The nonlinear dynamical stability of infrared modifications of gravity

Alexandra Terrana

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In collaboration with Richard Brito, Vitor Cardoso and Matthew Johnson

Why Study Modifications to Gravity?

General relativity elegantly explains all observed phenomena on solar-system scales.

On cosmological scales, ΛCDM requires dark energy to explain the accelerated expansion of the universe.

Unresolved theoretical issues: cosmological constant problem

This has driven interest in **infrared** modifications of GR.

Challenge: Screening

Want a large distance modification only – predictions must be reconciled with solar system tests of GR

Any extra degrees of freedom must be active on cosmological scales, but screened on sufficiently short scales

This can be accomplished via the **Vainshtein mechanism** which relies on non-linearities of the theory, and is manifest in theories with **galileon symmetry**.

Challenge: Screening

Various static spherically symmetric screening solutions are known.

But are these solutions accessed *dynamically*?

How dynamically stable are these solutions?

What range of initial data will evolve to the screening solutions?

How do the screening solutions react to dynamical sources?

Class of Theories Under Consideration

$$\mathcal{L}_\pi = c_2\mathcal{L}_2 + \frac{c_3}{\Lambda^3}\mathcal{L}_3 + \frac{c_4}{\Lambda^6}\mathcal{L}_4 + \frac{c_5}{\Lambda^9}\mathcal{L}_5 + \frac{\xi\pi T}{M_4} + \frac{\alpha}{M_4\Lambda^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu},$$

$\pi =$ scalar degree of freedom

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$$\begin{aligned}
\mathcal{L}_2 &= (\partial\pi)^2 \\
\mathcal{L}_3 &= (\partial\pi)^2[\Pi] \\
\mathcal{L}_4 &= (\partial\pi)^2([\Pi]^2 - [\Pi^2]) \\
\mathcal{L}_5 &= (\partial\pi)^2([\Pi]^3 - 3[\Pi][\Pi]^2 + 2[\Pi^3])
\end{aligned}$$

$\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$ and $[\Pi] = \eta^{\mu\nu}\Pi_{\mu\nu}$

Galileon symmetry: $\pi \to \pi + c + b_\mu x^\mu$

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- Captures the decoupling limit of **DGP**[1] and ghost-free **dRGT Massive Gravity**[2] for a specific choice of c_n
- Focus today on **DGP Model** (although not phenomenologically viable!!) so that the analysis is simplified
- ...can qualitatively generalize our results to all galileon theories of this form

[1] G. Dvali, G. Gabadadze, and M. Porrati, Phys.Lett. B485, 208 (2000), arXiv:hep-th/0005016 [hep-th]

[2] C. de Rham, G. Gabadadze, and A. J. Tolley, Phys.Rev.Lett. 106, 231101 (2011), arXiv:1011.1232[hep-th]

The Decoupling Limit

- We work in the **decoupling limit** in which we can decouple the tensor $h_{\mu\nu}$ and scalar π degrees of freedom: $\mathcal{L} = \mathcal{L}_{h_{\mu\nu}} + \mathcal{L}_{\pi}$.
- Perturbing around flat space: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$.
- We retain the full dynamics of the scalar mode

The Decoupling Limit:

$$m \to 0, \qquad M_4, \;\; T_{\mu\nu} \to \infty, \qquad \Lambda, \;\; \frac{T_{\mu\nu}}{M_4} \sim \text{constant}$$

$m =$ graviton mass in dRGT, $\quad m = M_5^3/M_4^2$ in DGP

$M_4 =$ 4D Planck Mass,

$T_{\mu\nu} =$ energy-momentum tensor for the matter source

$\Lambda = (M_4 m^2)^{1/3} =$ strong coupling scale

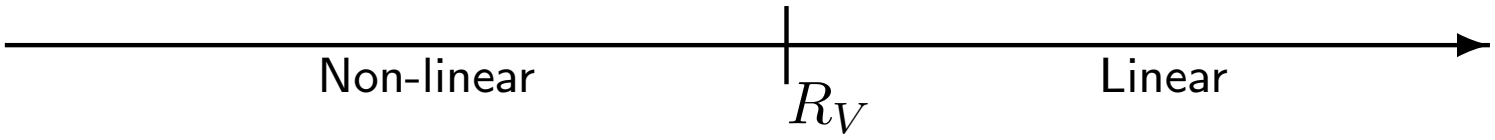
The Decoupling Limit

Let's keep in mind...

- The scalar mode does not gravitate
- Solutions only valid in the weak field regime
- Can describe the important non-linear dynamics of the theory (Vainshtein mechanism)
- Solutions in the decoupling limit may not capture all features of the full theory

Vainshtein Mechanism

For this class of theories, the scalar *fifth force* is screened inside of the **Vainshtein Radius**



$r \ll R_V$:

$$\left|\frac{F_\pi}{F_g}\right| \sim \left(\frac{r}{R_V}\right)^{3/2}$$

$r \gg R_V$:

$$\left|\frac{F_\pi}{F_g}\right| \sim 1$$

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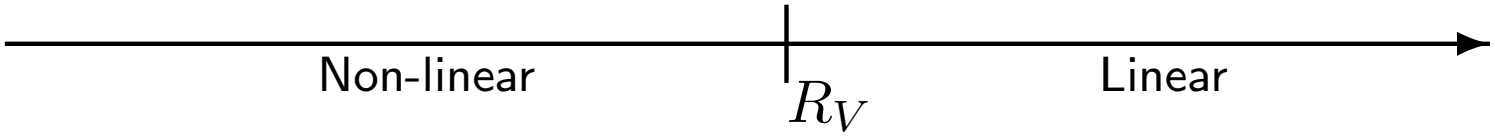

$$\boxed{R_V \equiv \frac{1}{\Lambda}\left(\frac{M(r\to\infty)}{M_4}\right)^{1/3}}$$

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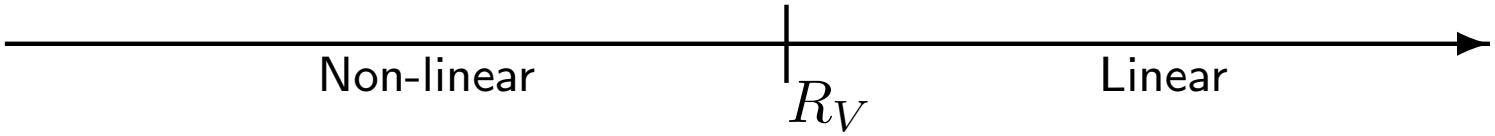

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$$T^{00} = \rho \exp\left(-\frac{r^2}{R_0^2}\right) \qquad \Rightarrow \qquad R_V = \sqrt{\pi}\rho^{1/3}R_0$$

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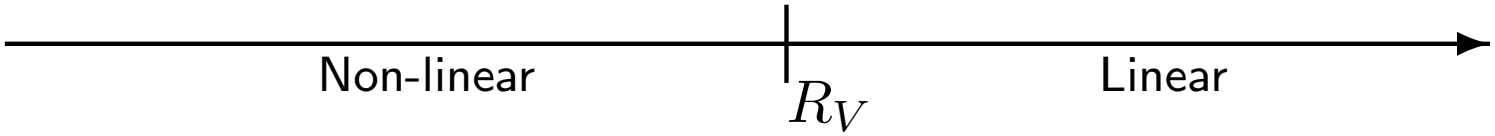

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$$\frac{\pi}{\Lambda^3} \to \pi \qquad\qquad \frac{M}{\Lambda^3 M_4} \to M$$

DGP in the Decoupling Limit

$$\mathcal{L}_\pi = -3\mathcal{L}_2 - \frac{1}{\Lambda^3}\mathcal{L}_3 + \frac{1}{2M_4}\pi T$$

Time-dependent Equation of Motion

$$\ddot{\pi}\left(6 + 8\frac{\pi'}{r} + 4\pi''\right) = \frac{1}{2}T + 12\frac{\pi'}{r} + 6\pi'' + 4(\dot{\pi}')^2 + 8\frac{\pi'\pi''}{r} + 4\frac{(\pi')^2}{r^2}$$

DGP in the Decoupling Limit

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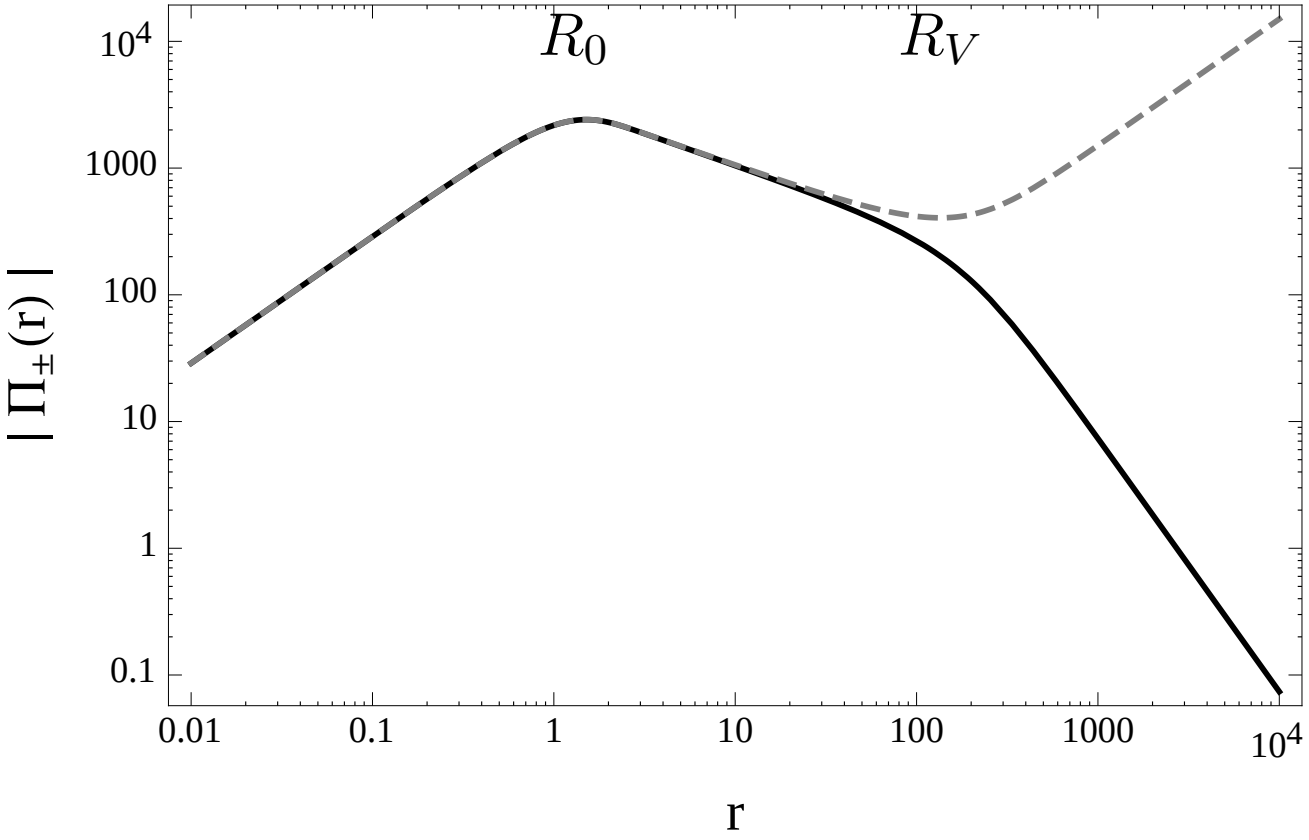
Static Equation of Motion

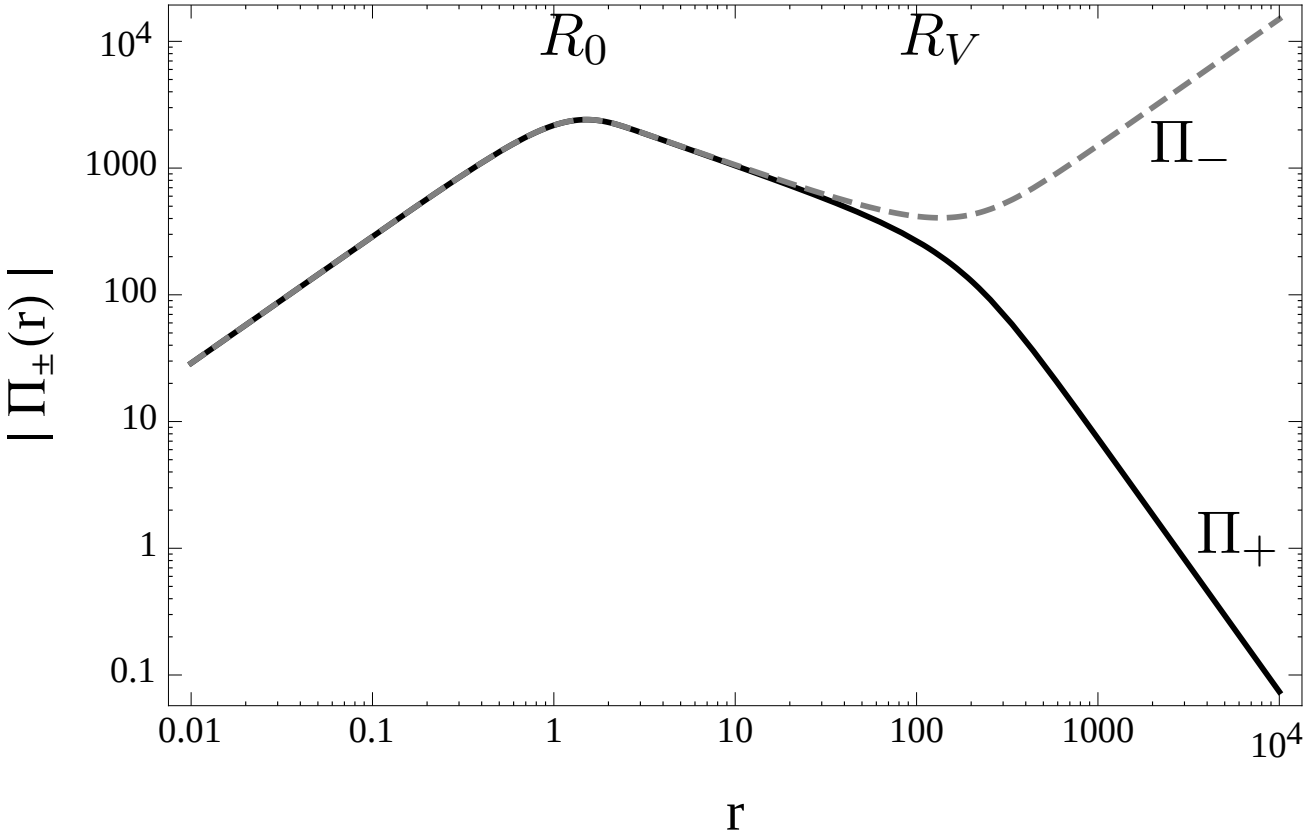
$$6r^2\Pi + 4r\Pi^2 = \frac{M(r)}{8\pi}$$

$$\Pi \equiv \pi'$$

quadratic in Π $\Rightarrow$ two real solutions



$$\Pi_\pm(r) = -\frac{3r}{4} \pm \frac{\sqrt{rM(r) + 18\pi r^4}}{4\sqrt{2\pi}r}$$



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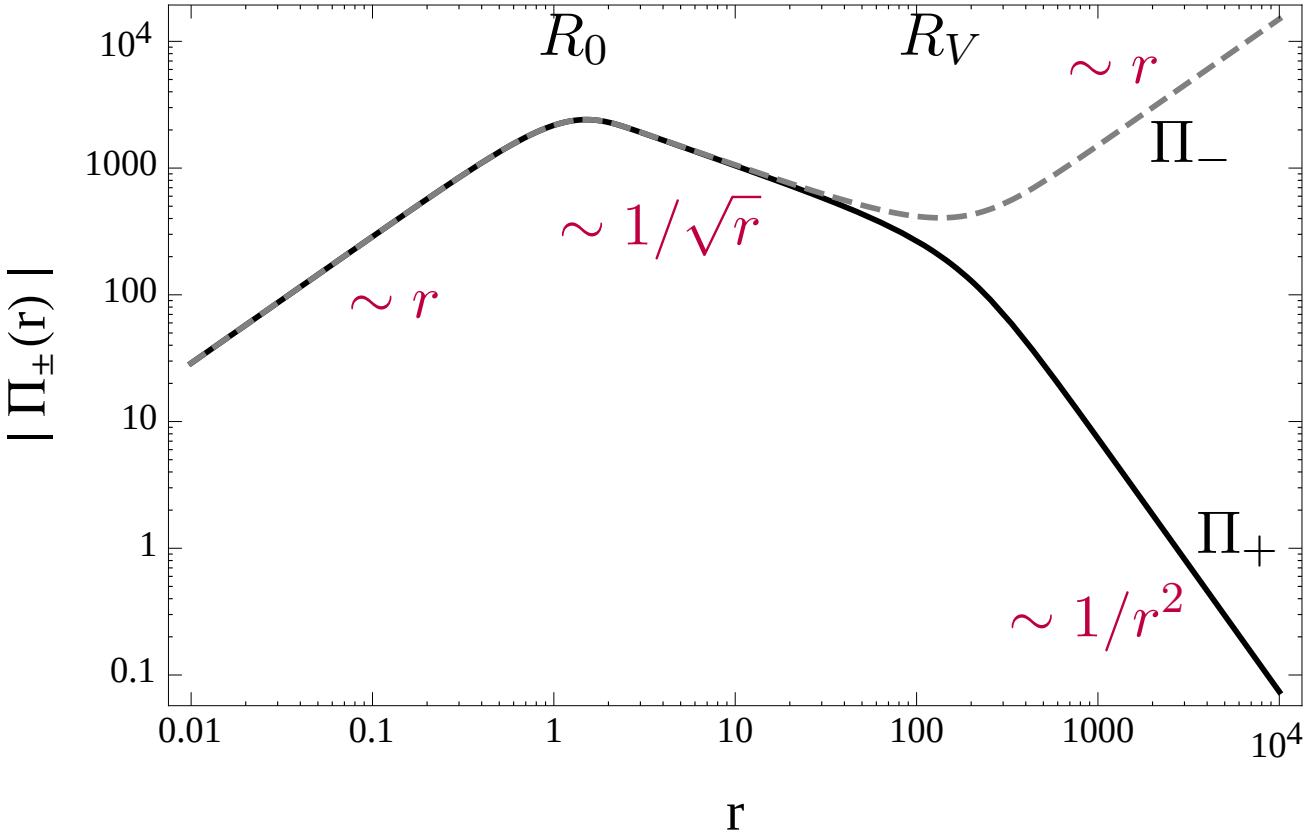

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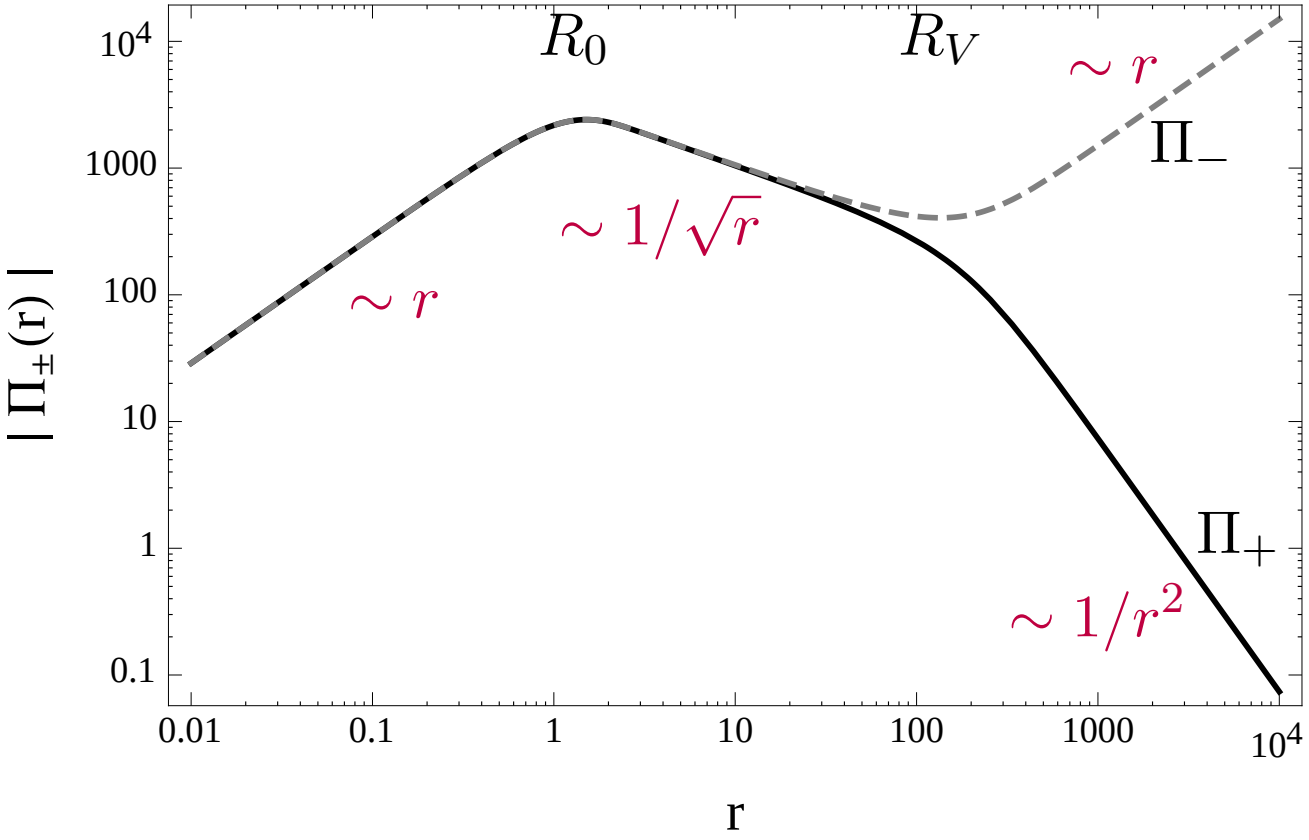

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Numerics

$$\ddot{\pi}\left(6+8\frac{\pi'}{r}+4\pi''\right)=\frac{1}{2}T+12\frac{\pi'}{r}+6\pi''+4(\dot{\pi}')^2+8\frac{\pi'\pi''}{r}+4\frac{(\pi')^2}{r^2}$$

- A numerical treatment is necessary to fully explore the stability of screening solutions and investigate the general properties of time evolution

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- Integrate the resulting ODEs using a fourth order explicit **Runge-Kutta method**

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- A numerical treatment is necessary to fully explore the stability of screening solutions and investigate the general properties of time evolution
- Evolve using **method of lines** with a second order differencing scheme
- Integrate the resulting ODEs using a fourth order explicit **Runge-Kutta method**
- Choose parameters ρ and R_0 to give sufficient hierarchy between R_0 and $R_V = \sqrt{\pi}\rho^{1/3}R_0$ so that we can resolve the screening regime.

Physically realistic values $R_V \sim 10^9 R_0$ are numerically inaccessible.
We use $R_V \gtrsim 10R_0$

Results

1. What initial configurations evolve to the screening solution?
   - **a.** Perturbations
   - Quasinormal modes and tails
   - **b.** Starting from vacuum
2. How does the screening solution react to dynamical sources?
   - **c.** Collapsing source
   - **d.** Exploding source
3. Cauchy Breakdown

❶ **a.** Perturbing the static screening solutions

Initial Condition:

$$\begin{aligned}\pi(r,0) &= \pi_{\pm}(r) + \frac{A}{r}\exp\left(-\frac{(r-r_w)^2}{2\sigma^2}\right)\\ \dot{\pi}(r,0) &= -\frac{A(r-r_w)}{\sigma^2 r}\exp\left(-\frac{(r-r_w)^2}{2\sigma^2}\right)\end{aligned}$$

We expect that the wave will dissipate away in a stable manner, leaving behind the original static screening solution at late times.

