Nonlocal Gravity and Structure in the Universe

Sohyun Park
Penn State University

Co-author: Scott Dodelson

Based on

August 25, 2014 Chicago, IL
Cosmo 2014
Current cosmic acceleration:
a surprise, not explained by GR

- Einstein equation (General Relativity)

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

does not account for the observed acceleration of the Universe, so we need

New substance: Dark Energy or New formulation: Modified Gravity

\[ G_{\mu\nu} = 8\pi G (T_{\mu\nu} + \Delta T_{\mu\nu}) \quad G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

- Models in both camps can reproduce the observed expansion history.
- An emerging method to distinguish these models is to study

Growth of structure for a fixed expansion history

We apply this technique to a particular modified gravity model, which in conclusion fails to pass this test, but we learned an important lesson regarding why it doesn't work.
A nonlocally modified gravity model

- Deser and Woodard, PRL 99 (2007) 111301, 0706.2151

\[
\mathcal{L} = \frac{1}{16\pi G} \sqrt{-g} R \left[ 1 + f \left( \Box^{-1} R \right) \right] \rightarrow G_{\mu\nu} + \Delta G_{\mu\nu}(f) = 8\pi G T_{\mu\nu}
\]

- f can be fitted to produce the expansion history w/o \( \Lambda \) or DE. (See the next slide.)

- Advantages, main features, theoretical motivations, ...
  - \( \Box^{-1} R \) is dimensionless:
    - no new mass parameter (typically required to be very small \( 10^{-33} eV \)) is needed.
  - \( R \approx 0 \) during rad-dom & \( \Box^{-1} R \) grows slowly (logarithmically) during mat-dom:
    - the modification does not affect the expansion history until recently, exactly the type of modification we need for the current epoch of acceleration!
  - \( \Box^{-1} R \) is small in the Solar System: the model passes local tests of gravity
  - For more general discussion on the issues of screening and the stability of the model: Deser and Woodard, JCAP 11 (2013) 036, 1307.6639
  - Theoretical motivation for nonlocal terms might arise from a quantum theory: See for example, Polyakov, PLB 103, 207 (1981)
Fit $\Lambda$CDM expansion history w/o $\Lambda$

$$G_{\mu\nu} + \Delta G_{\mu\nu}(f) = 8\pi G T_{\mu\nu}$$

Specialize the modified field eqn to the FRW (homogeneous, isotropic, spatially flat) geometry and determine $f$ so as to match with the $\Lambda$CDM expansion history, which is given as

$$H(t) = H_0\sqrt{\Omega_\Lambda + \Omega_m / a^3 + \Omega_r / a^4} = \frac{d \ln a}{dt}$$

Note: once $H_0$ & $\Omega$ values are given, $H(t)$ is fixed.

- **Friedmann Eq.**

$$H^2(t) = \frac{8\pi G}{3} \rho = \frac{8\pi G}{3} (\rho_\Lambda + \rho_m + \rho_r)$$

To get exactly the same $H(t)$ w/o $\rho_\Lambda$, need to make Newton’s constant $G$ growing with time:

$$H^2(t) = \frac{8\pi G}{3} \rho = \frac{8\pi G_{\text{eff}}(t)}{3} (\rho_m + \rho_r)$$

**Problem:**
Data says growth is a bit lower than what’s expected in the $\Lambda$CDM model.

Can we make perturbations behave opposite way so as to suppress growth of structure? $G_{\text{eff}}(t) > G$

Generically $f(\Box^{-1}R), f(R), \cdots$ any modification to GR
Perturbation Eqs. & growth of structure

To see the growth of structure, perturb the metric around the FRW background;

\[ ds^2 = -(1 + 2\Psi(t, \bar{x}))dt^2 + a^2(t)(1 + 2\Phi(t, \bar{x}))dx^2 \]

- 4 evolution Eqs. for 4 perturbations, \( \Psi, \Phi, \delta, \theta \)

**General Relativity**

\[
(\Phi + \Psi) = 0 \\
\frac{k^2}{a^2} \Phi = 4\pi G\bar{\rho}\delta \\
\dot{\delta} + H\theta = 0, \\
H\dot{\theta} + (\dot{H} + 2H^2)\theta - \frac{k^2}{a^2} \Psi = 0
\]

