A numerical calculation of the gravitational wave signal in the low frequency regime produced by binary supermassive black holes

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Supermassive black holes (SMBHs)

- Black holes which have a mass $\gtrsim 10^6 M_\odot$
- Found at the centers of most galaxies at $z = 0$
- Quasars at $z \lesssim 7$
- Masses correlate remarkably closely with the properties of their host galaxies
- Famous scaling relations: $M_{\text{BH}}-\sigma$ and $M_{\text{BH}}-L_{\text{bulge}}$

**Figure:** $M_{\text{BH}}-L_{\text{bulge}}$ scaling relation

Kormendy & Ho 2013
Do galaxy mergers produce SMBH binaries?

- Large galaxies experience many mergers over their lifetimes.
- When two galaxies merge, the SMBHs fall to the center of the remnant through dynamical friction.
- The final parsec problem: can they get close enough to form a binary?
- We don’t see a lot of galaxies with two SMBHs a parsec apart.
- Could be solved for galaxies with non-spherical symmetry or in gas-rich mergers.

**Figure:** NGC 6240, the result of a galaxy merger, with two active black holes at its center.
Credit: NASA/CXC/MPE/S.Komossa et al.
Binary SMBHs produce gravitational radiation

- If sufficiently tight binaries form, they will emit gravitational waves at twice their orbital frequency.
- This emission tightens the binary: after $\sim 100$ Myr, the black holes will merge.
- Observed gravitational wave amplitude (‘strain’) is a function of:
  - Separation (orbital frequency)
  - Chirp mass $M_{\text{chirp}} = \mu^{3/5} M^{2/5}$
  - Distance to the observer

Figure: Artist’s conception of binary black holes emitting gravitational radiation. Credit: T. Carnahan (NASA GSFC)
The gravitational wave background from SMBH binaries

- Due to their orbital periods, binaries should emit a lot of gravitational waves at frequencies between $\sim 10^{-9}$ Hz and $\sim 10^{-7}$ Hz.
- The characteristic signal produced by SMBH binaries is an incoherent sum of the strains from all sources.
- It is traditionally represented as a 2/3 power law, but the reality is more complicated.

**Figure**: A 2/3 power law vs. more realistic signals.
Sesana et al. 2008
Pulsar Timing arrays

PTAs look for slight variations in the periods of millisecond pulsars to see if gravitational waves are passing the earth.
PTAs will measure low-frequency gravitational waves.

Pulsar timing arrays are sensitive to nHz frequencies. Sensitive to the SMBH binary gravitational wave background.

Figure:
NANOgrav pulsar timing array sensitivity. Arzoumanian et al. 2014
Large-scale dark matter simulations

We use halo merger trees from:

**Bolshoi**
- 250 Mpc/h box size
- WMAP5 cosmology
- $10^8 \, M_\odot$/h mass resolution

**Dark Sky**
- 1000 Mpc/h box size
- Planck cosmology
- $10^{10} \, M_\odot$/h mass resolution

*Figure:* A snapshot of the Bolshoi simulation at $z = 0$.
Credit: Stefan Gottlober (AIP)

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Gravitational waves from SMBH binaries
Dark matter halos

Halo mass functions

- **Dark Sky**
- **Bolshoi**

<table>
<thead>
<tr>
<th>Halo mass functions</th>
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<tbody>
<tr>
<td>Num density (Mpc$^{-3}$)</td>
</tr>
<tr>
<td>$10^{-8}$</td>
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</tbody>
</table>

- **Dark Sky**
  - $z = 0$
  - $z = 1$

- **Bolshoi**
  - $z = 0$
  - $z = 1$

$M_{\text{halo}}$ (M$_{\odot}$)

- **Elinore Roebber**
- **Gravitational waves from SMBH binaries**
Placing galaxies in halos

**Figure:** Empirically calibrated stellar mass–halo mass relation. Behroozi et al. 2013

**Figure:** Resulting stellar mass functions.

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**Simulations**

**From halos to black holes**

Building a population of SMBH binaries

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Gravitational waves from SMBH binaries
Placing black holes in galaxies

**Figure:** Relation between galaxy bulge mass and black hole mass. Kormendy & Ho 2013

**Figure:** Resulting black hole mass functions.

From halos to black holes
Building a population of SMBH binaries

Calculations

Simulations

Introduction

To better emphasize the slight variation in $M_{\bullet}/M_{\text{bulge}}$ we plot this ratio expressed as a percent against bulge mass in Figure 18. A direct fit to these points gives

$$
M_{\bullet} = a + b \times M_{\text{bulge}}
$$

with $a = 0.14 \pm 0.08$ and intrinsic scatter $= 0.14$ dex.