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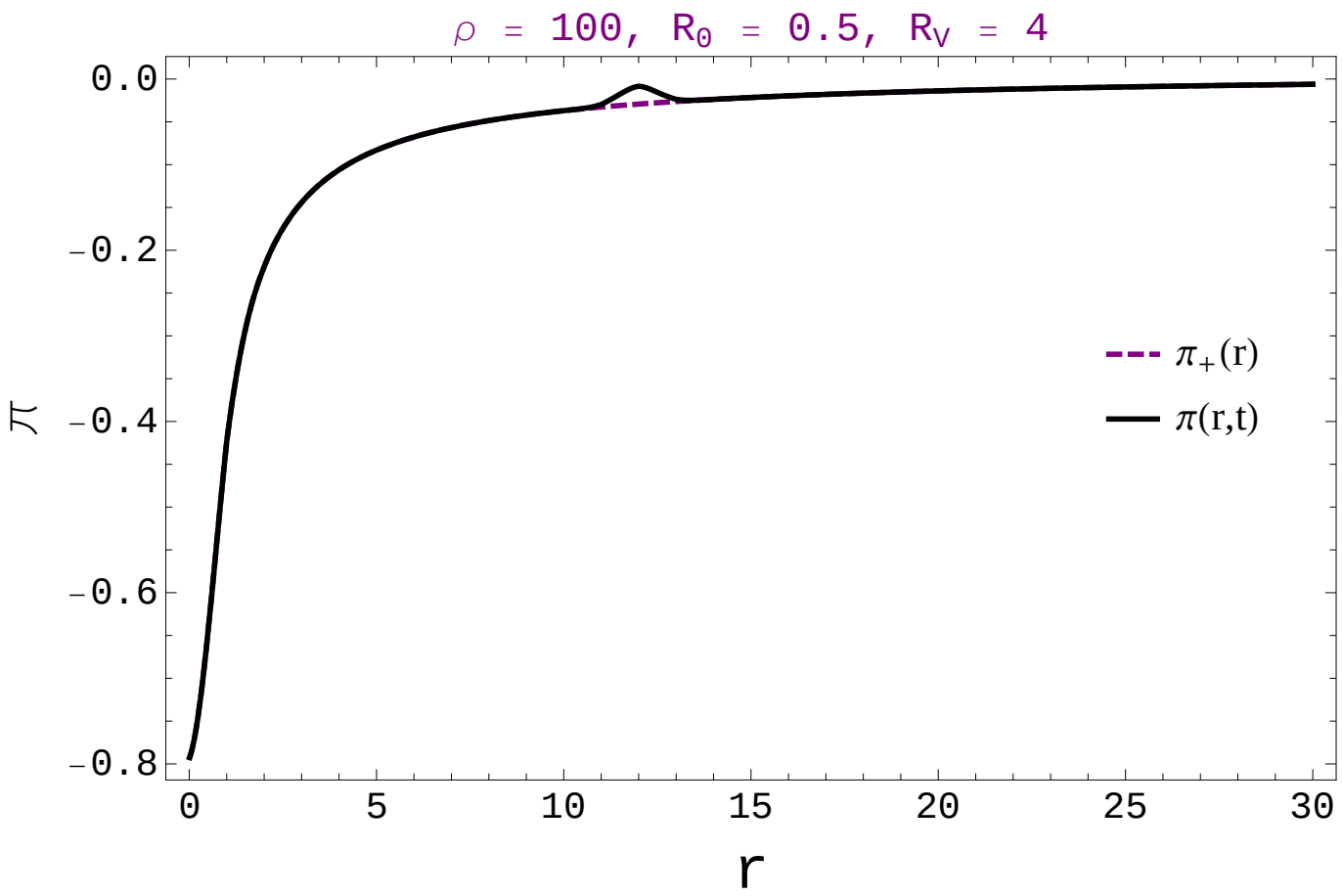

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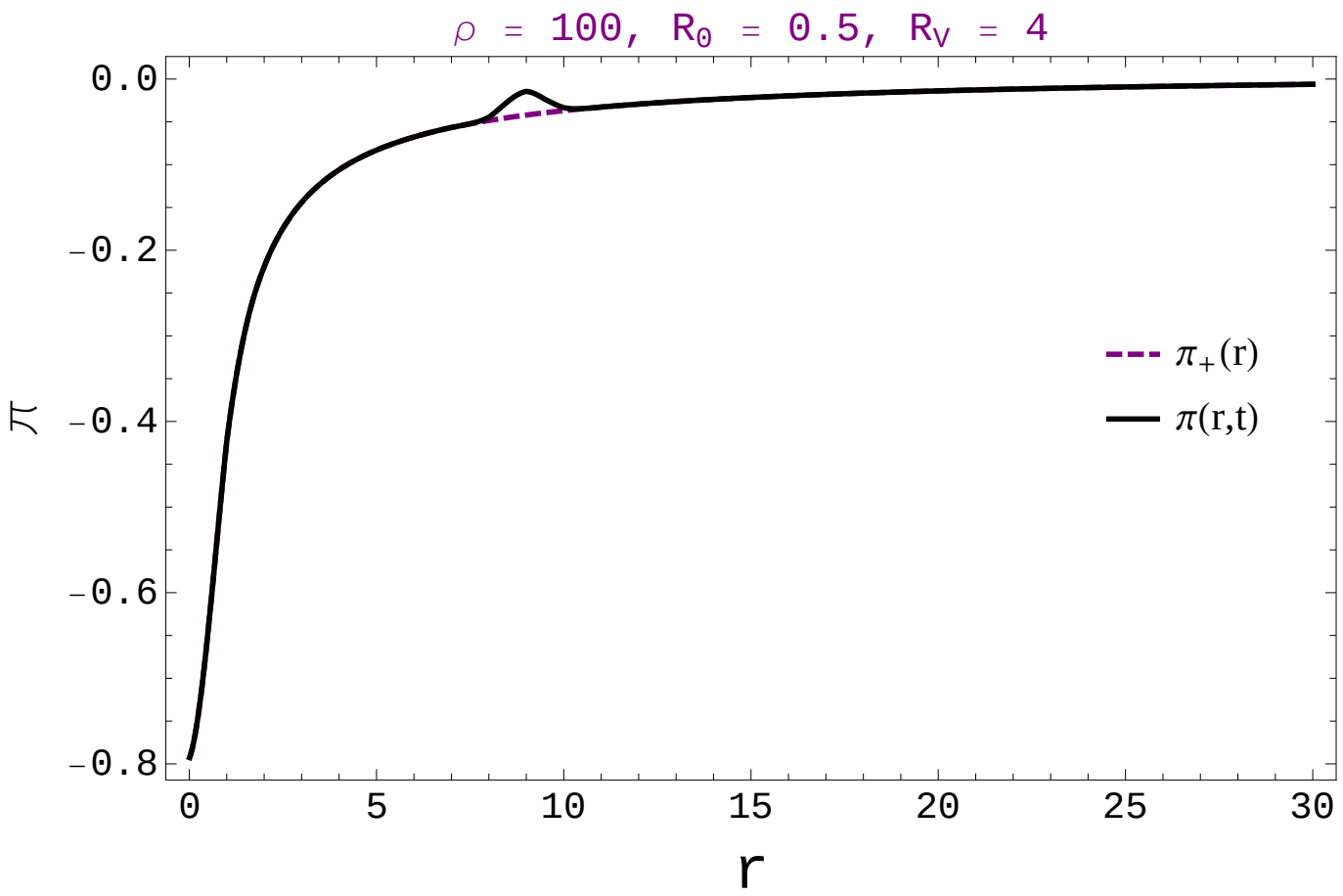

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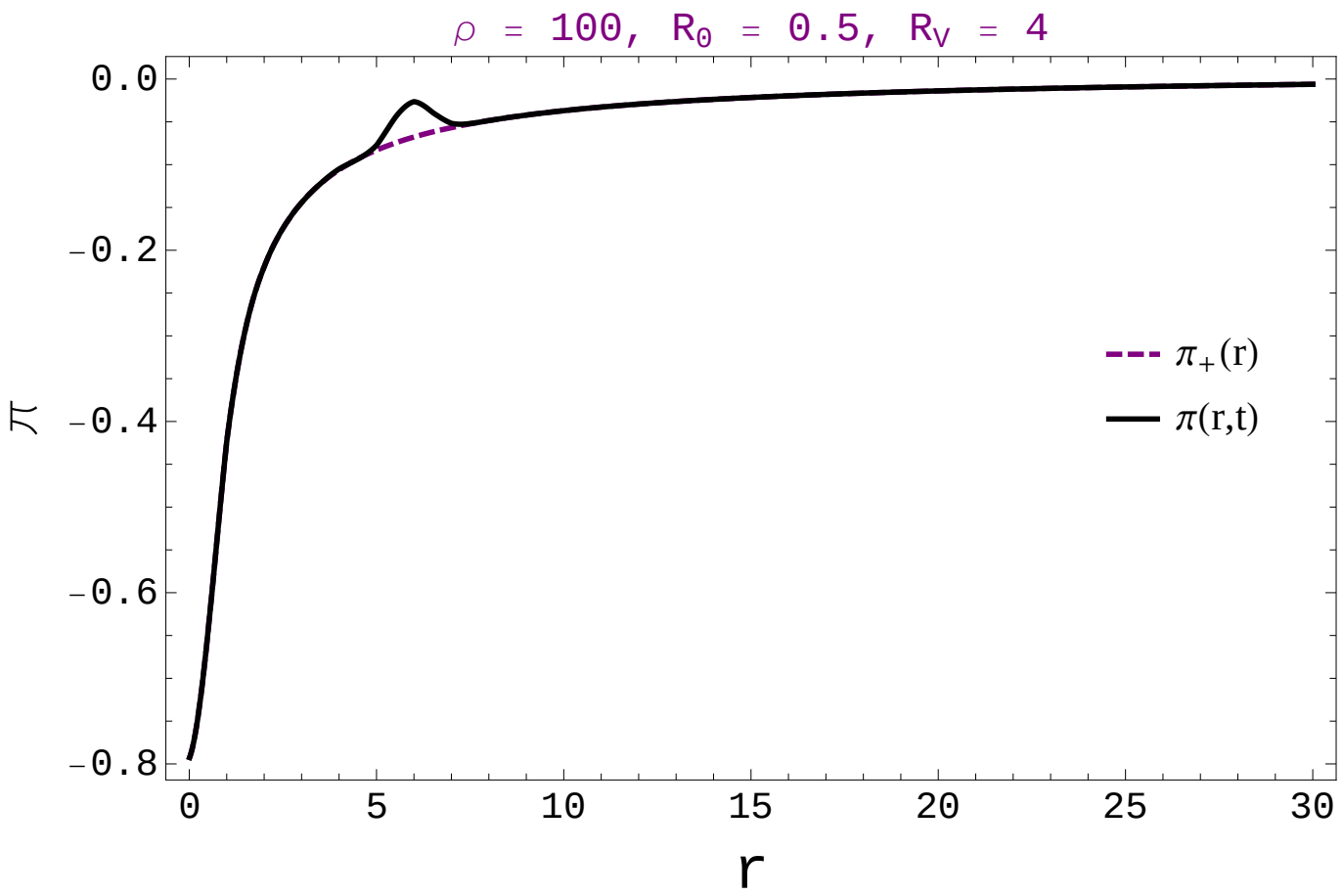

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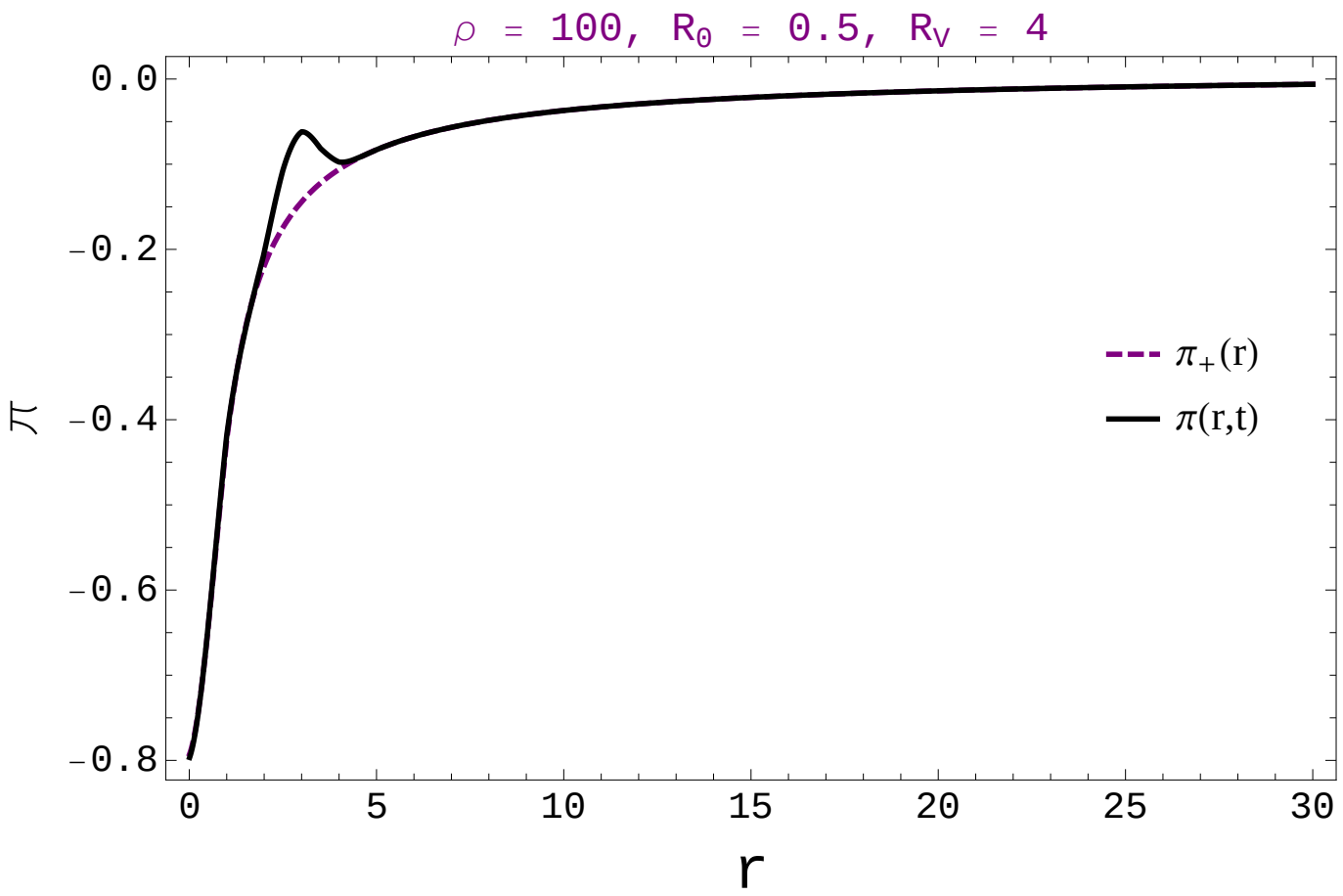

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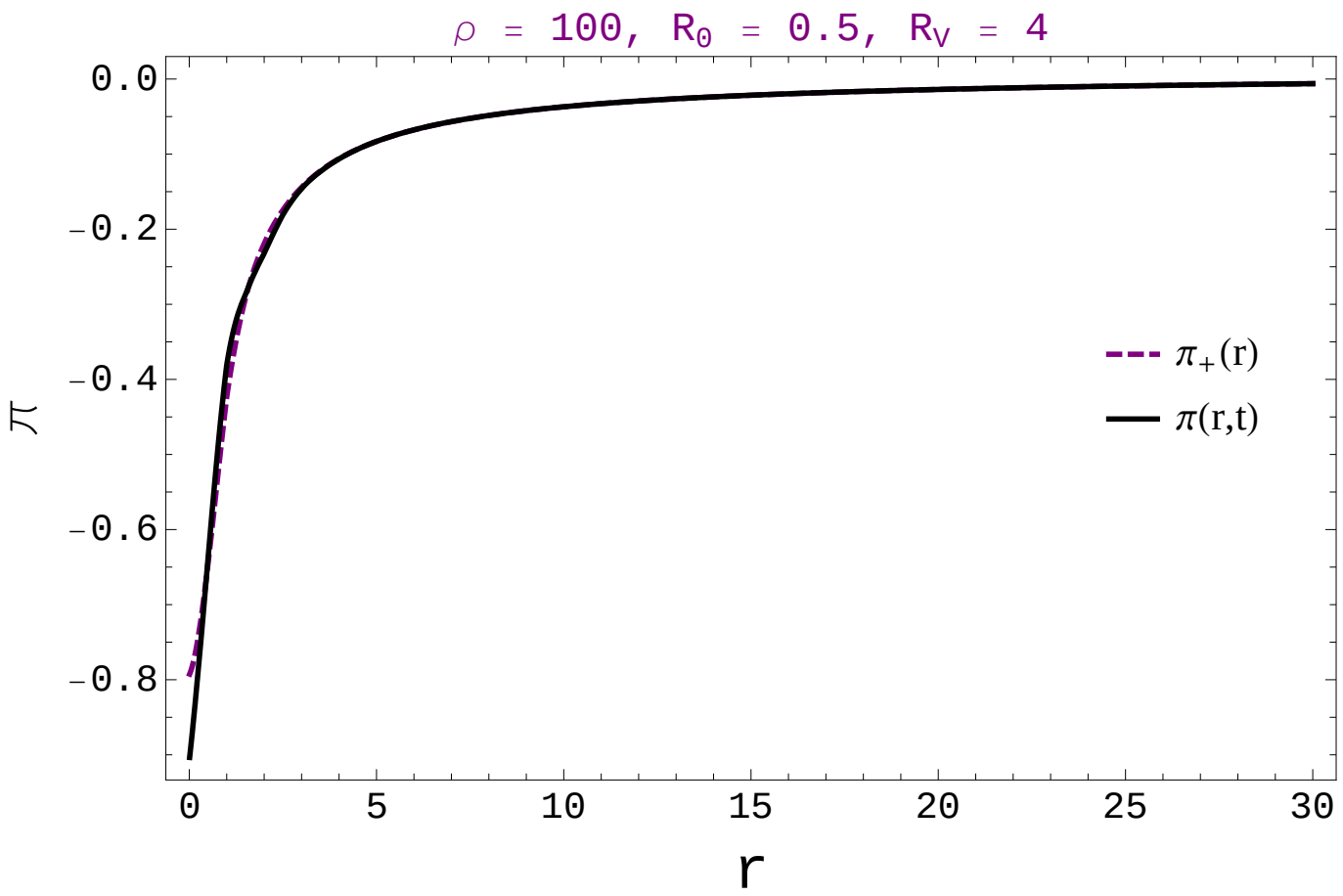

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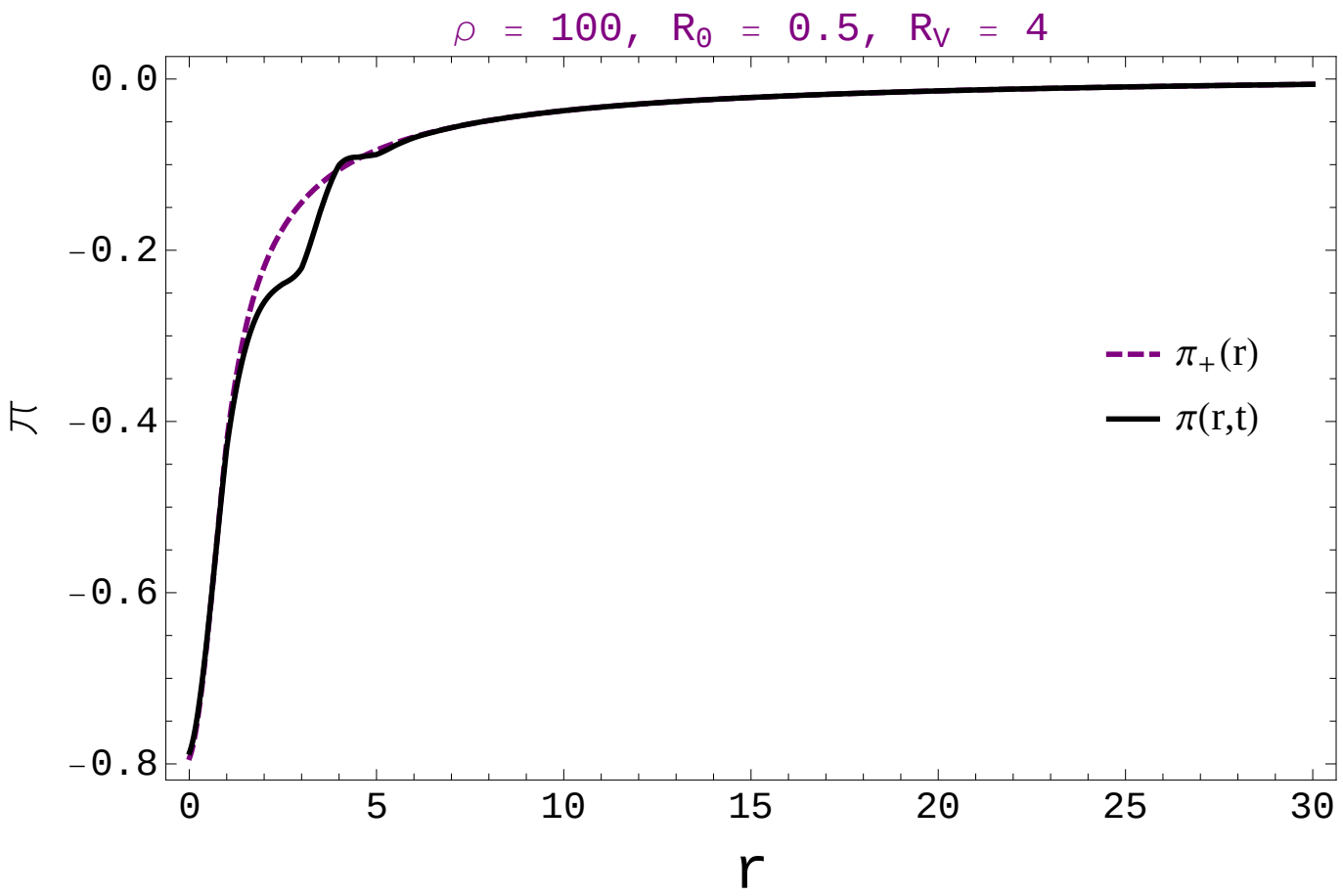

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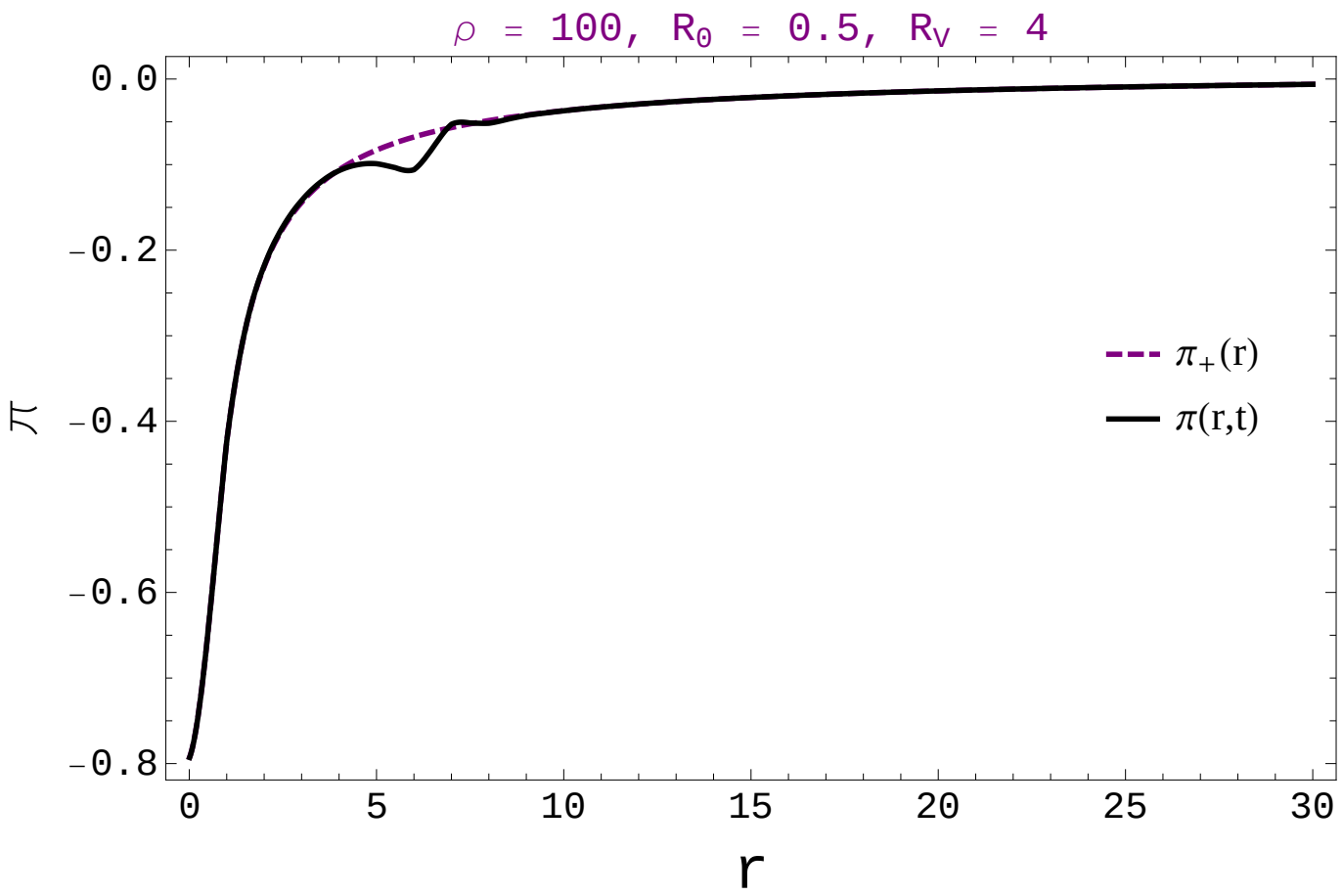