**Nonlocal Gravity**

\[
(\Phi + \Psi) = -(\Phi + \Psi) \left\{ f(\bar{X}) + \frac{1}{\bar{R}} \left[ \bar{R}f'(\bar{X}) \right] \right\} - \left\{ f'(\bar{X}) \frac{1}{\bar{R}} \delta R + \frac{1}{\bar{R}} \left[ f'(\bar{X}) \delta R \right] \right\} \\
\frac{k^2}{a^2} \Phi + \frac{k^2}{a^2} \left[ \Phi \left\{ f(\bar{X}) + \frac{1}{\bar{R}} \left[ \bar{R}f'(\bar{X}) \right] \right\} \right] + \frac{1}{2} \left\{ f'(\bar{X}) \frac{1}{\bar{R}} \delta R + \frac{1}{\bar{R}} \left[ f'(\bar{X}) \delta R \right] \right\} = 4\pi G\bar{\rho}\delta
\]

Stress-energy conservation

\[ \nabla^\mu \Delta G_{\mu\nu} = 0 \]

still holds in this nonlocal model

Blue < 0: time only, At 0th order, only Blue matters.

Red > 0: time and space, At 1st order, both blue and red matter.

\[ \bar{X} \equiv \bar{R}^{-1} R \]
Parameterization of the deviations from GR

We solve the system of the 4 integro-differential eqs. for $\Psi$, $\Phi$, $\delta$, $\theta$ (numerically)

$$(\Phi + \Psi) = -(\Phi + \Psi) \left\{ f(\bar{X}) + \frac{1}{a} \left[ \bar{R} f'(\bar{X}) \right] \right\} - \left\{ f'(\bar{X}) \frac{1}{a} \delta R + \frac{1}{4} \left[ f'(\bar{X}) \delta R \right] \right\}$$

$$\frac{k^2}{a^2} \Phi + \frac{k^2}{a^2} \left[ \Phi \left\{ f(\bar{X}) + \frac{1}{a} \left[ \bar{R} f'(\bar{X}) \right] \right\} \right] + \frac{1}{2} \left\{ f'(\bar{X}) \frac{1}{a} \delta R + \frac{1}{4} \left[ f'(\bar{X}) \delta R \right] \right\} \equiv \frac{k^2}{a^2} \Phi + \frac{k^2}{a^2} E[\Phi, \Psi] = 4\pi G \bar{\rho} \delta$$

$$\dot{\delta} + H\theta = 0,$$

$$H\dot{\theta} + (\dot{H} + 2H^2) \theta - \frac{k^2}{a^2} \Psi = 0$$

and the following parameterizations are useful for analyzing the solutions:

$$\eta \equiv \frac{\Phi + \Psi}{\Phi}$$

$$\frac{G_{\text{eff}}}{G} \equiv \frac{k^2 \Phi}{4\pi G \bar{\rho} a^2} = \frac{1}{1 + \frac{E[\Phi, \Psi]}{\Phi}}$$

$$\Psi \equiv (1 + \mu) \Psi_{\text{GR}} \quad \text{or} \quad \Psi - \Phi \equiv (1 + \Sigma) \left[ \Psi_{\text{GR}} - \Phi_{\text{GR}} \right]$$

It turns out

$$\eta < 0 \quad \therefore \quad \Phi > 0, \quad \Psi < 0, \quad |\Psi| > |\Phi| \quad \text{related by} \quad 1 - \eta > 1$$

$$\frac{G_{\text{eff}}}{G} < 1 \quad \therefore \quad E[\Phi, \Psi] > 0 \quad \text{i.e. Red} > 0 \text{ is dominant over Blue} < 0$$

$$\square = -\partial_t^2 - 3H \partial_t + \frac{\nabla^2}{a^2}$$
Combining the 4 evolution eqns. we have an eqn for $\delta$:

**General Relativity**

\[
\frac{d^2 \delta}{da^2} + \left[ \frac{d \ln(H)}{da} + \frac{3}{a} \right] \frac{d\delta}{da} - \frac{3}{2} \frac{\Omega_m}{h^2(a)a^5} \delta = 0
\]