BH mass ratios range from true to $\sim$ with NGC xx8zB and NGC xx77 standing out at true and xx77 respectively. The systematic variation in $M_{\bullet}/M_{\text{bulge}}$ with $M_{\text{bulge}}$ is one reason why AGN feedback has little effect on galaxy structure at low BH masses and instead becomes important at the largest BH masses (Section 8).

Conversion from $L_K$, bulge to $M_{\text{bulge}}$ does not make much difference for old stellar populations.

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Gravitational waves from SMBH binaries
Binary black holes

Putting it all together:
- Halos are populated with galaxies.
- Galaxies are populated with black holes.
- The simulation tells us which halos merge.

From this, we can determine the population of SMBH binaries born between each redshift snapshot.

Figure: Binary SMBH mass functions
Constructing a representative sample of ‘observed’ binaries

- Begin with the population of SMBH binaries derived for each simulation within a redshift slice.
- Transform the population in each simulation box to the population we expect to see on the sky.
- Draw binaries from this population:
  - Chirp Mass of the binary
  - Frequency (separation)
  - Distance from the observer
- Calculate the resulting gravitational wave signal for this population!

**Figure**: The strain for a single population of binaries.
What does the gravitational wave background look like?

Example: Bolshoi

![Graph showing the gravitational wave strain as a function of frequency (Hz). The x-axis represents frequency in the range from $10^{-17}$ to $10^{-8}$ Hz, and the y-axis represents the GW strain in the range from $10^{-17}$ to $10^{-14}$. The graph shows a decrease in GW strain with increasing frequency.]
What does the gravitational wave background look like?

Example: Bolshoi
What does the gravitational wave background look like?

Example: Bolshoi

![Graph showing GW strain as a function of frequency (f) for the Bolshoi experiment. The graph displays a logarithmic scale with GW strain on the y-axis (10^-17 to 10^-14) and frequency (Hz) on the x-axis (10^-8 to 10^-7). The median strain is indicated by a black arrow.](image_url)
What does the gravitational wave background look like?

Example: Bolshoi

- Contains 68% of the signal amplitude
- Contains 95% of the signal amplitude

The gravitational wave background

Introduction
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Results
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Gravitational waves from SMBH binaries
What does the gravitational wave background look like?

Example: Bolshoi

![Graph showing the gravitational wave strain vs. frequency](image-url)

- The gravitational wave background is depicted with a power law spectrum.
- The graph illustrates the GW strain (solid line) and the rms strain (dashed line) across different frequencies.
- The spectrum shows a decrease in strain with increasing frequency.
What does the gravitational wave background look like?

Example: Bolshoi

![Graph showing gravitational wave strain vs frequency for different experiments: PPTA, EPTA, NANOgrav. The graph depicts the median strain and rms strain over a range of frequencies from $10^{-8}$ to $10^{-7}$ Hz. The strain values are given in terms of $10^{-17}$ to $10^{-14}$.](image)
Amplitude of the signal

- The rms strain closely resembles the expected 2/3 power law.
- Its amplitude is a factor of 2–3 lower than the current upper limits.
- Consistent with predictions from ‘empirical’ models (Sesana 2013, Ravi et al. 2014).
- We see a ~25% difference between simulations, including the effect of different cosmologies.
- Variance due to astrophysics is somewhat larger (Sesana 2013).

**Figure:** The rms amplitudes for each simulation, compared to a 2/3 power law.
Scatter in the signal

- No single realization of the gravitational wave spectrum will look like a power law.
- Scatter increases with frequency: at low frequencies, a single realization of the strain will be close to the rms signal; at high frequencies it will be much smaller with occasional individual bright sources.
Conclusions

- As a result of galaxy mergers, a large population of supermassive black hole binaries is expected to exist.
- This population is difficult to probe with conventional astrophysics, and is a poorly understood aspect of galaxy evolution.
- BUT: a strong source of gravitational waves out to $z \sim 1$.
- Should be detectable by pulsar timing arrays soon!