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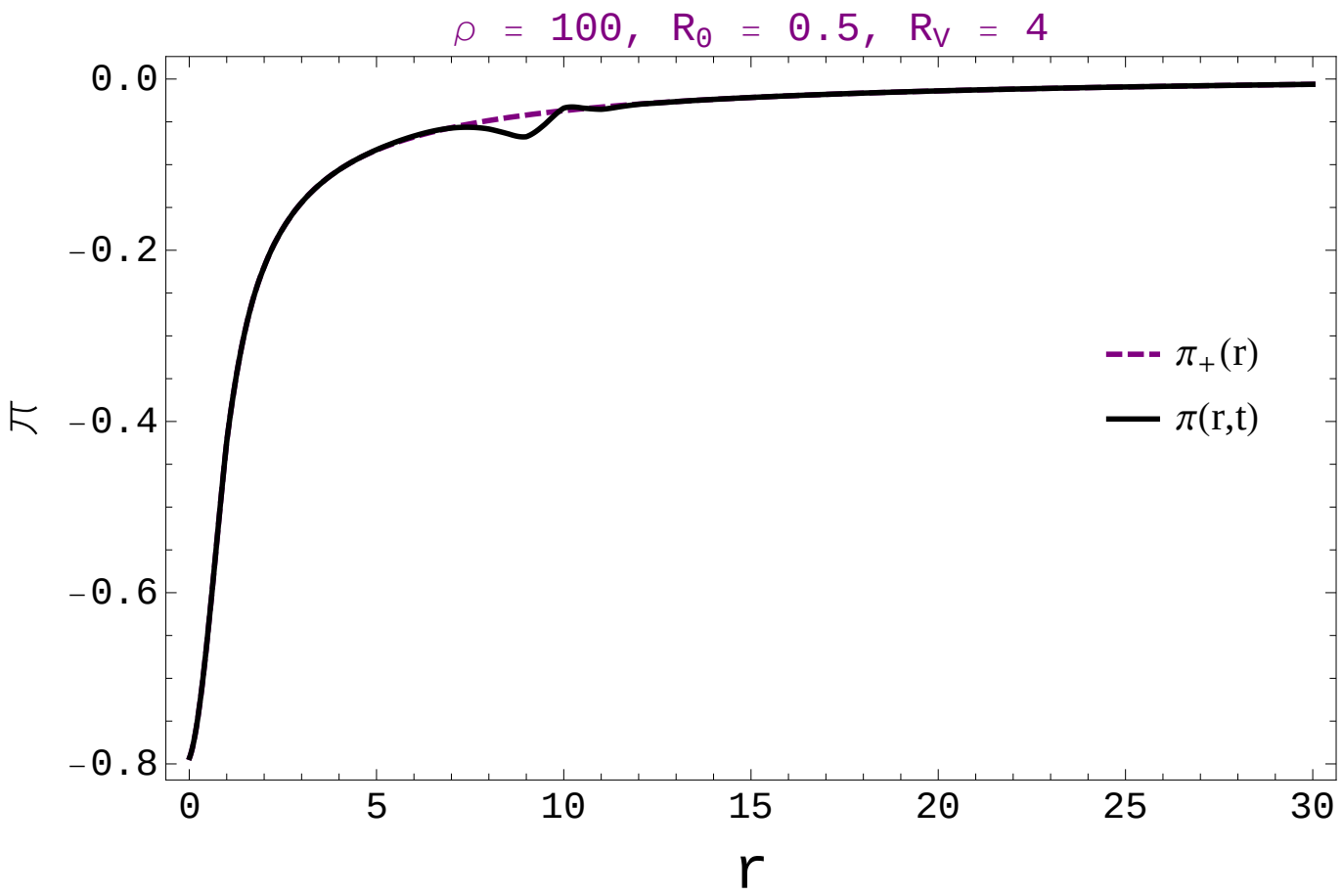

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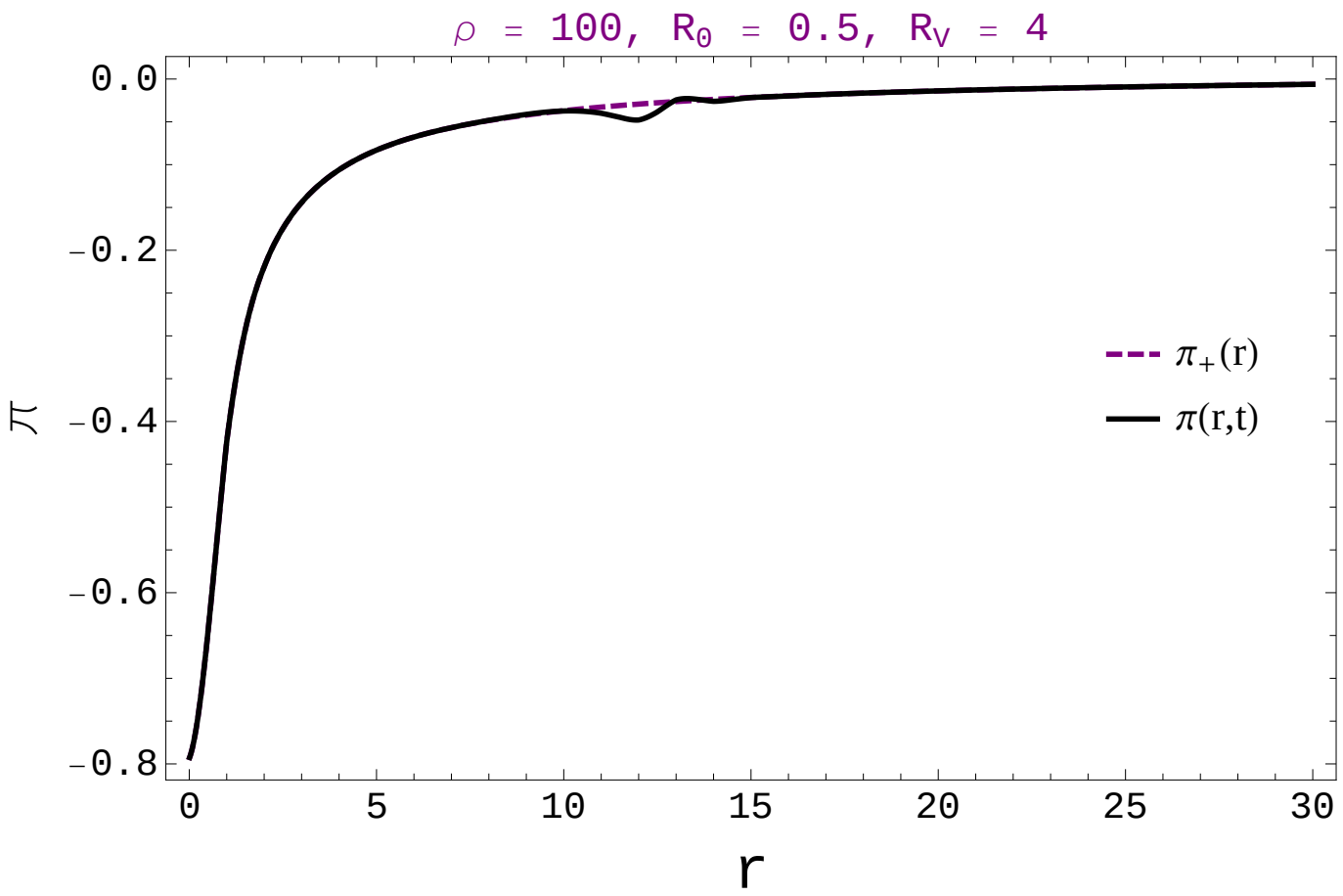

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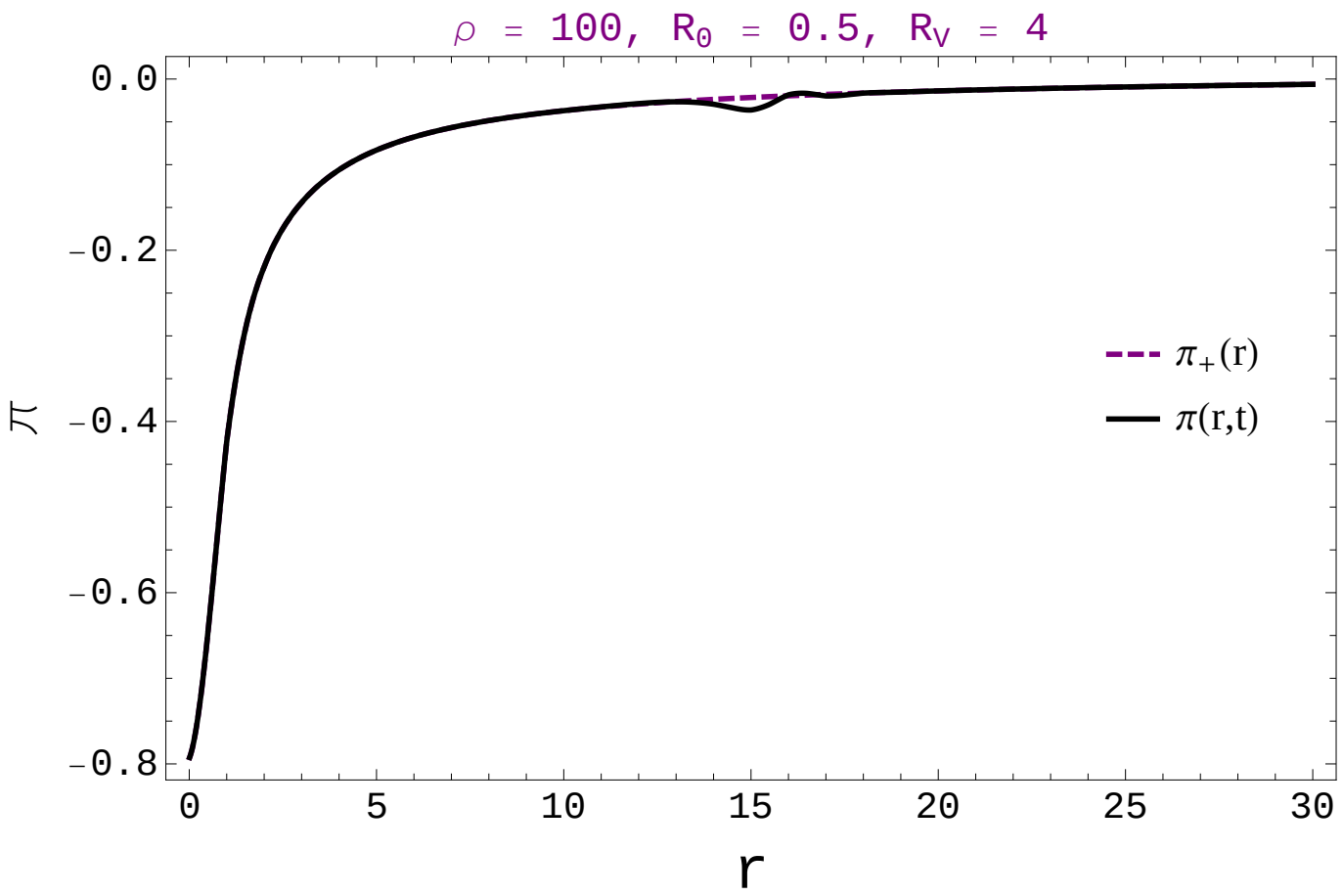

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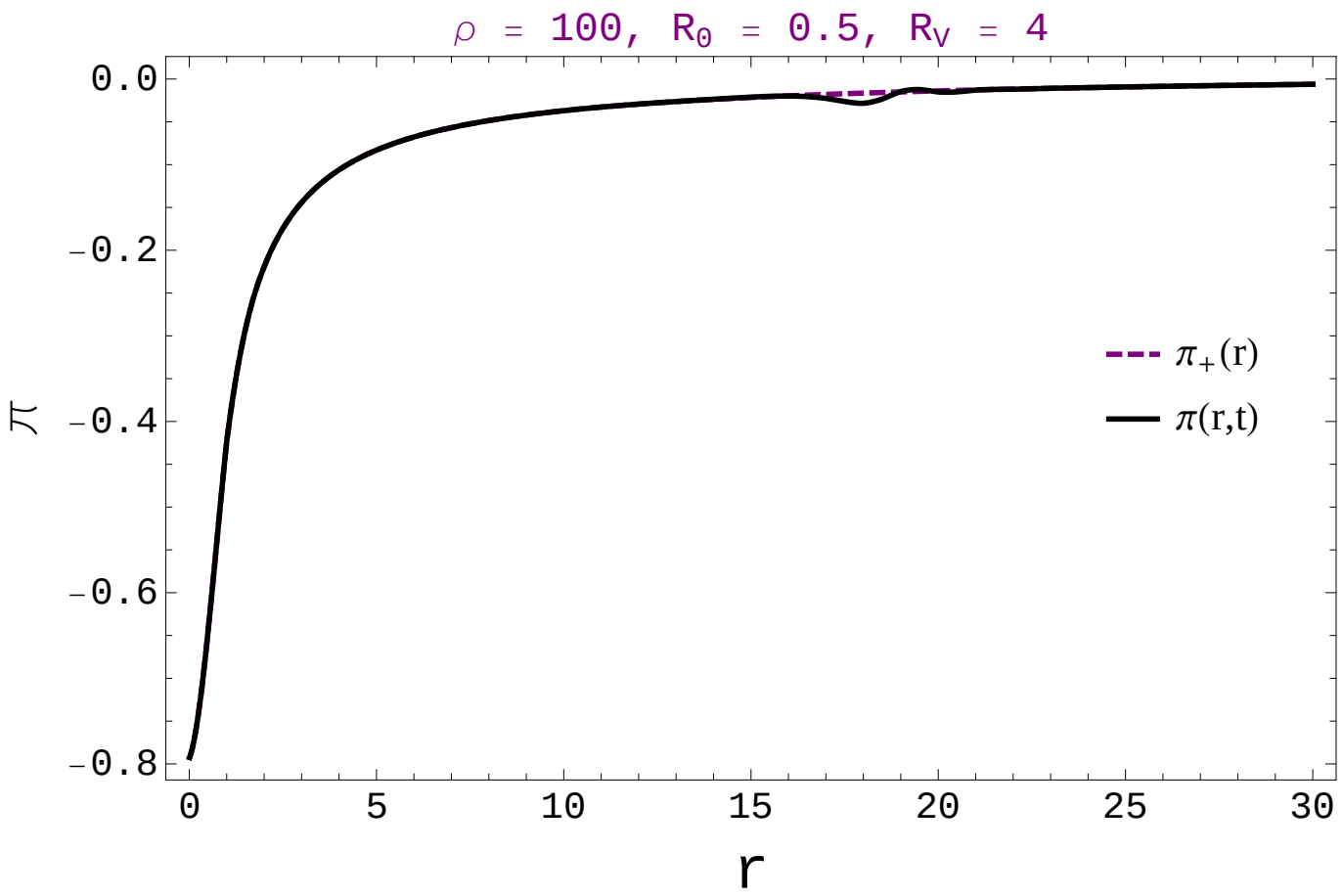

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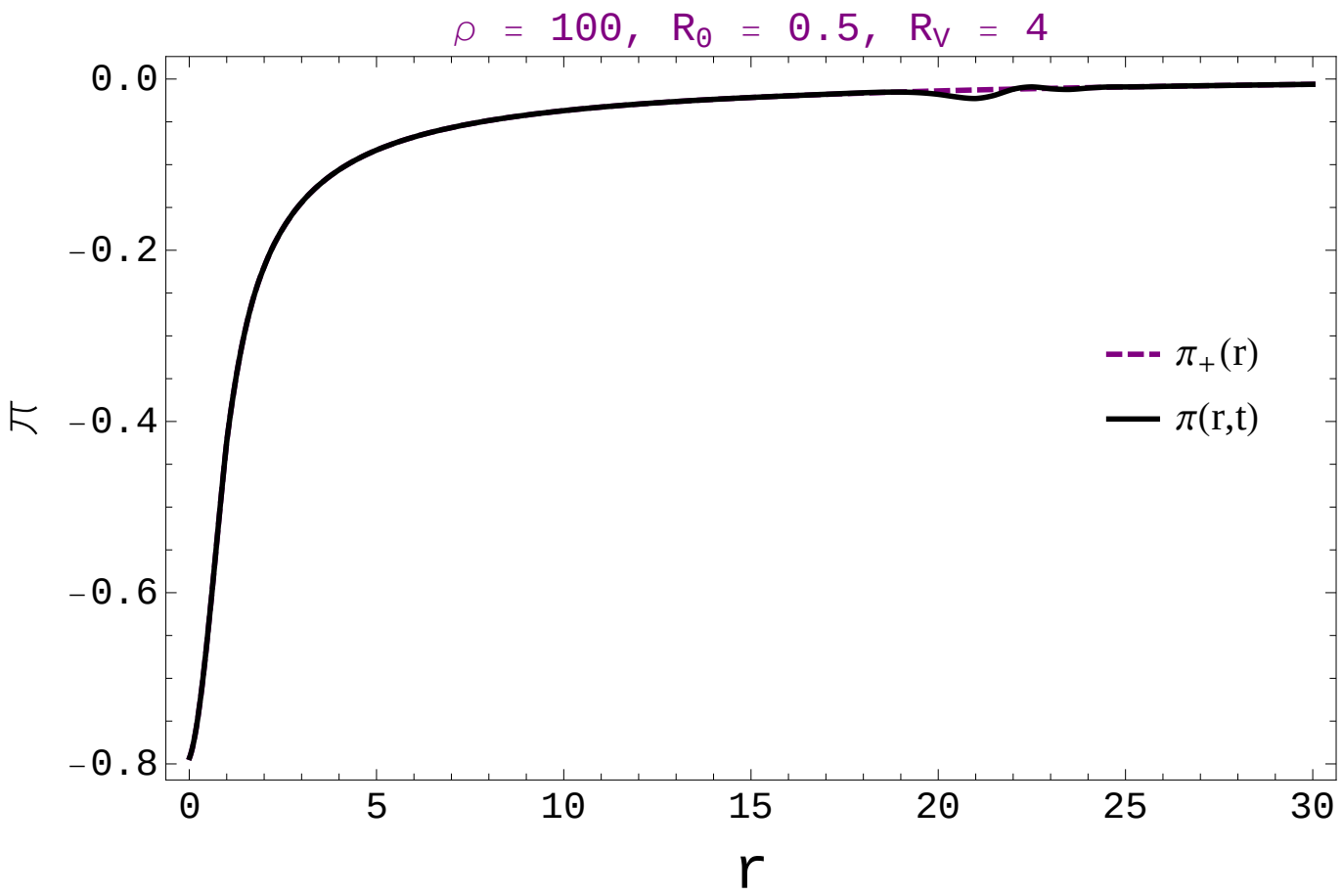


After the initial response to the primary scattering of the wave packet, we observed that the screening solution *vibrates* and eventually *settles* to the original static solution.

To understand this behaviour more thoroughly, we performed an analysis of small fluctuations of the screening solution...

Fluctuations Propagate in an Effective Metric

Small perturbations $\delta\pi$ around the screening solution propagate in an **effective metric** $Z^{\mu\nu}$:

$$S_{\delta\pi} = \int d^4x \left[\frac{1}{2} Z^{\mu\nu} \partial_\mu \delta\pi \partial_\nu \delta\pi + \frac{1}{2M_4} \delta\pi T\right]$$

Effective metric components:

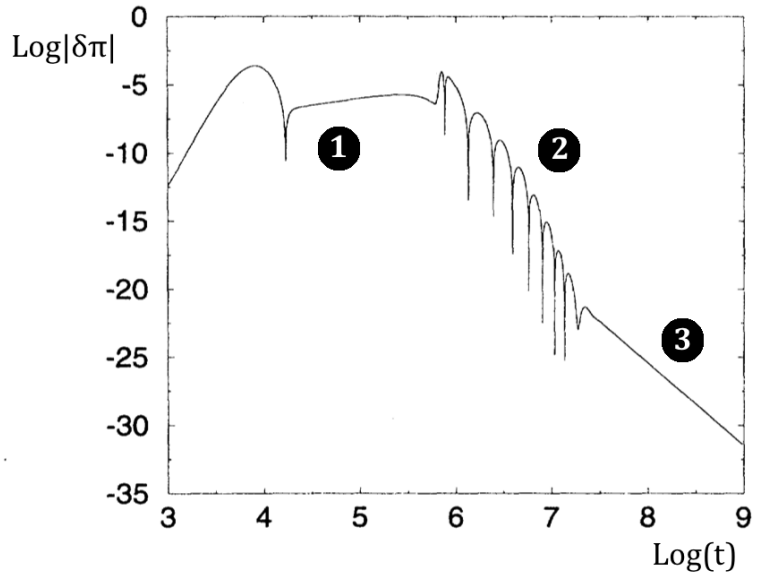
$$Z^{tt} = -\left(6 + 8\frac{\Pi}{r} + 4\Pi'\right), \qquad Z^{rr} = 6 + 8\frac{\Pi}{r}$$

$$r^2 Z^{\theta\theta} = r^2 \sin^2\theta Z^{\phi\phi} = 6 + 4\frac{\Pi}{r} + 4\Pi'.$$

Quasinormal Modes and Tails

Waveforms propagating in gravitational systems have three universal stages:

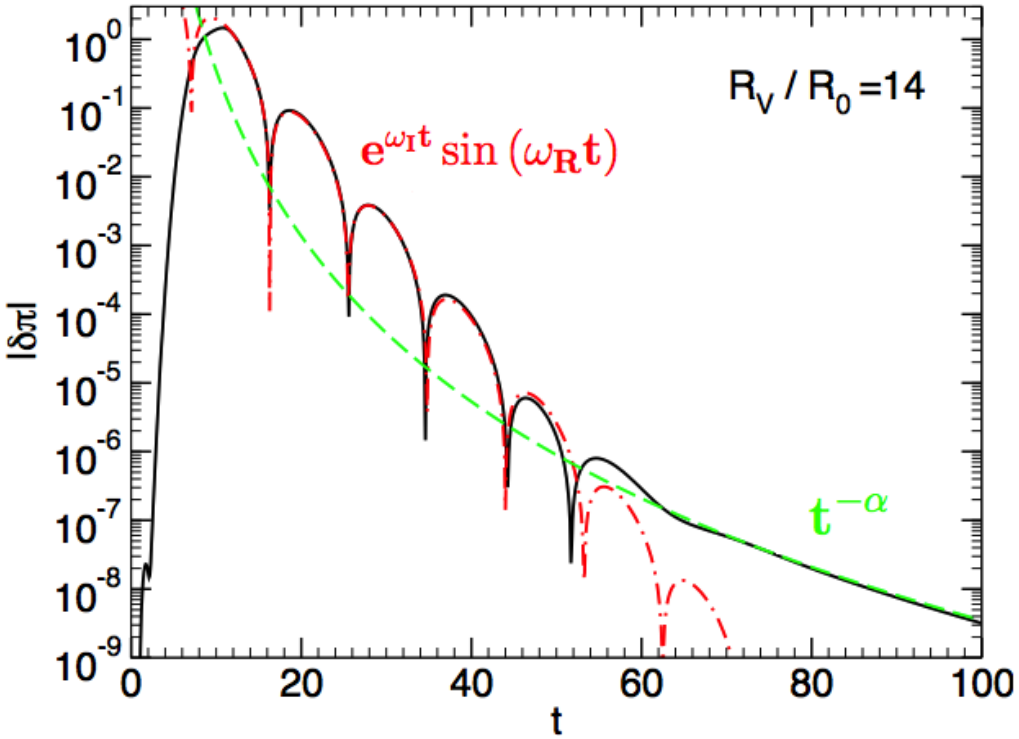
1. Early times: prompt response due to the primary scattering
2. Intermediate times: exponentially damped sinusoids, termed **"quasinormal modes"**
3. Late times: power-law decay **"tails"**



E. Ching, et al, Phys.Rev. D52, 2118 (1995).

This same phenomena is observed and well understood in perturbed stars and black holes.

Quasinormal Modes and Tails

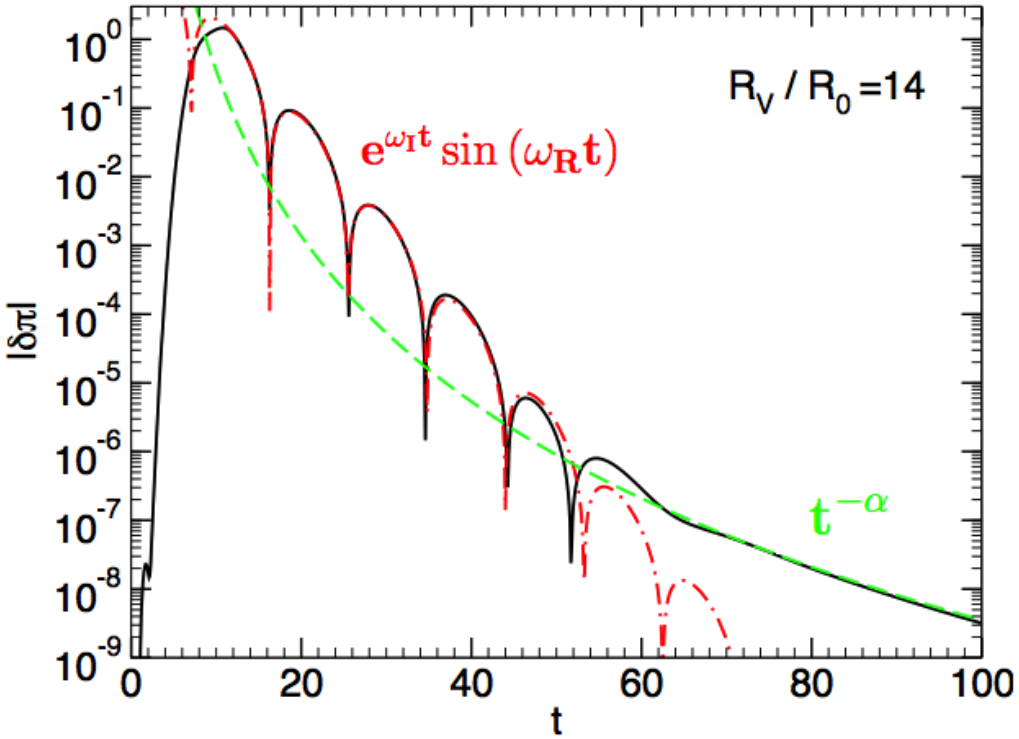


QNM frequencies: Sinusoidal oscillations: $\omega_R \sim 1/R_V$

Exponential decay: $\omega_I \sim -1/R_V$

Late time power-law tail: $t^{-\alpha}$ with $\alpha = 8$

Quasinormal Modes and Tails



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Exponential decay: $\omega_I \sim -1/R_V$

Late time power-law tail: $t^{-\alpha}$ with $\alpha = 8$

Not a property of our source!