**Nonlocal Gravity**

\[
\frac{d^2 \delta}{da^2} + \left[ \frac{d \ln(H)}{da} + \frac{3}{a} \right] \frac{d\delta}{da} - \frac{3}{2} (1 + \mu) \frac{\Omega_m}{h^2(a)a^5} \delta = 0
\]

\[1 + \mu = (1 - \eta) \frac{G_{\text{eff}}}{G} > 1 \quad \text{Growth gets enhanced}
\]

\[1 + \mu = (1 - \eta) \frac{G_{\text{eff}}}{G} < 1 \quad \text{Growth gets suppressed}
\]

Who wins?

\[\frac{G_{\text{eff}}}{G} < 1 \quad \text{or} \quad 1 - \eta > 1 \quad \text{(equivalently } |\Psi| > |\Phi|)\]

Partially successful: $\frac{G_{\text{eff}}}{G} < 1$ but the effect of $\eta$ overwhelms it.
Figures for \[ 1 + \mu = (1 - \eta) \frac{G_{\text{eff}}}{G} > 1 \]
Weak Lensing

Fixing the redshift-distance relation and the initial amplitude of fluctuations, the power spectrum of the convergence of galaxies in two redshift bins is

\[
C_{ij}^{\chi} = \left( \frac{3\Omega_m H_0^2}{2} \right)^2 \int_0^\infty d\chi \frac{g_i(\chi)g_j(\chi)}{a^2(\chi)} P(l / \chi; \chi)[1 + \Sigma(\chi)]^2
\]

where

\[
g_i(\chi) \equiv \int_\chi^\infty d\chi' \frac{d{n_i}}{d\chi'} \left( 1 - \frac{\chi}{\chi'} \right)
\]

the weighting function in each redshift bin

\[
d{n_i}/d\chi
\]

the redshift distribution of source galaxies in bin \(i\)

\(\xi\) in three different redshift bins as measured in CFHTLenS (black points with error bars).

Top and bottom panels: correlation function in the high and low redshift bins resp.
Middle panel: the cross spectrum.

Both GR and nonlocal models have the redshift-distance relation corresponding to Planck parameters.

5.9-\(\sigma\) preference for GR over the nonlocal model
Redshift Space Distortions

Redshift space distortions probe the product of

the growth rate \( \beta \equiv d \ln D / d \ln a \) and \( \sigma_8(z) \) a measure of the clustering amplitude.

Here the growth function \( D(a) \) is the solution to the growth eq. with initial condition \( D(a) = a \).

Measurement of \( \beta \sigma_8 \)

This is directly measured in spectroscopic surveys capable of probing redshift space distortions.

Data points come from BOSS, 2dF, SDSS LRG, WiggleZ.

7.8-\( \sigma \) preference for GR over the nonlocal model.
Estimator of Gravity \( E_G \)

Gravitational lensing is sensitive to the combination, \( \Phi - \Psi \) while spectroscopic surveys are sensitive to the velocity field, which is related to \( \dot{\delta} \).

Combining the two, we have an estimator of gravity \( E_G \)

(Zhang, Liguori, Bean, and Dodelson, PRL 99, 141302 (2007), 0704.1932)

\[
E_G \equiv \frac{k^2 a (\Phi - \Psi)}{3 H_0^2 \beta \dot{\delta}} = \frac{\Omega_m [1 + \Sigma]}{\beta}
\]

Estimator of gravity \( E_G \) as a function of redshift in GR and the nonlocal model.

The data point is from Reyes et al. (2010)

The growth rate \( \beta \) is larger in nonlocal gravity, but \( \Sigma \) is positive; an interesting interplay between the two effects. On balance, the \( \Sigma \) enhancement wins, leading to larger values of \( E_G \) in the nonlocal model.
Take Home Message

- Modified gravity, aimed at reproducing the expansion history, tends to make gravity stronger at 0\textsuperscript{th} order.
- Growth of structure is observed to be a bit lower than expected in the simplest $\Lambda$CDM model.
- In a successful modified gravity model, perturbations would weaken gravity enough to overcome strengthened gravity in the background level.