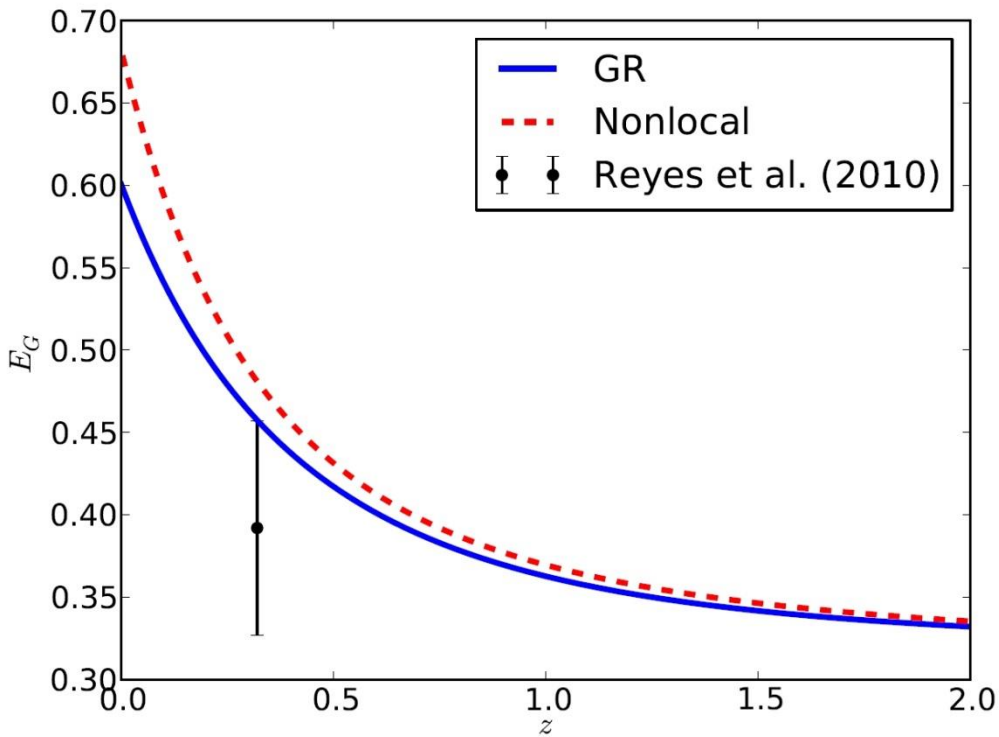
7.8-σ preference for
GR over the nonlocal model

Estimator of Gravity E_G

Gravitational lensing is sensitive to the combination, $\Phi - \Psi$ while spectroscopic surveys are sensitive to the velocity field, which is related to $\dot{\delta}$

Combining the two, we have an estimator of gravity E_G
(Zhang, Liguori, Bean, and Dodelson, PRL 99, 141302 (2007), 0704.1932)

$$E_G \equiv \frac{k^2 a(\Phi - \Psi)}{3H_0^2 \beta \delta} = \frac{\Omega_m \left[1 + \Sigma\right]}{\beta}$$



Estimator of gravity E_G as a function of redshift in GR and the nonlocal model.

The data point is from **Reyes et al. (2010)**

The growth rate β is larger in nonlocal gravity, but Σ is positive; an interesting interplay between the two effects. On balance, the Σ enhancement wins, leading to larger values of E_G in the nonlocal model.

Take Home Message

- **Modified gravity, aimed at reproducing the expansion history, tends to make gravity stronger at 0th order.**
- **Growth of structure is observed to be a bit lower than expected in the simplest ΛCDM model.**
- **In a successful modified gravity model, perturbations would weaken gravity enough to overcome strengthened gravity in the background level.**