The *screening solutions themselves* behave as a coherent object, much like a star or blackhole under linear perturbations, leaving behind the static solution at late times

❶ **b.** From vacuum to screening

In the absence of a source ($T = 0$), there are two vacuum solutions:

$$\pi(r, t = 0) = 0 \qquad\qquad \pi(r, t = 0) \propto r^2$$

In the presence of a source, the vacuum initial conditions will evolve.
We might expect that...

$$\pi(r, t = 0) = 0 \;\longrightarrow\; \pi_+(r) \qquad\qquad \pi(r, t = 0) \propto r^2 \;\longrightarrow\; \pi_-(r)$$

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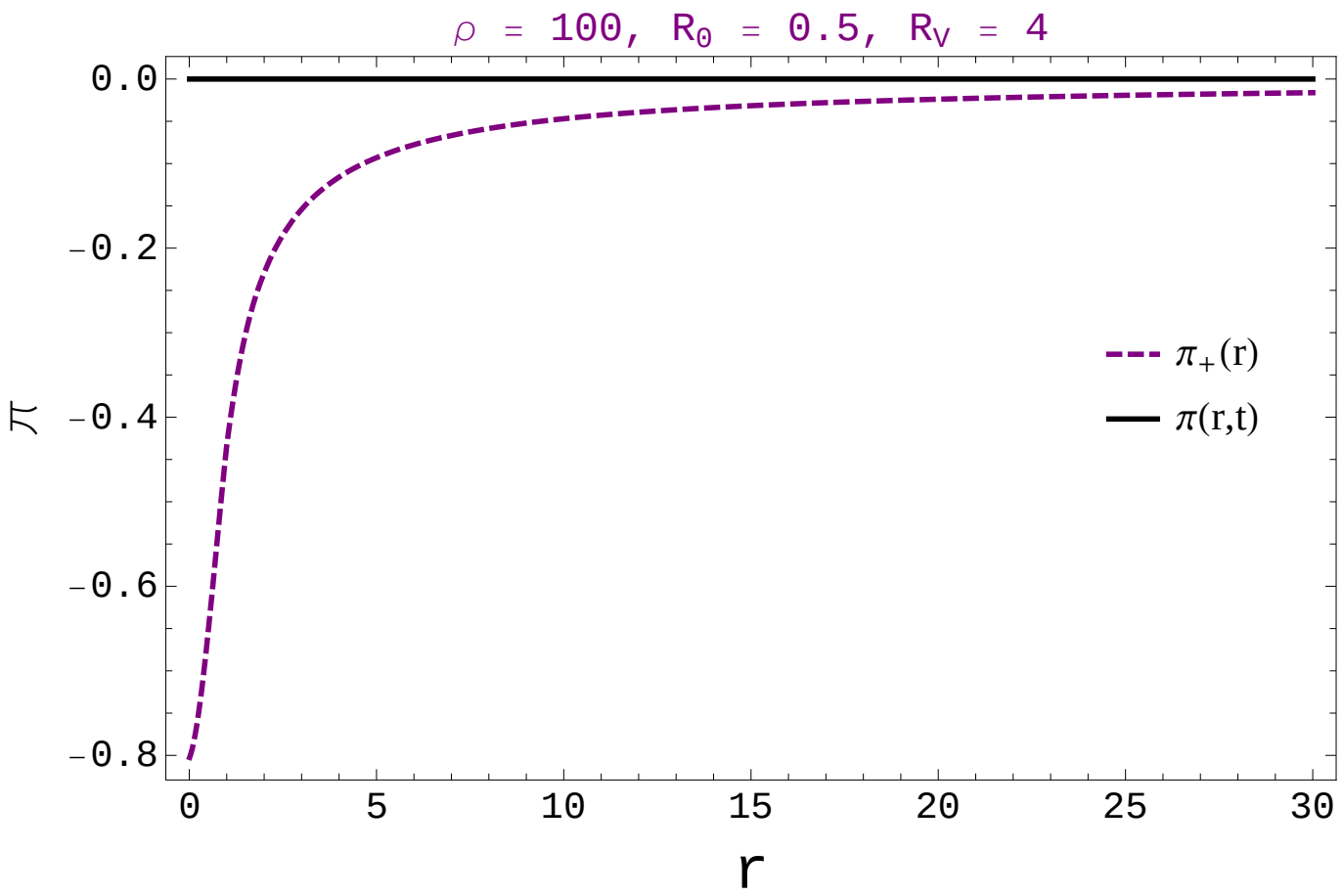

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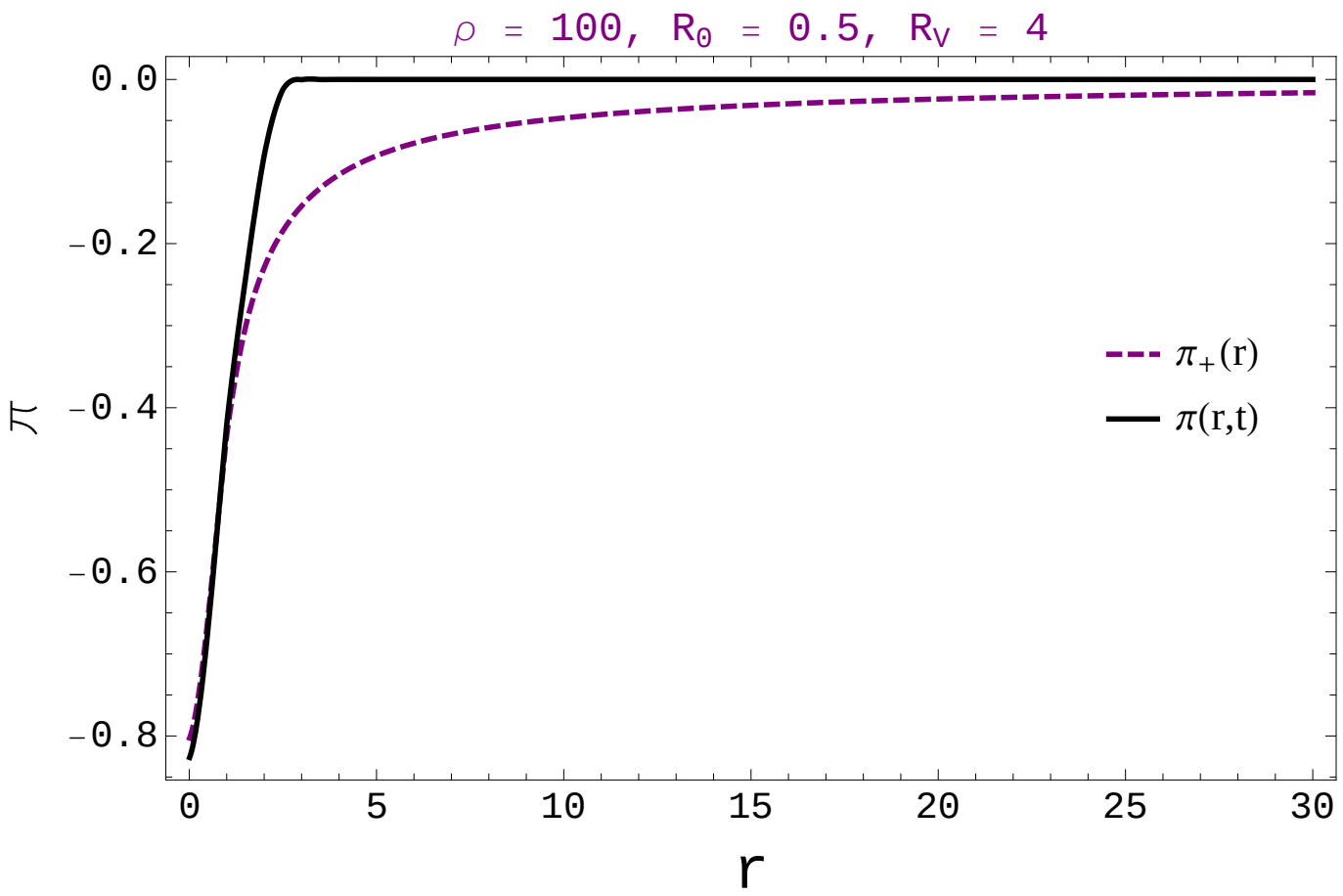

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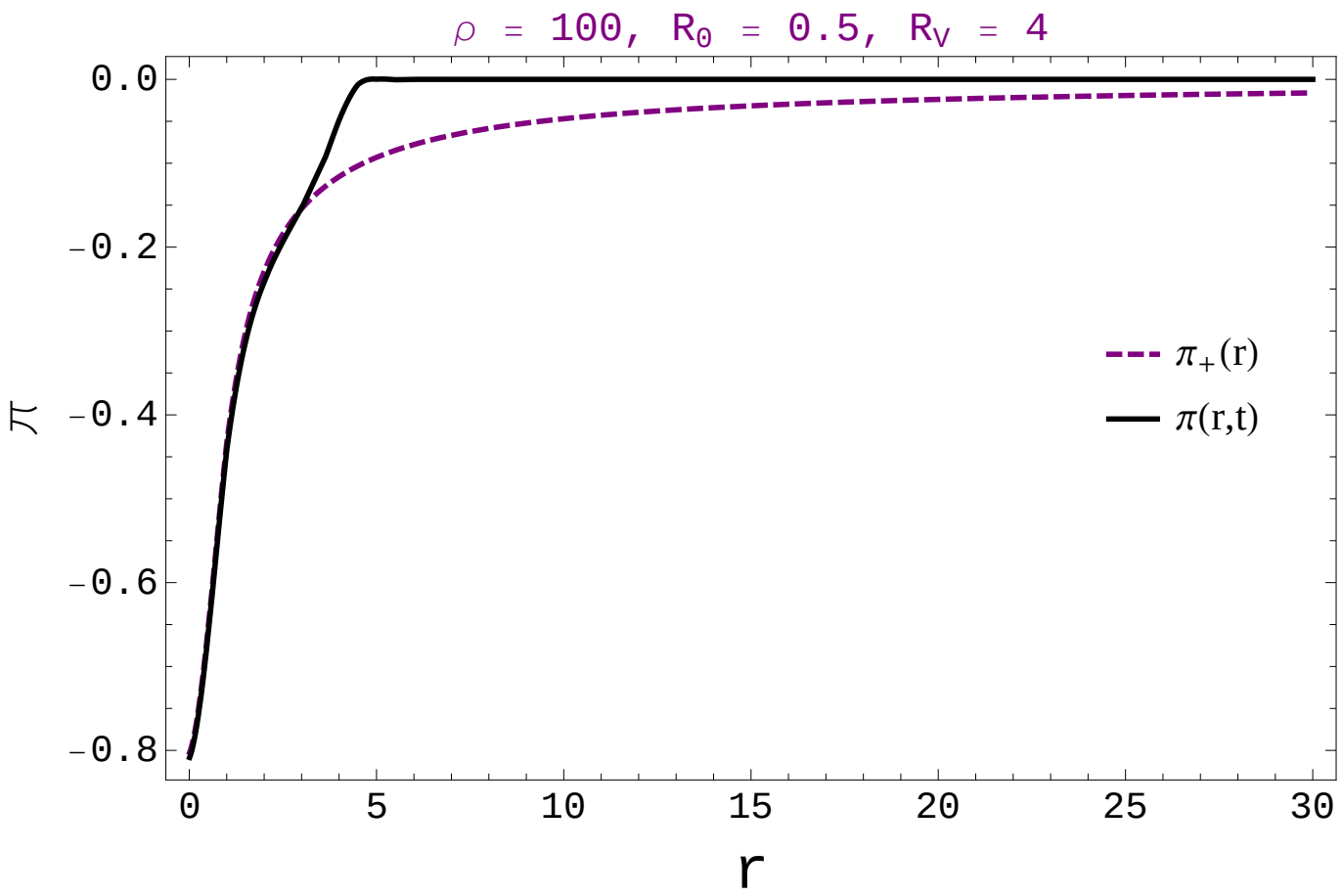

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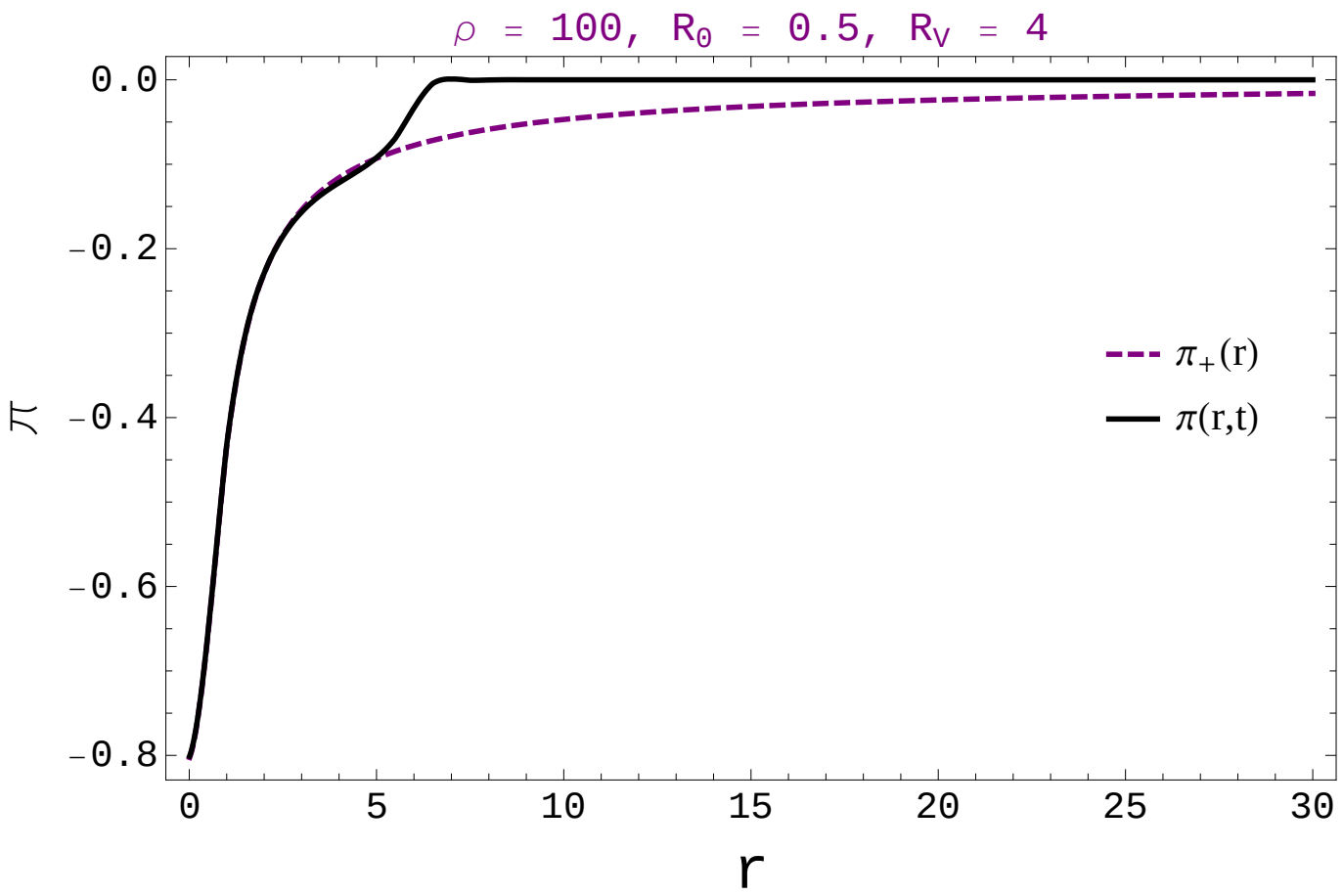

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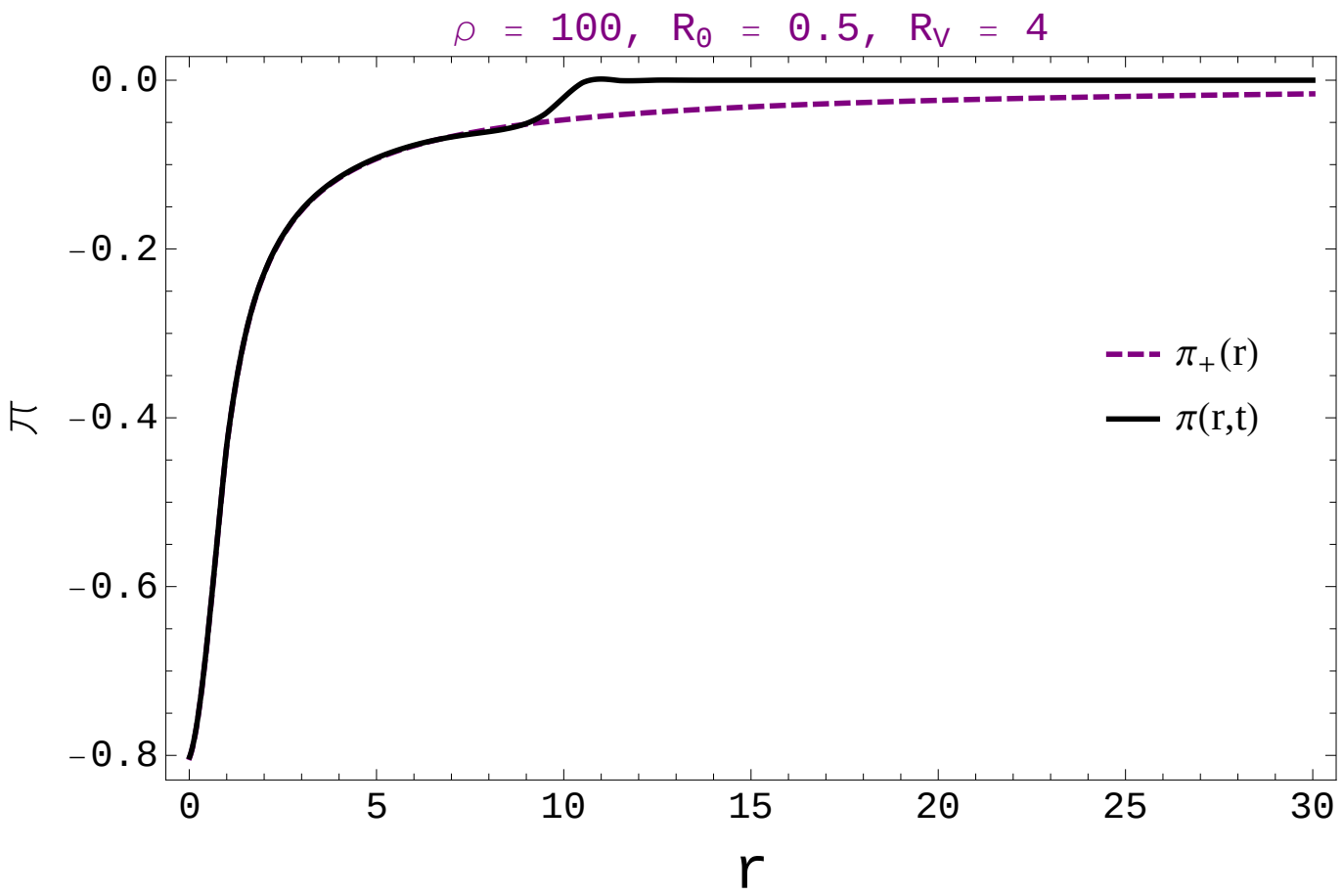

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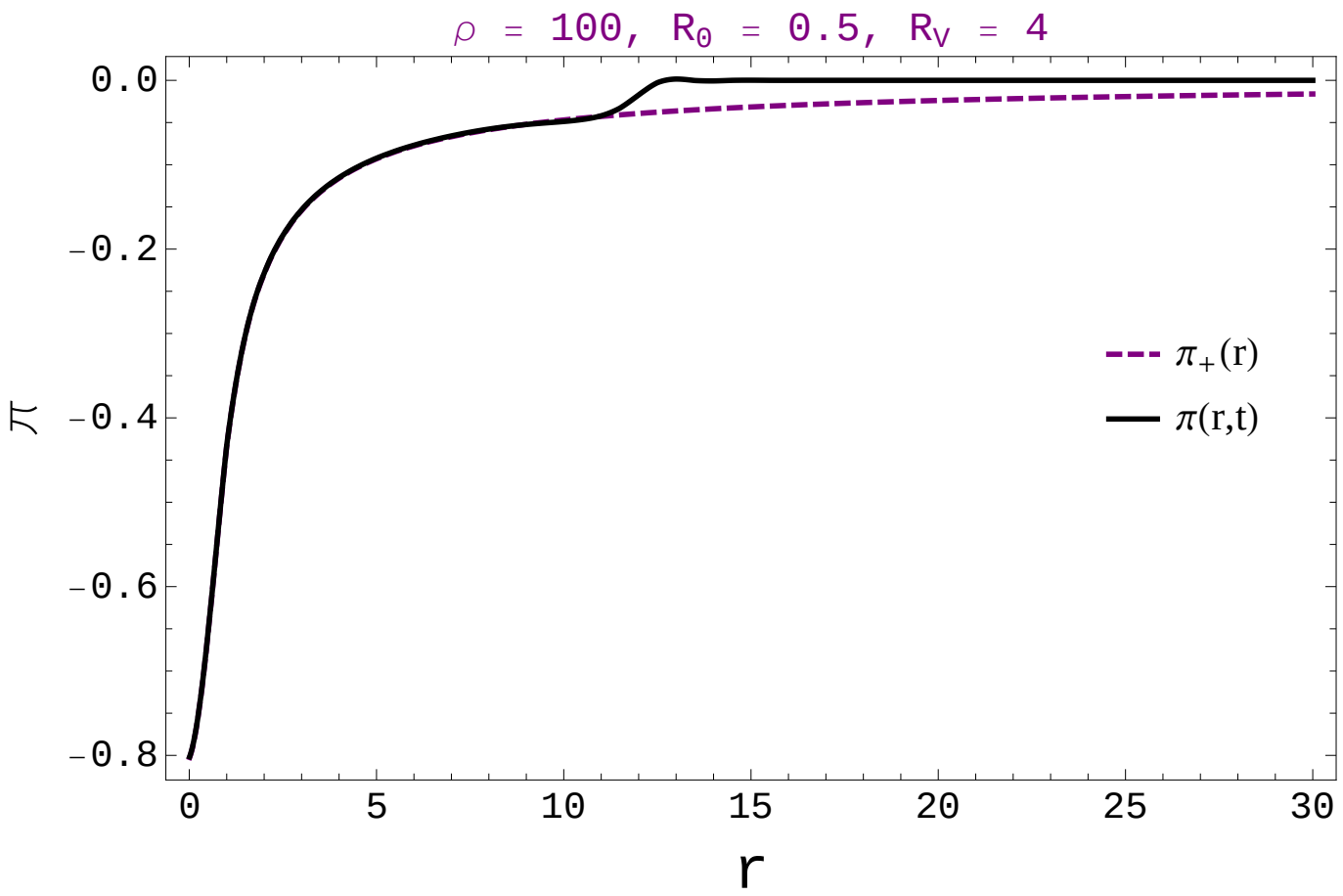

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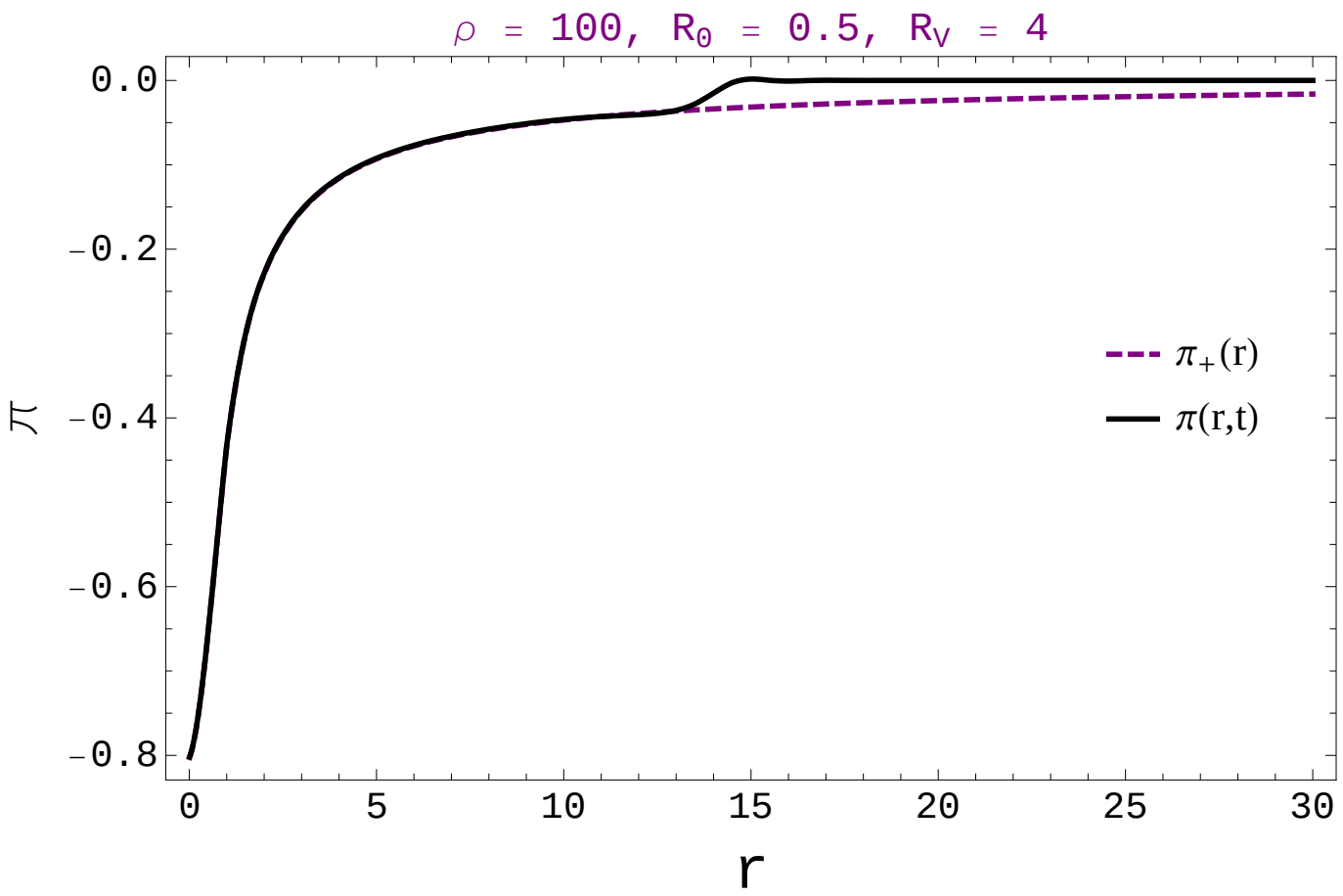

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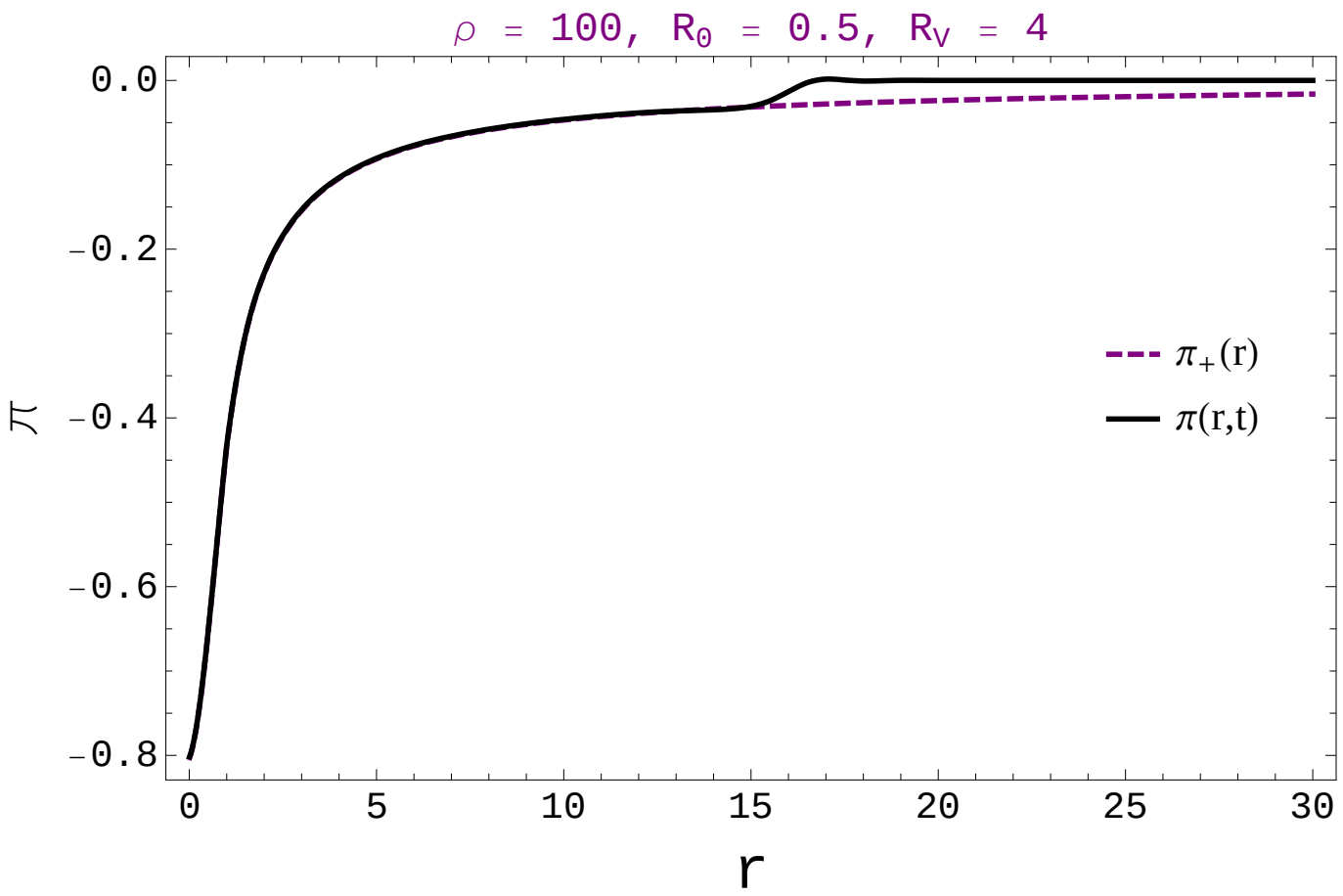

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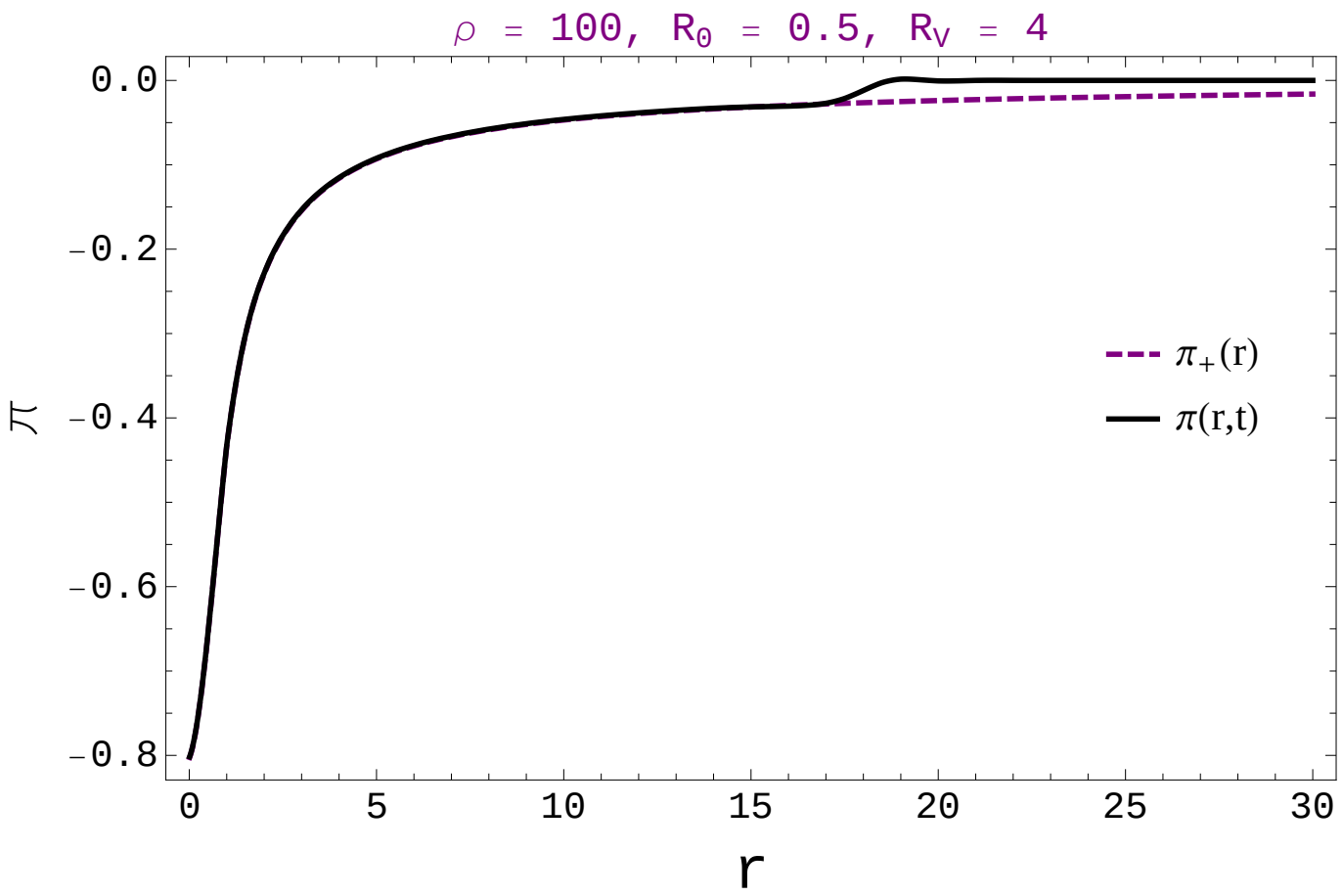

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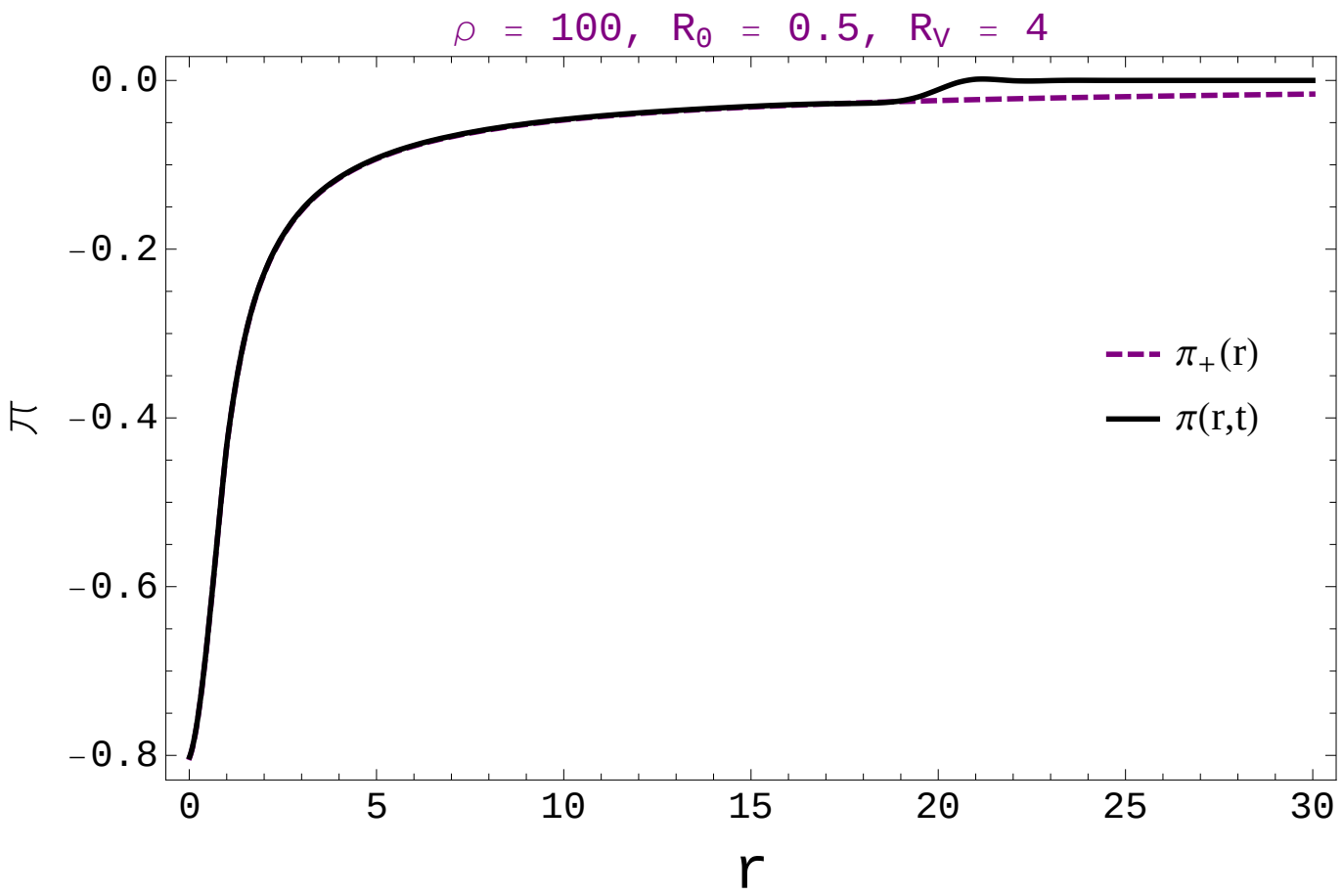

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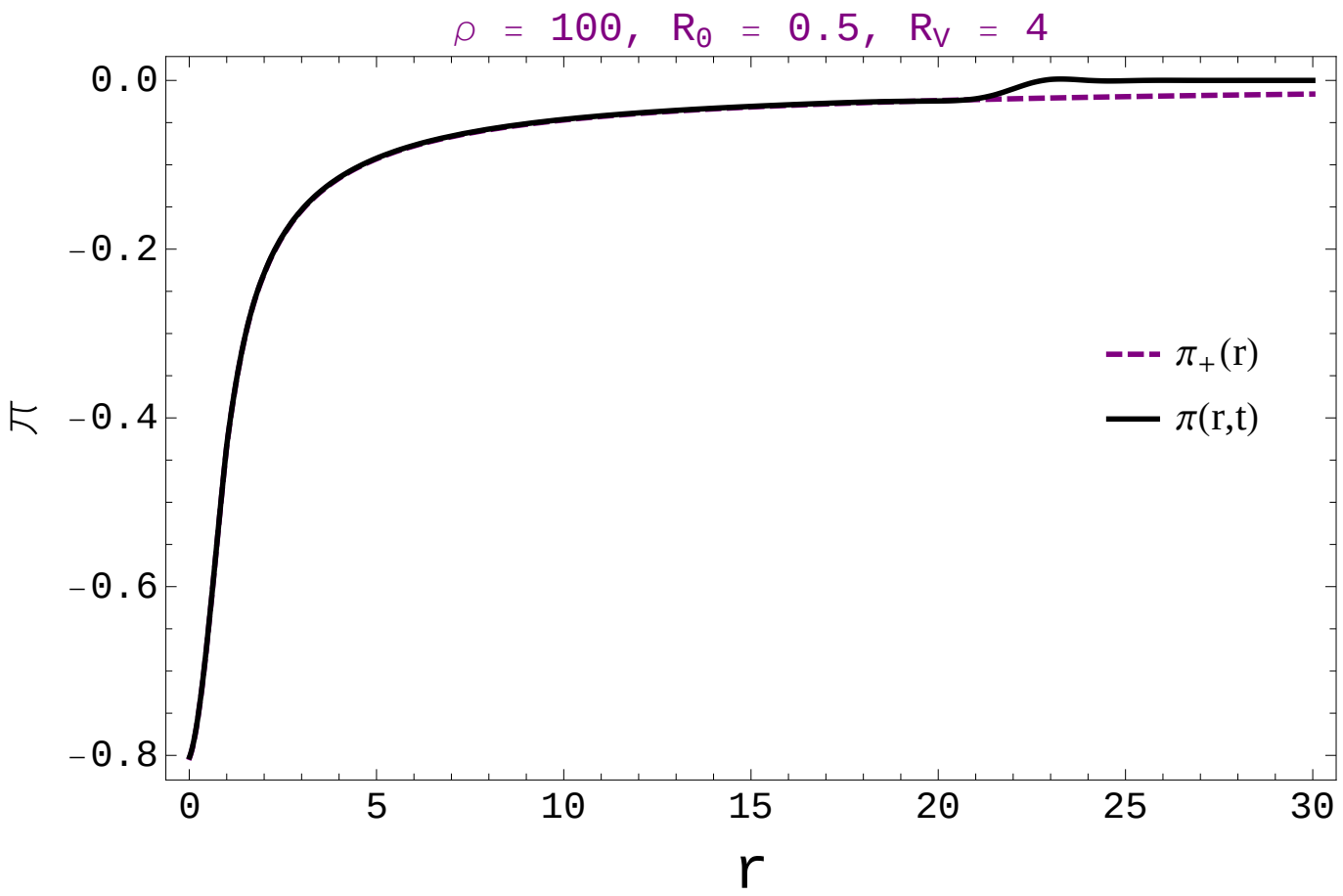

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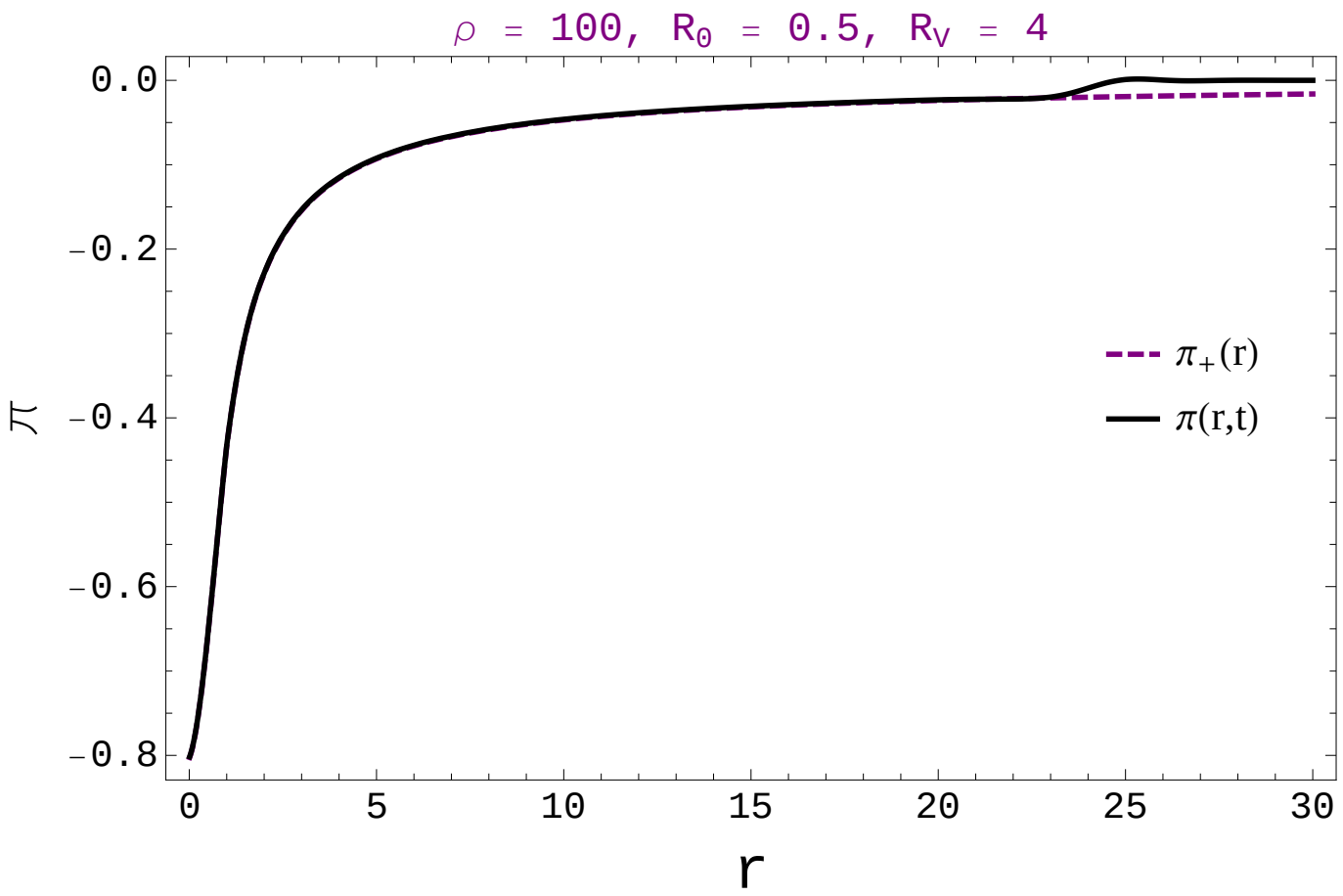

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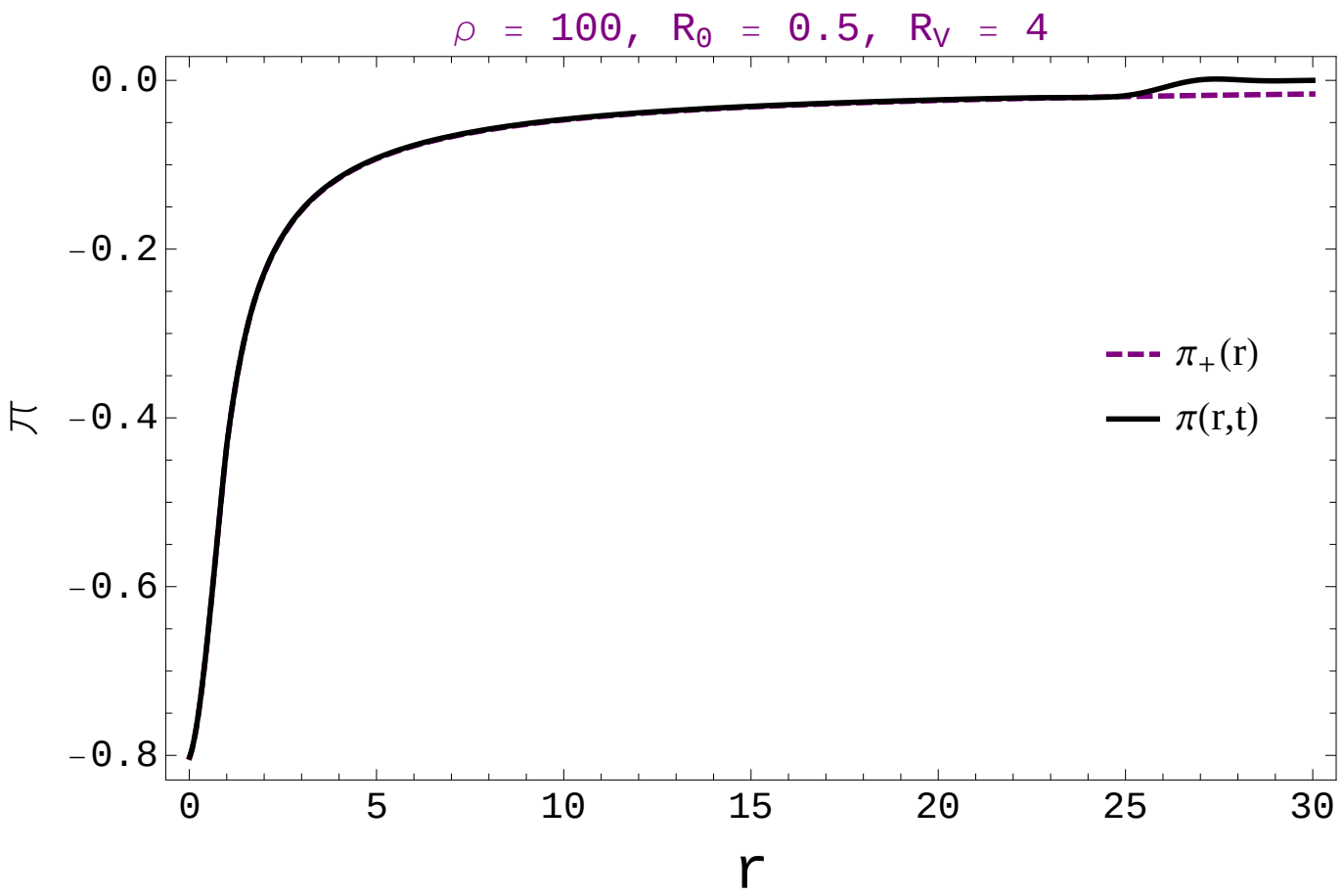

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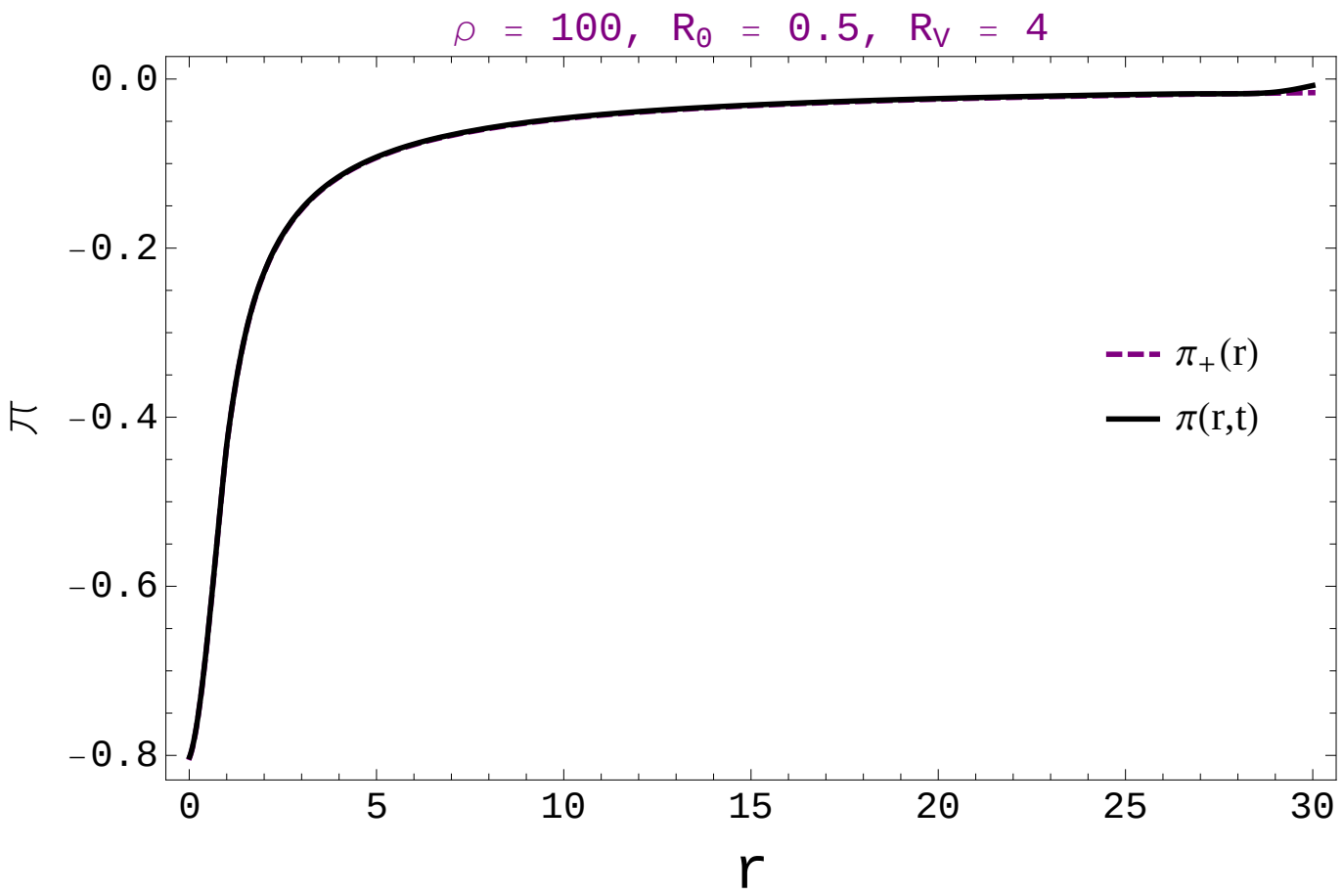


❷ c. Collapsing Source

The source undergoes gravitation collapse to form a relativistic object (e.g. neutron star) with equation of state $p = \rho/3$.

$$
\begin{aligned}
T^{00} =& \rho \exp\left(-\frac{r^2}{R_0^2}\right) \\
T^{ii} =& \frac{\rho}{3} \exp\left(-\frac{r^2}{R_0^2}\right)\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]
\end{aligned}
$$

Therefore the *trace* of $T^{\mu\nu}$ which controls the *coupling* of π to matter is:

$$
T = -\rho \exp\left(-\frac{r^2}{R_0^2}\right) \exp\left(-\frac{t}{\tau}\right) \longrightarrow 0
$$

τ is the **timescale** of the collapse

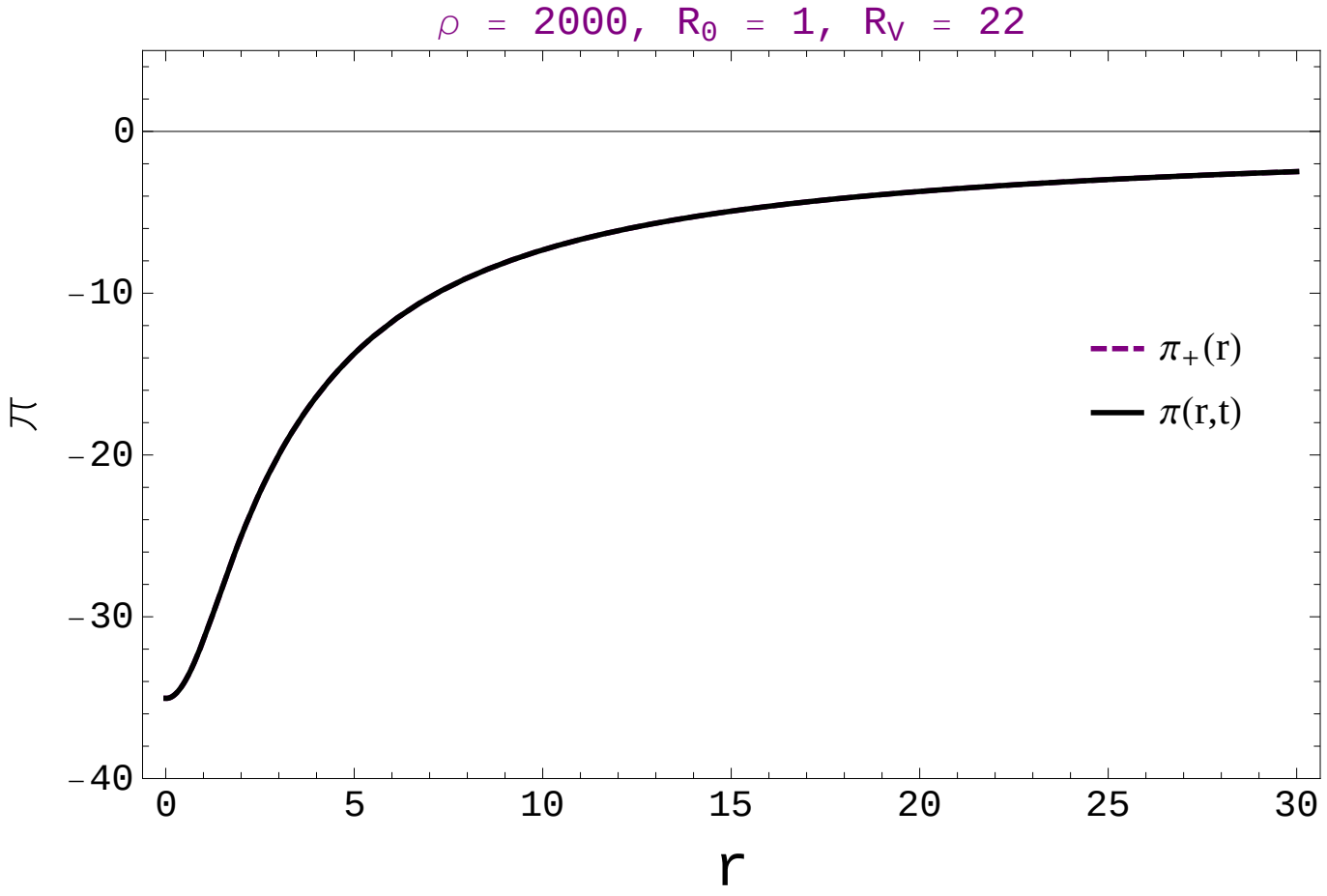
❷ c. Collapsing Source

We expect that if we start from the screening solutions, they should evolve to the vacuum solutions:

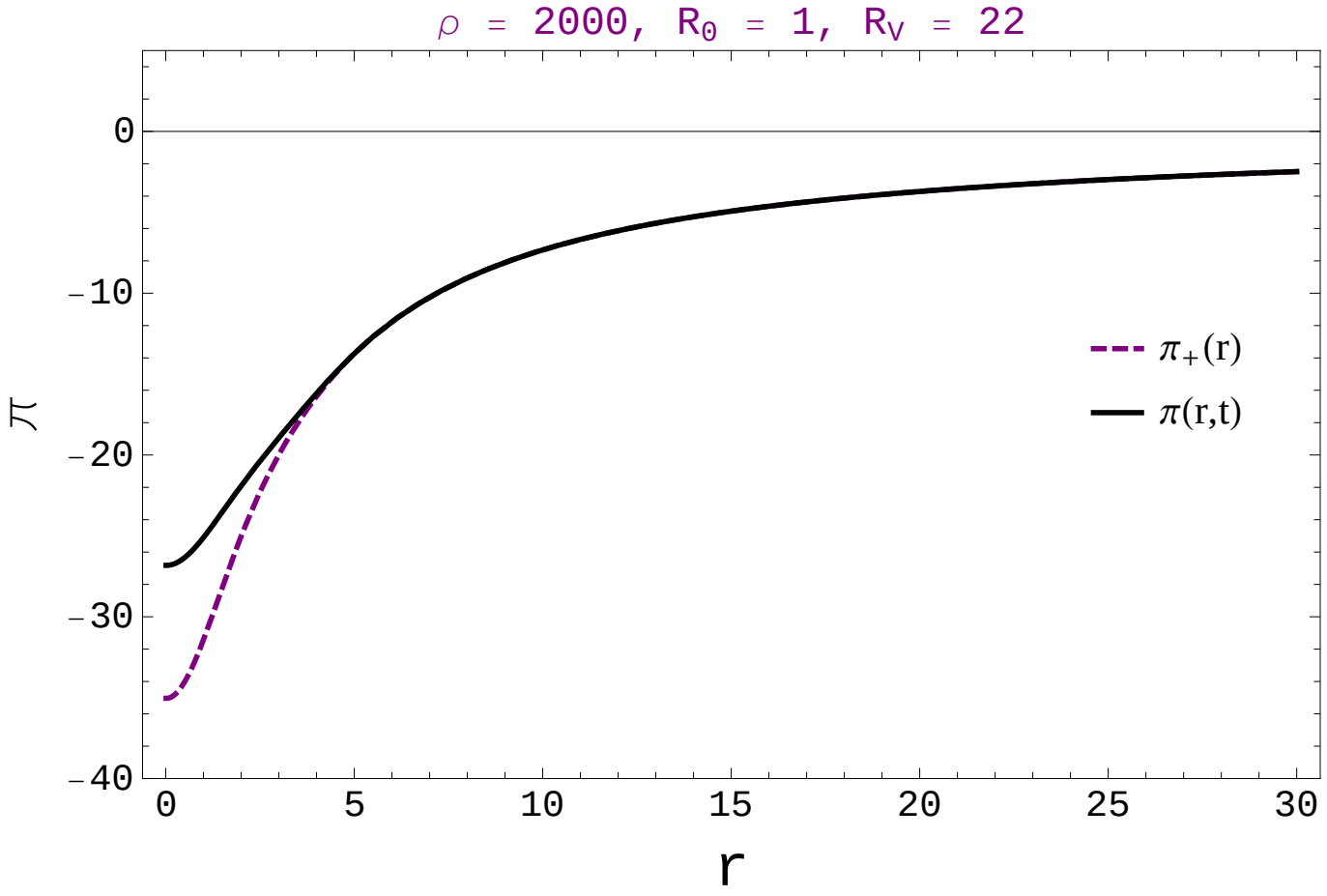
$$\pi_+(r) \longrightarrow \pi(r,t) = 0 \qquad\qquad \pi_-(r) \longrightarrow \pi(r,t) \propto r^2$$

For large timescales this was trivially the case.

❷ c. Collapsing Source



❷ c. Collapsing Source



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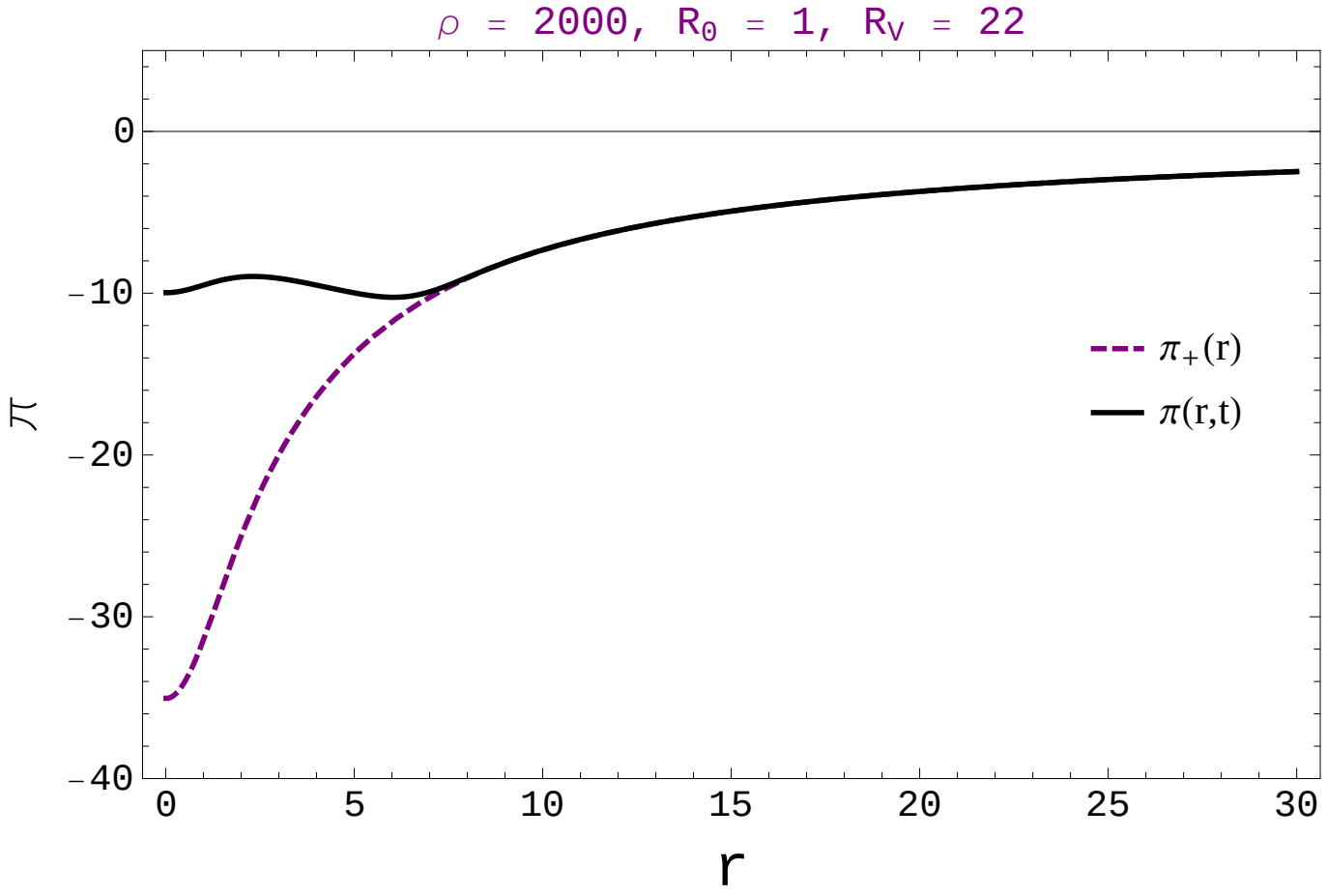

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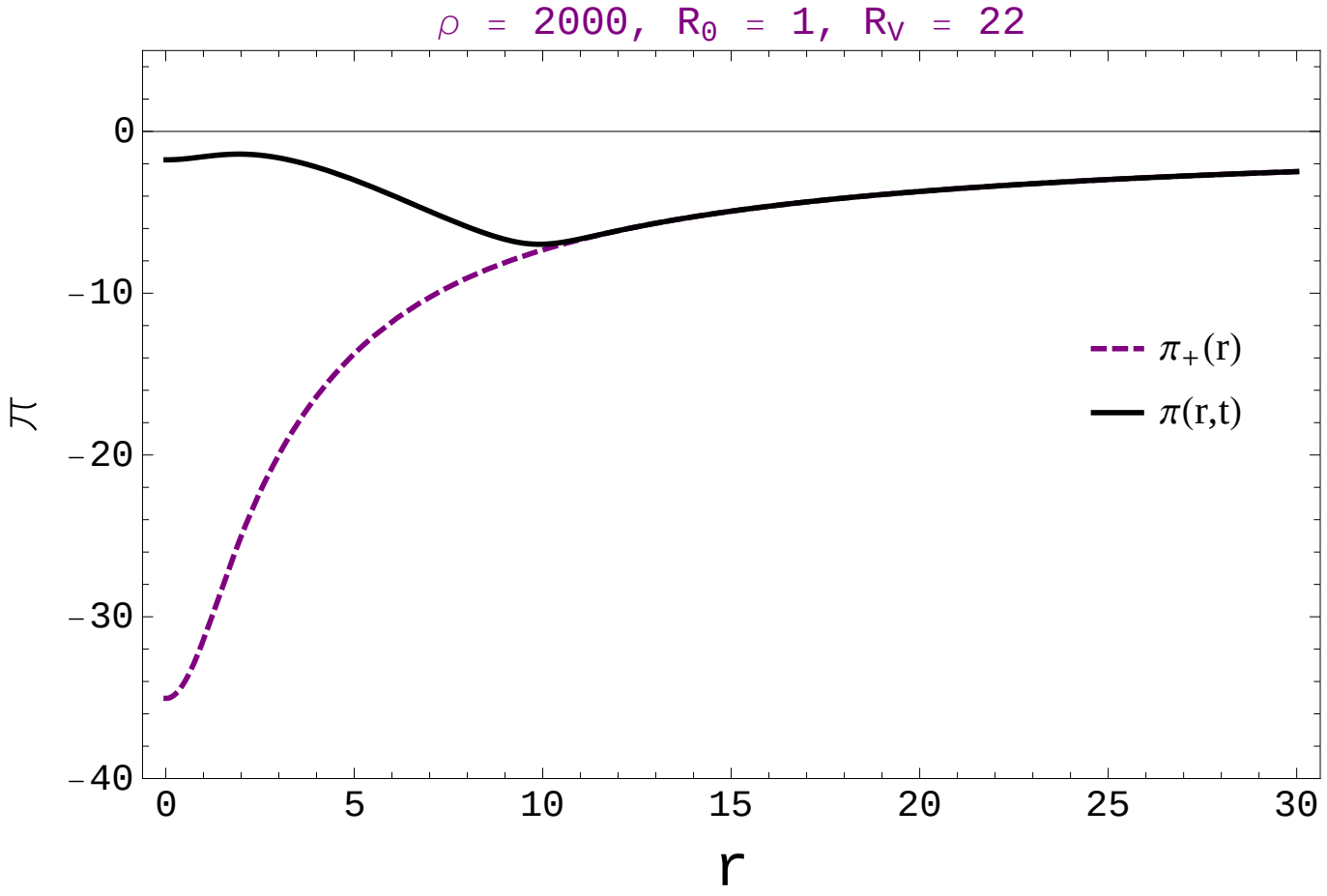

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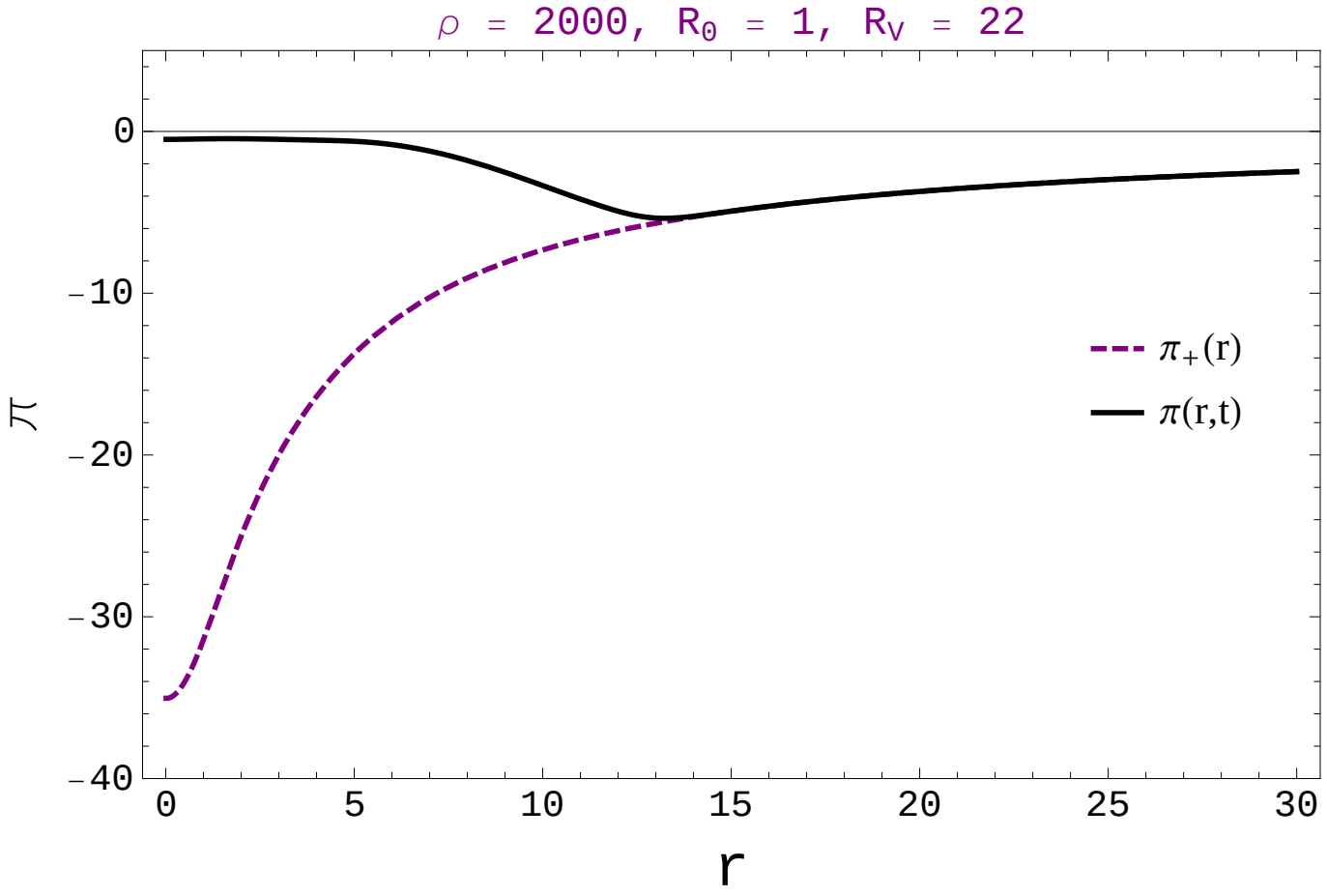

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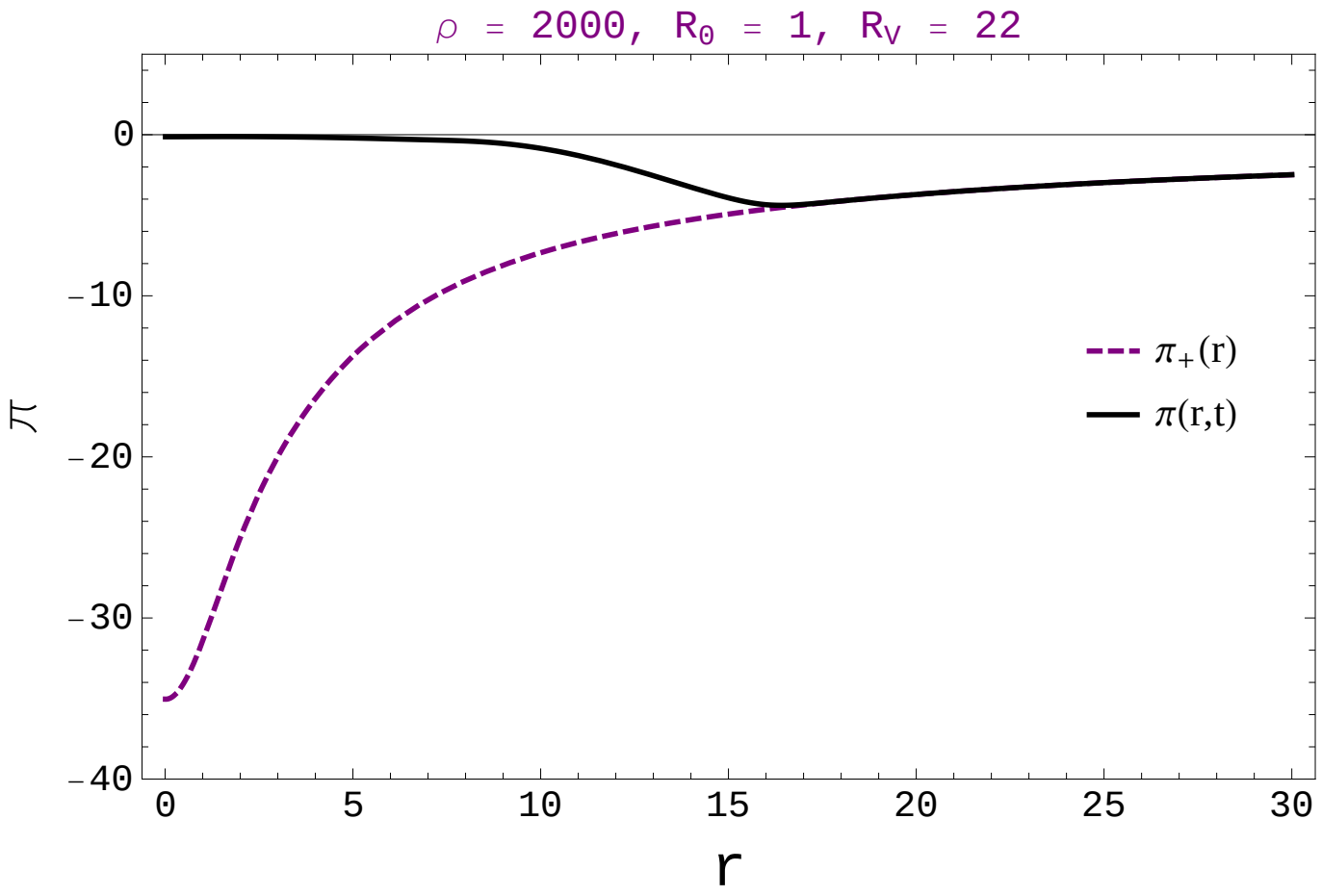

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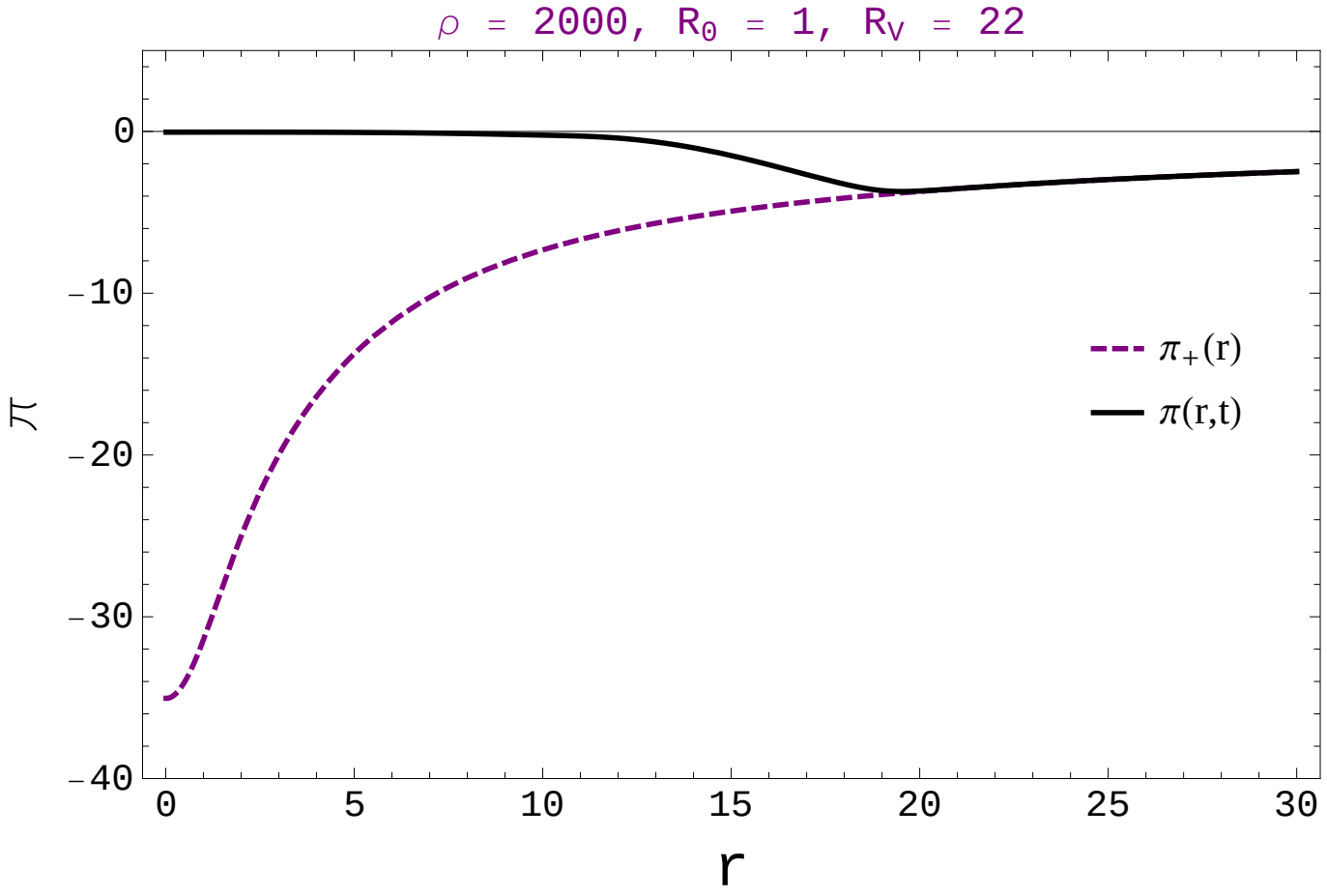

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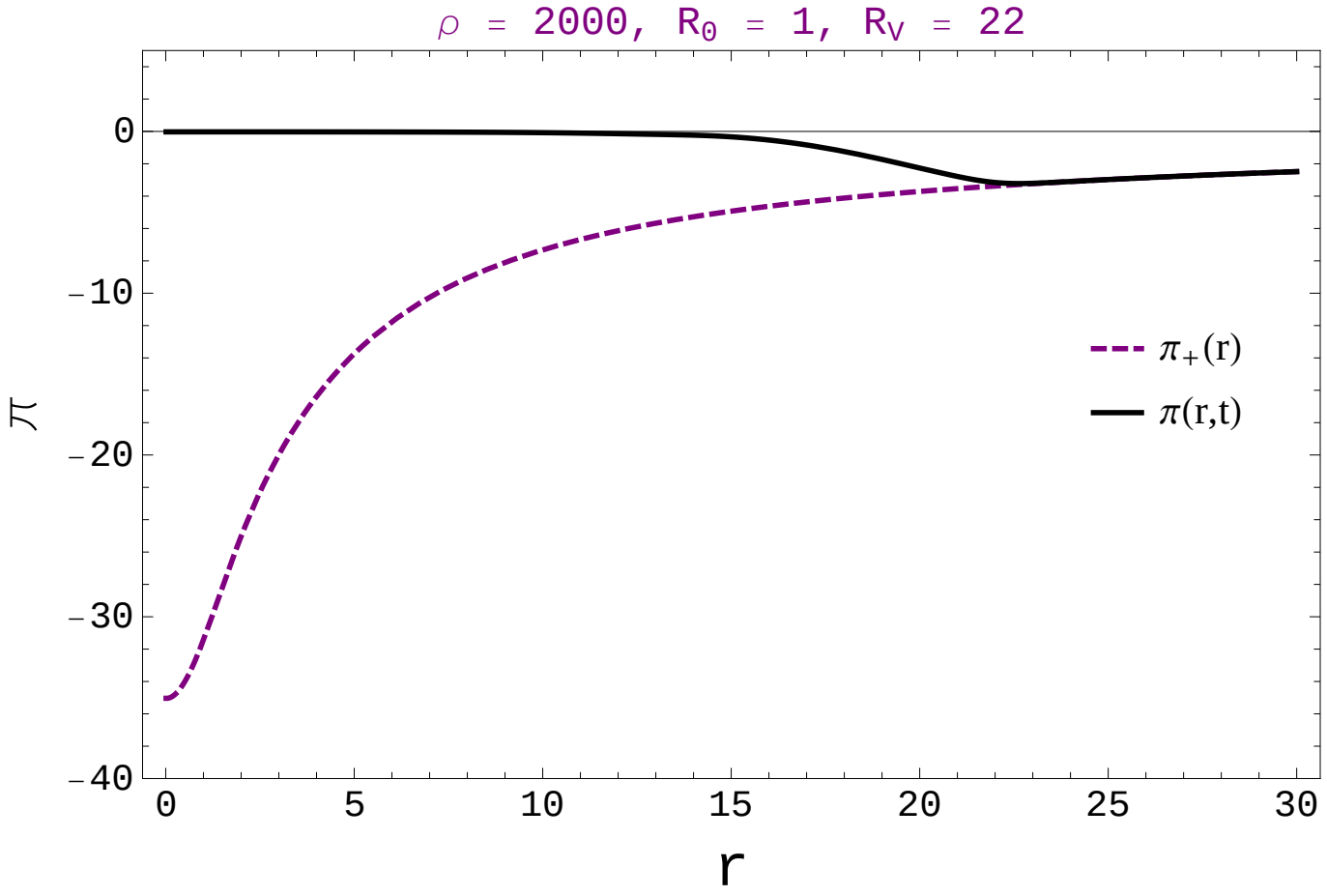
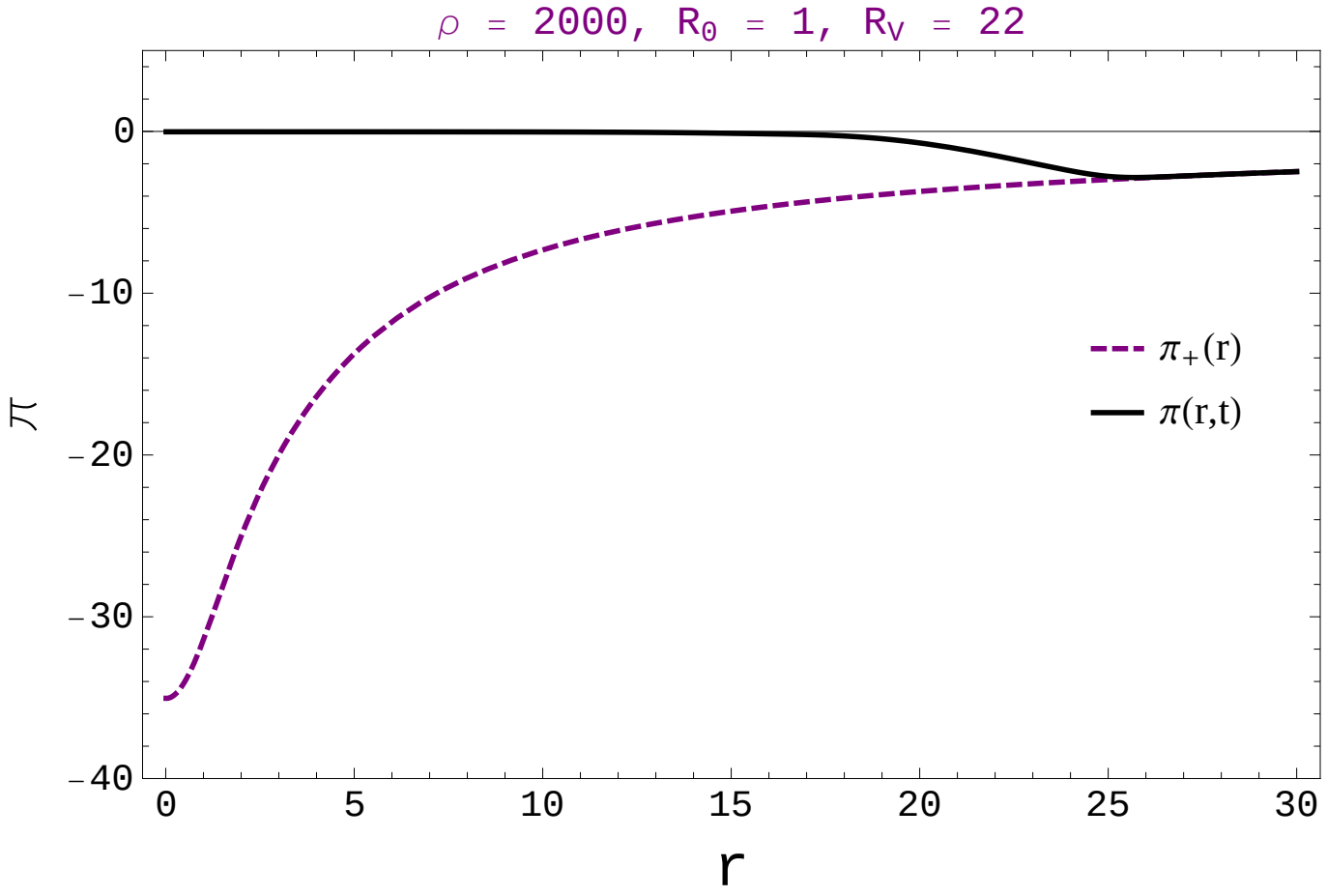


❷ c. Collapsing Source



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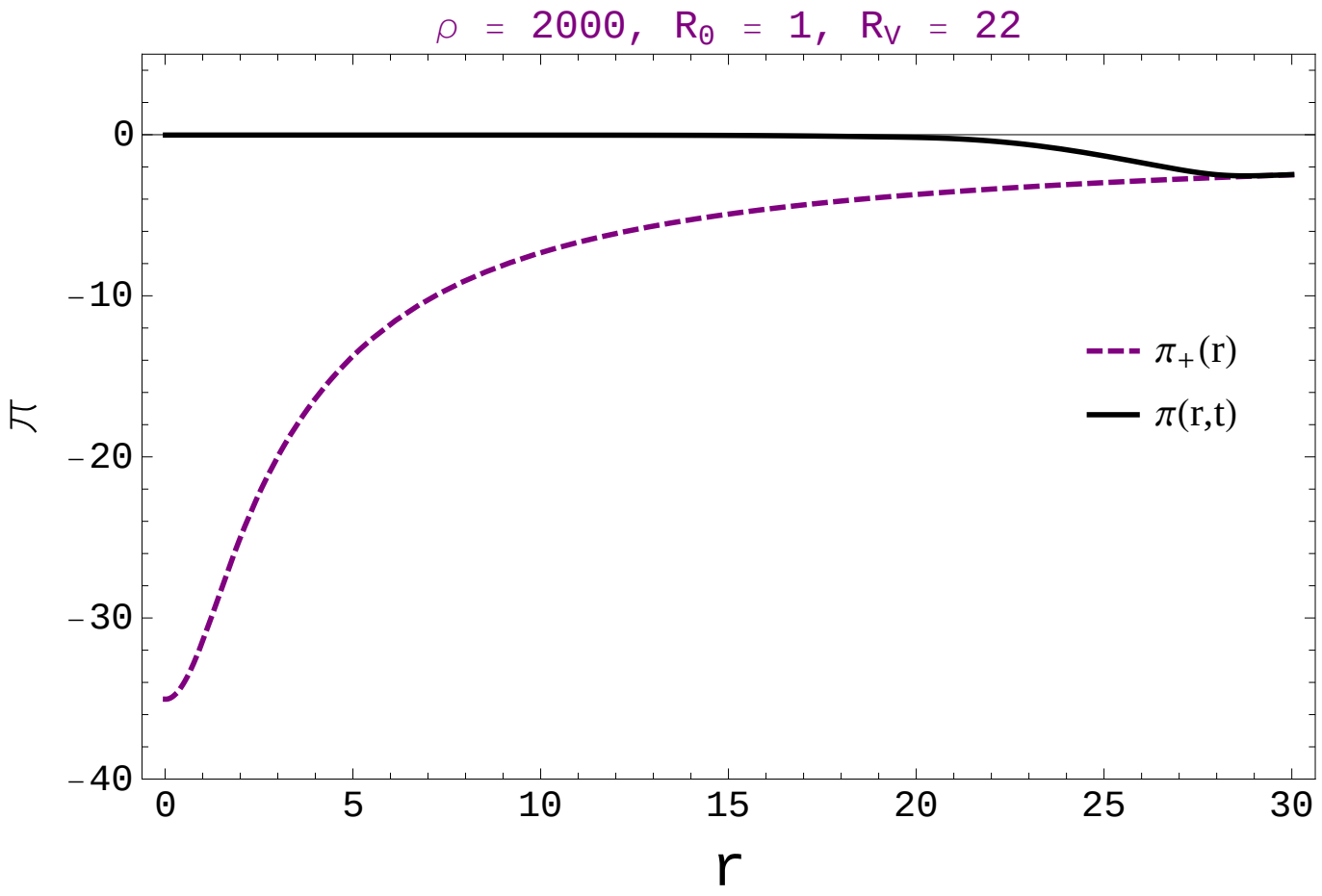

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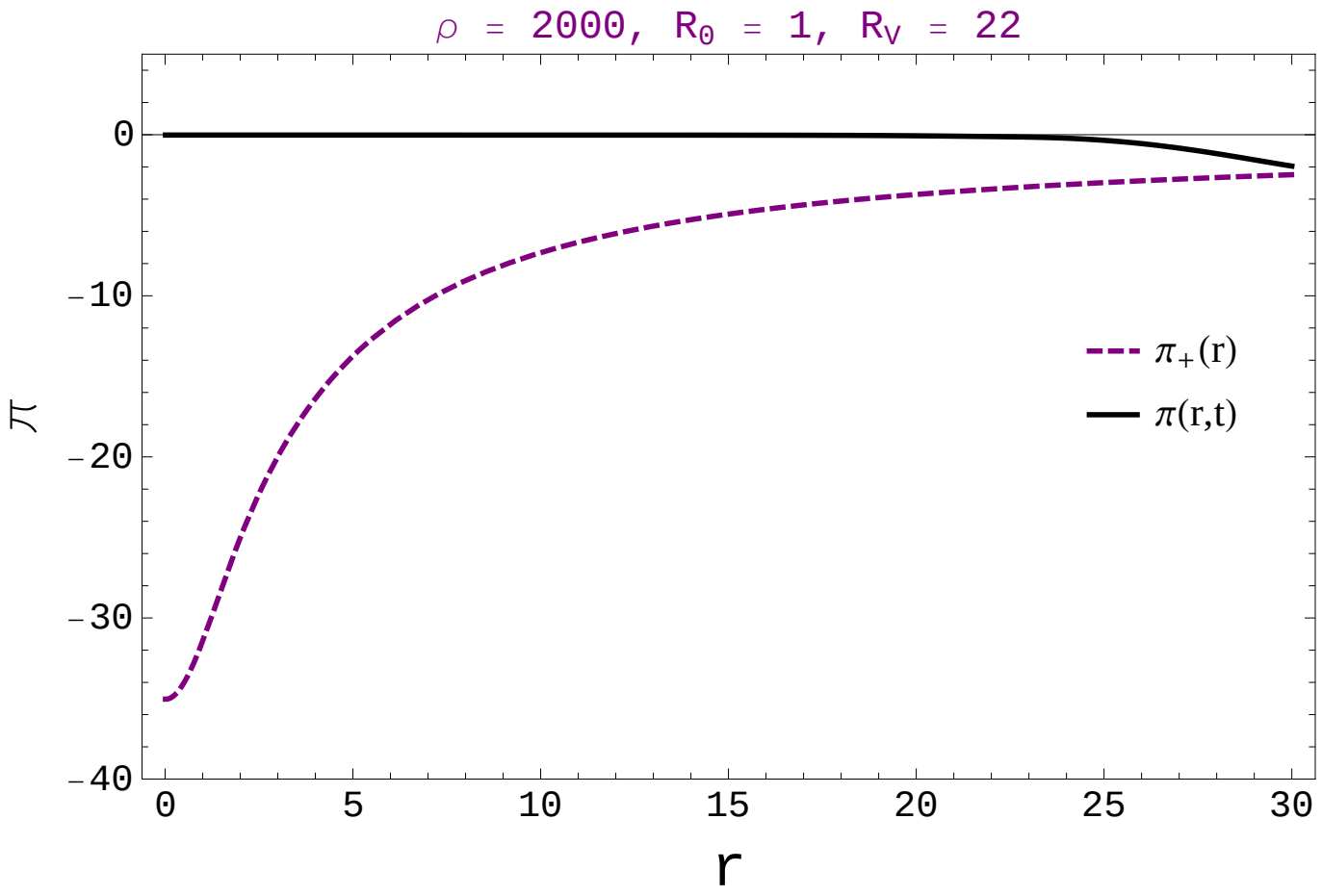

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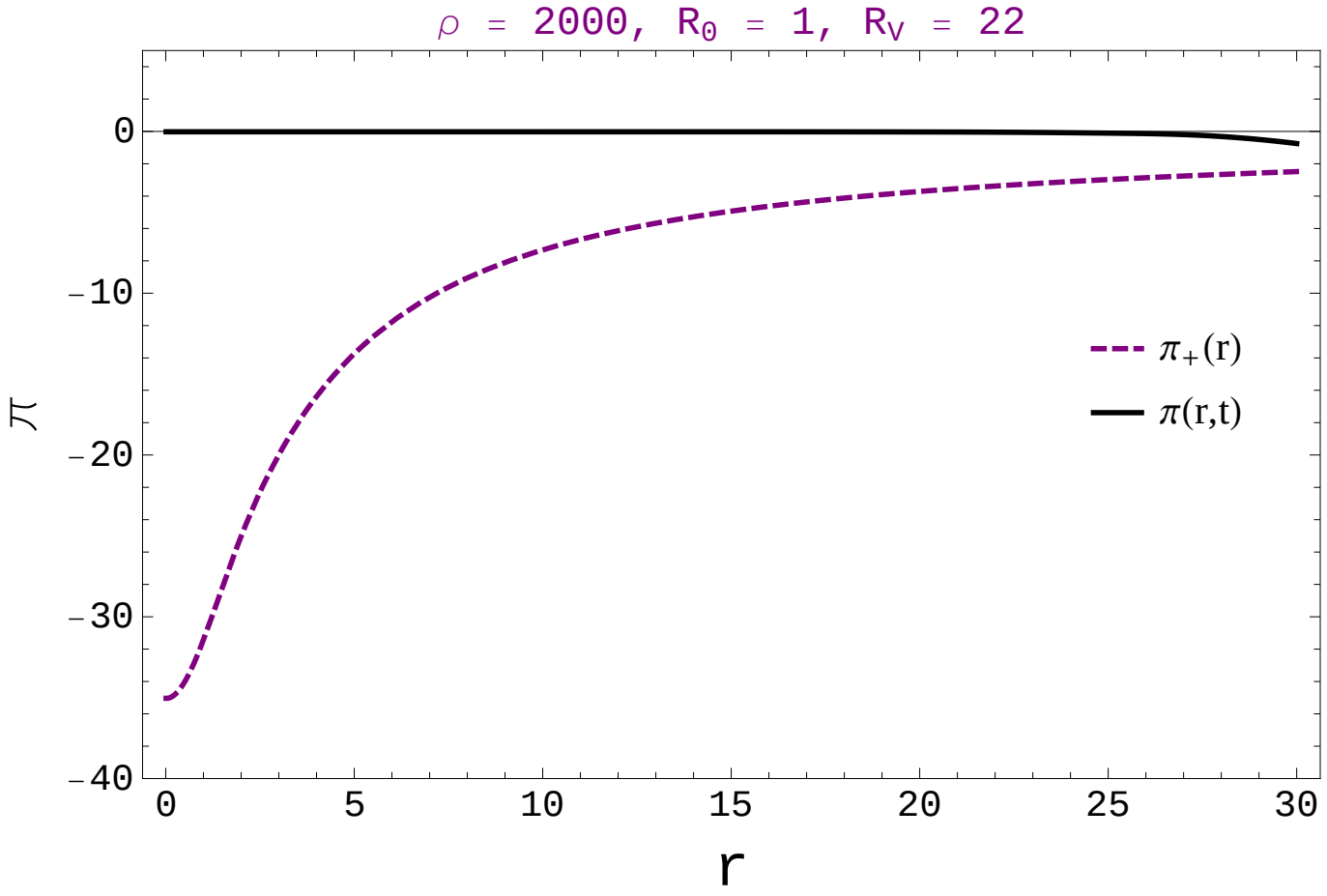

❷ **d.** Exploding Source

An exploding source (e.g. supernova), modelled as an outgoing spherical shell of dust

$$T^{\mu\nu} = \mathrm{diag}\left(\frac{\rho}{f(t)}\exp\left(-\frac{(r-t)^2}{R_0^2}\right), 0, 0, 0\right)$$

where $f(t)$ is defined so that $M(r \to \infty) = \pi^{3/2}\rho R_0^3$.

The vacuum solution is dynamically reached as a wave packet traces the source as it propagates off to infinity.

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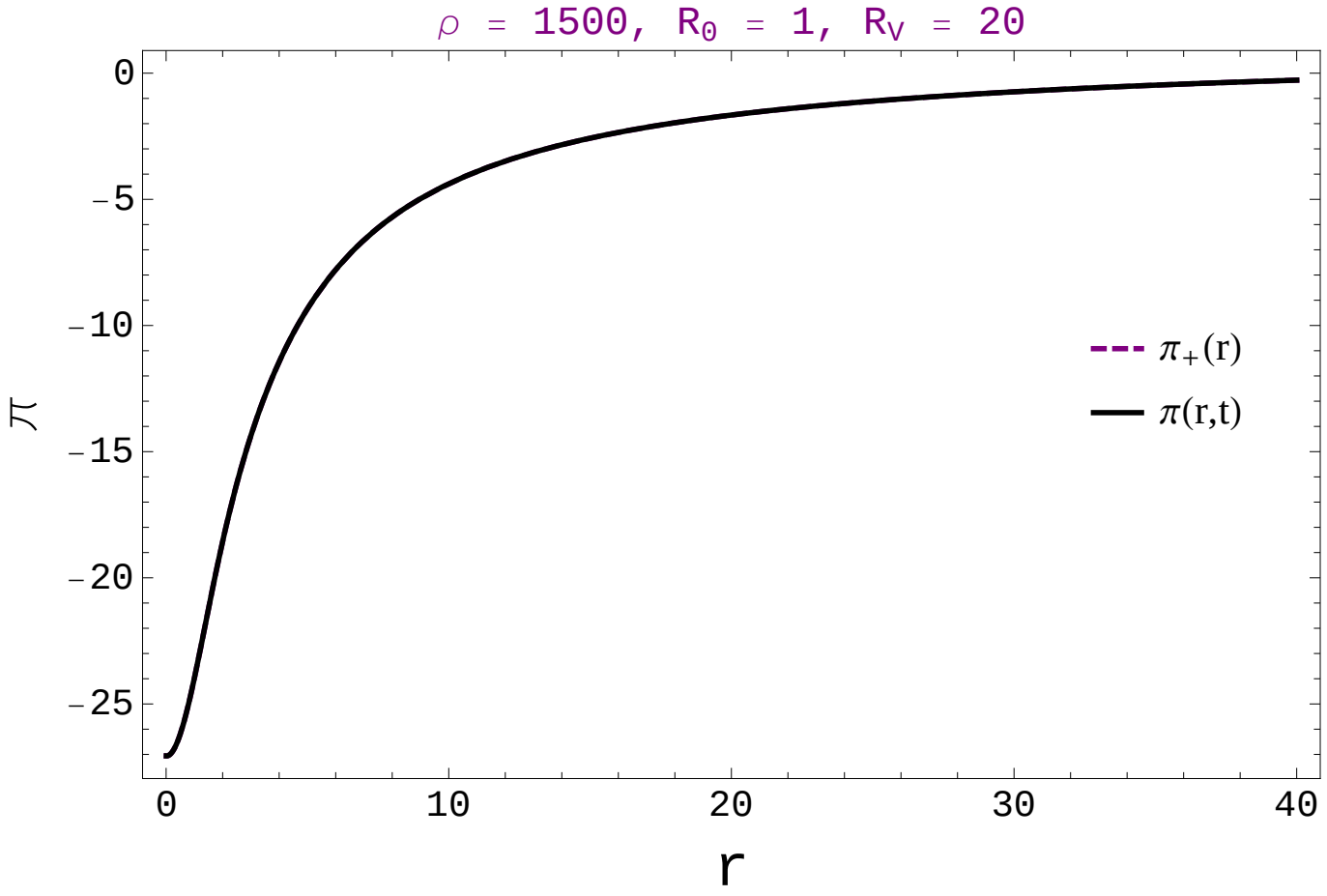
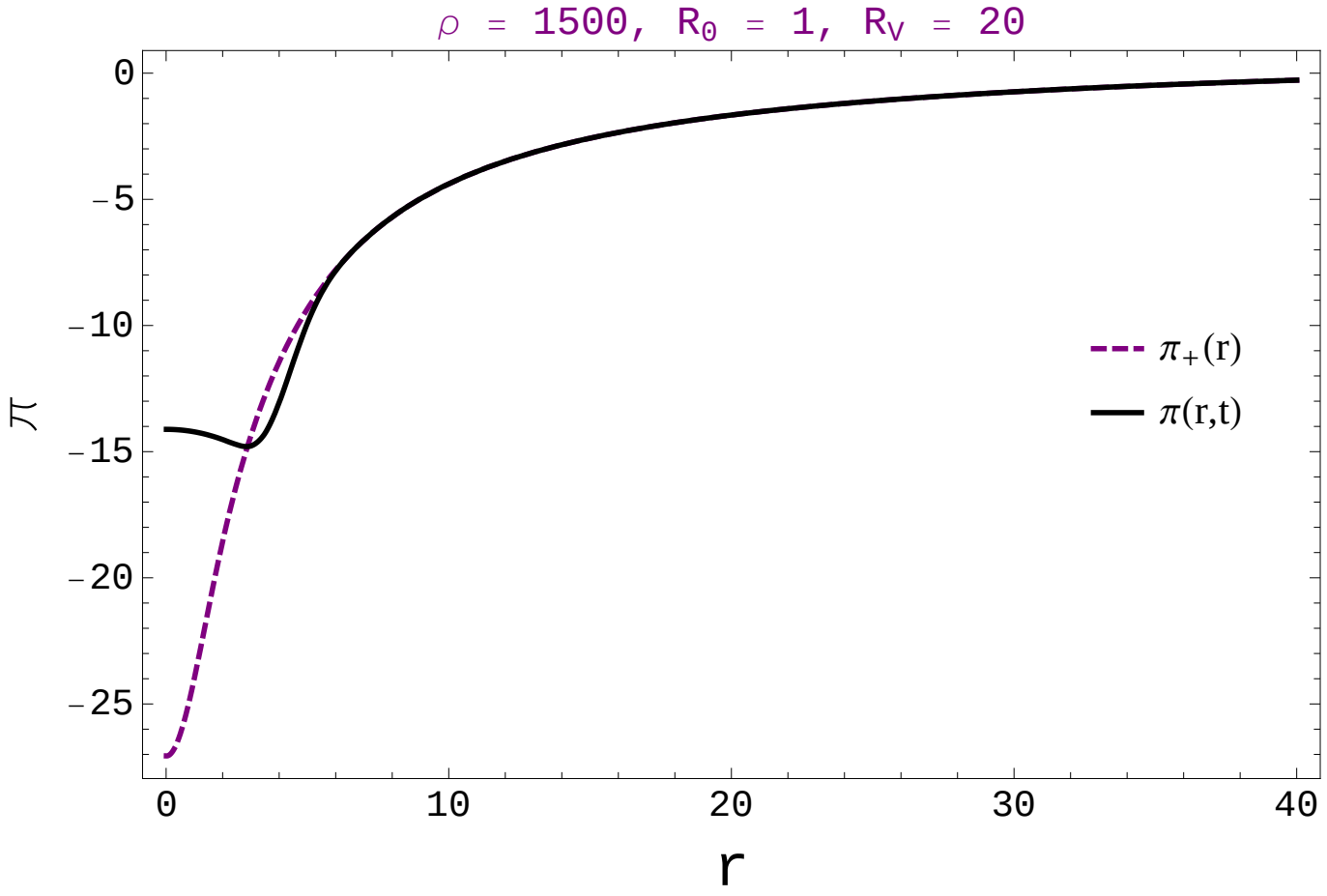
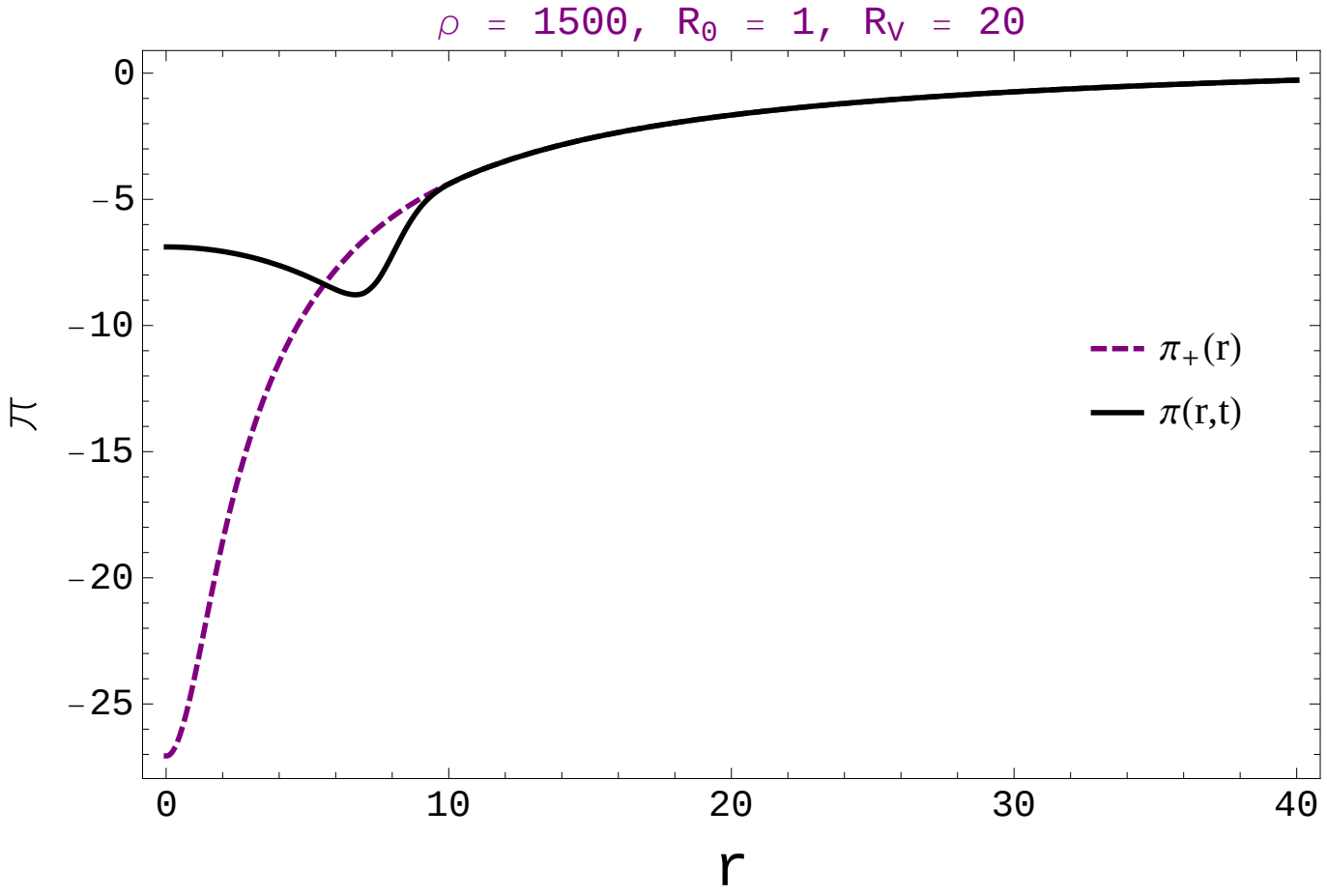


❷ d. Exploding Source



❷ d. Exploding Source



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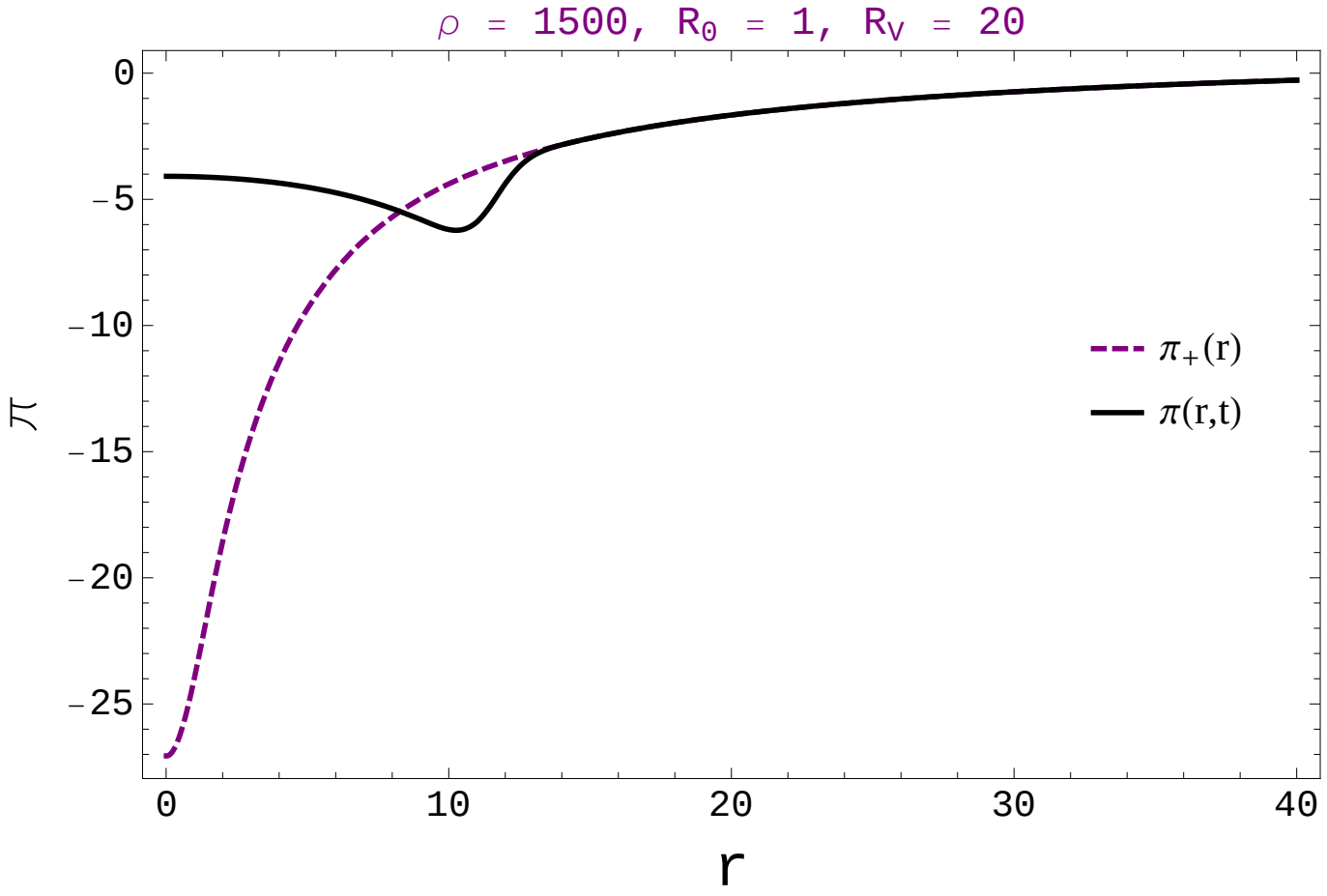

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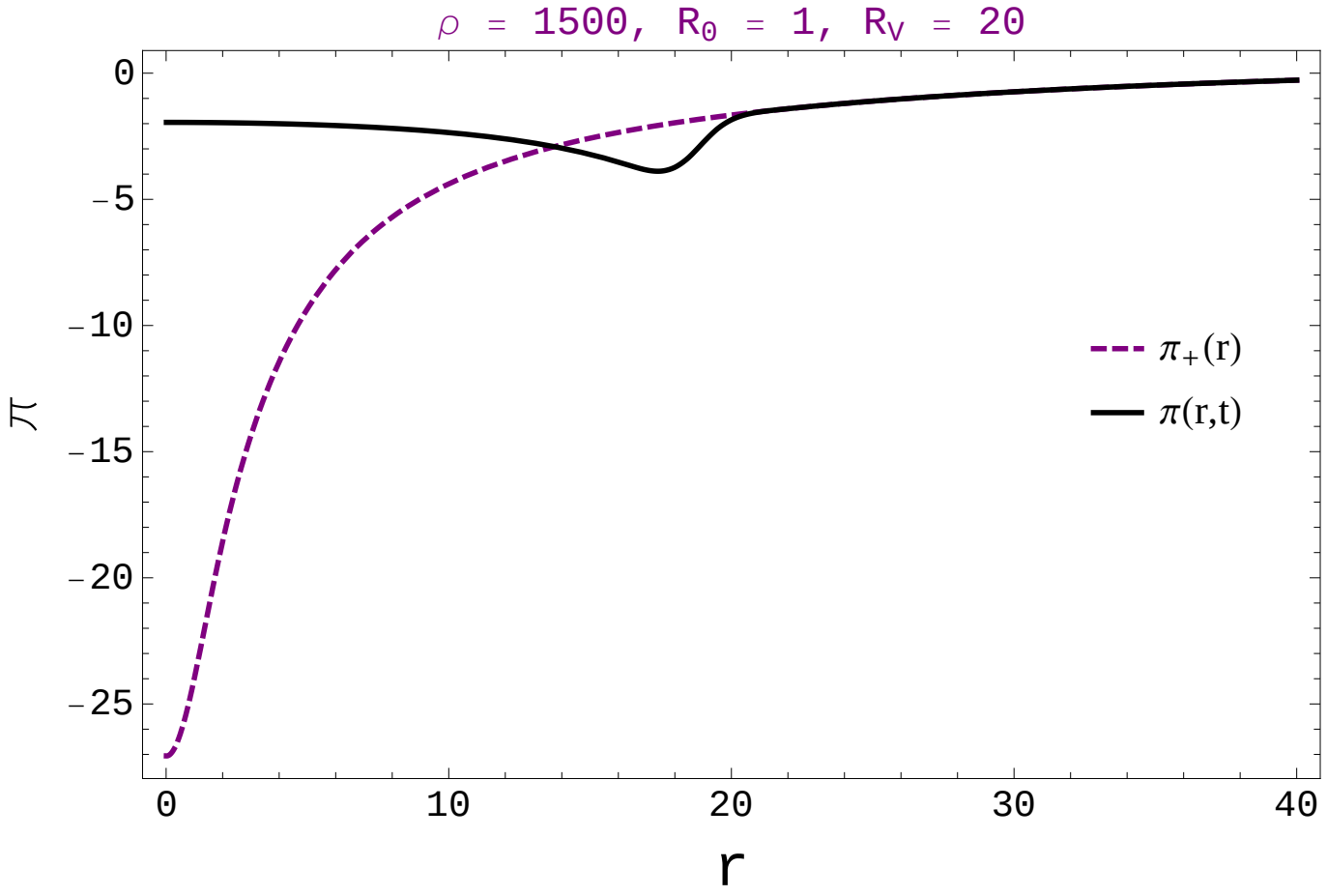

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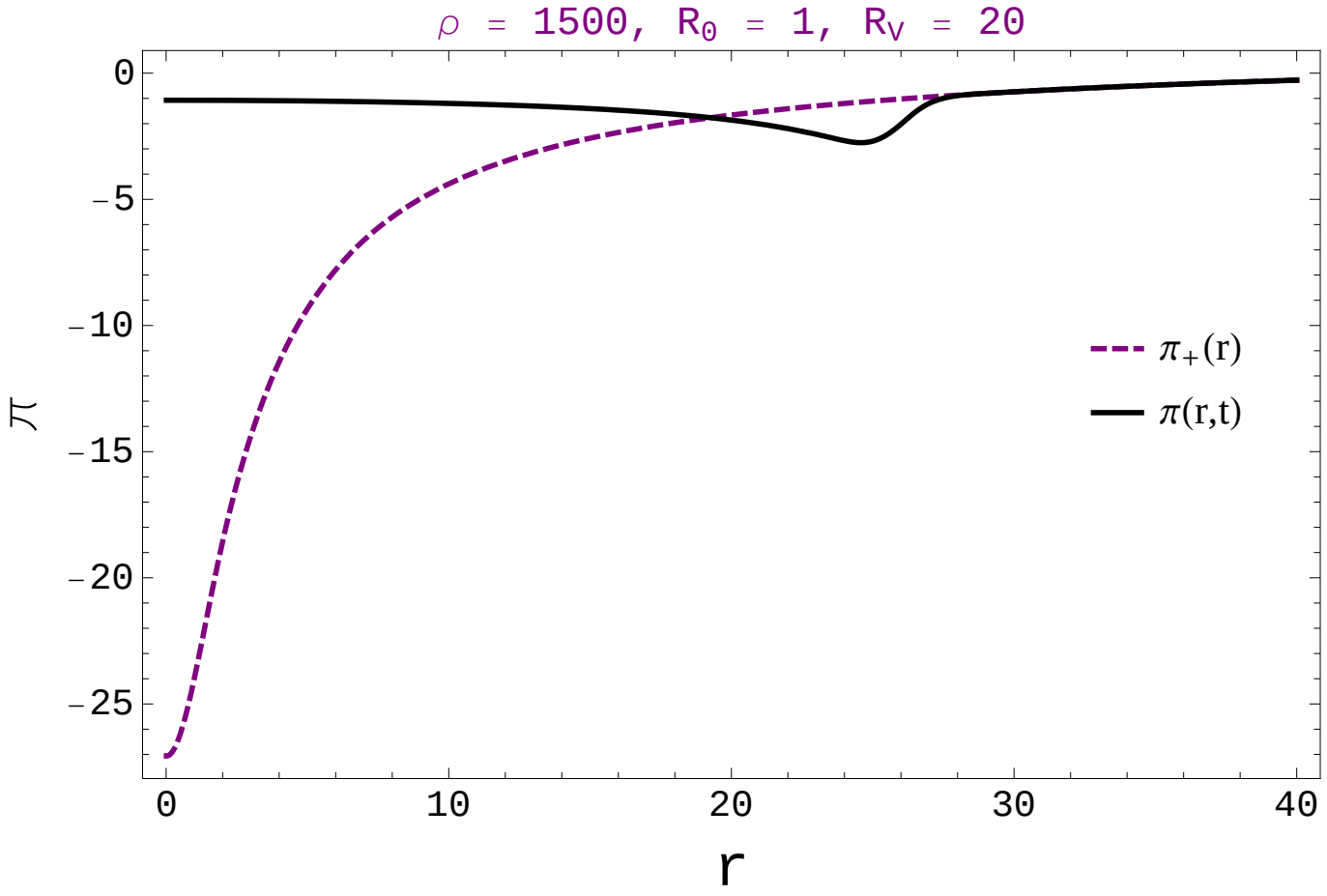

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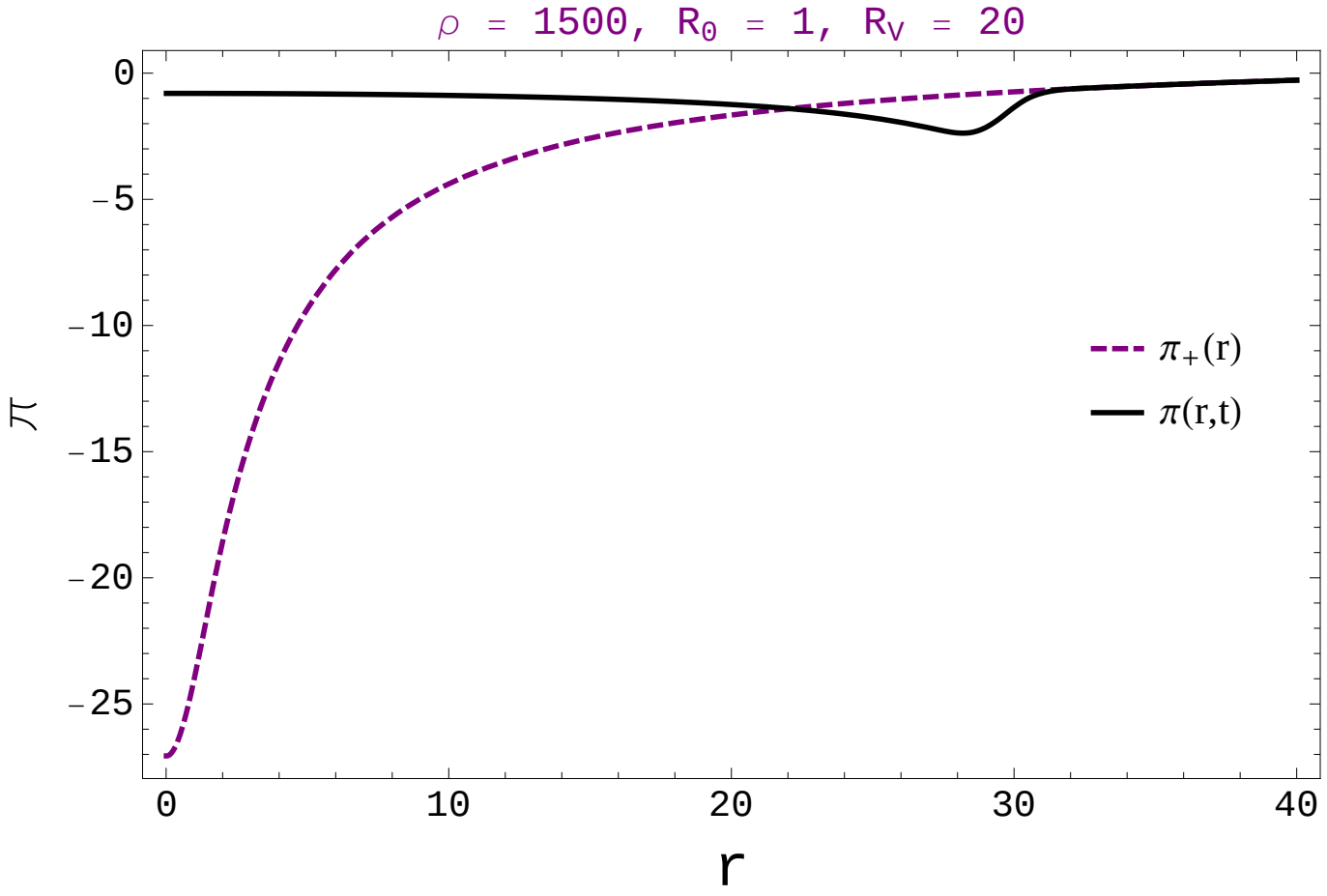

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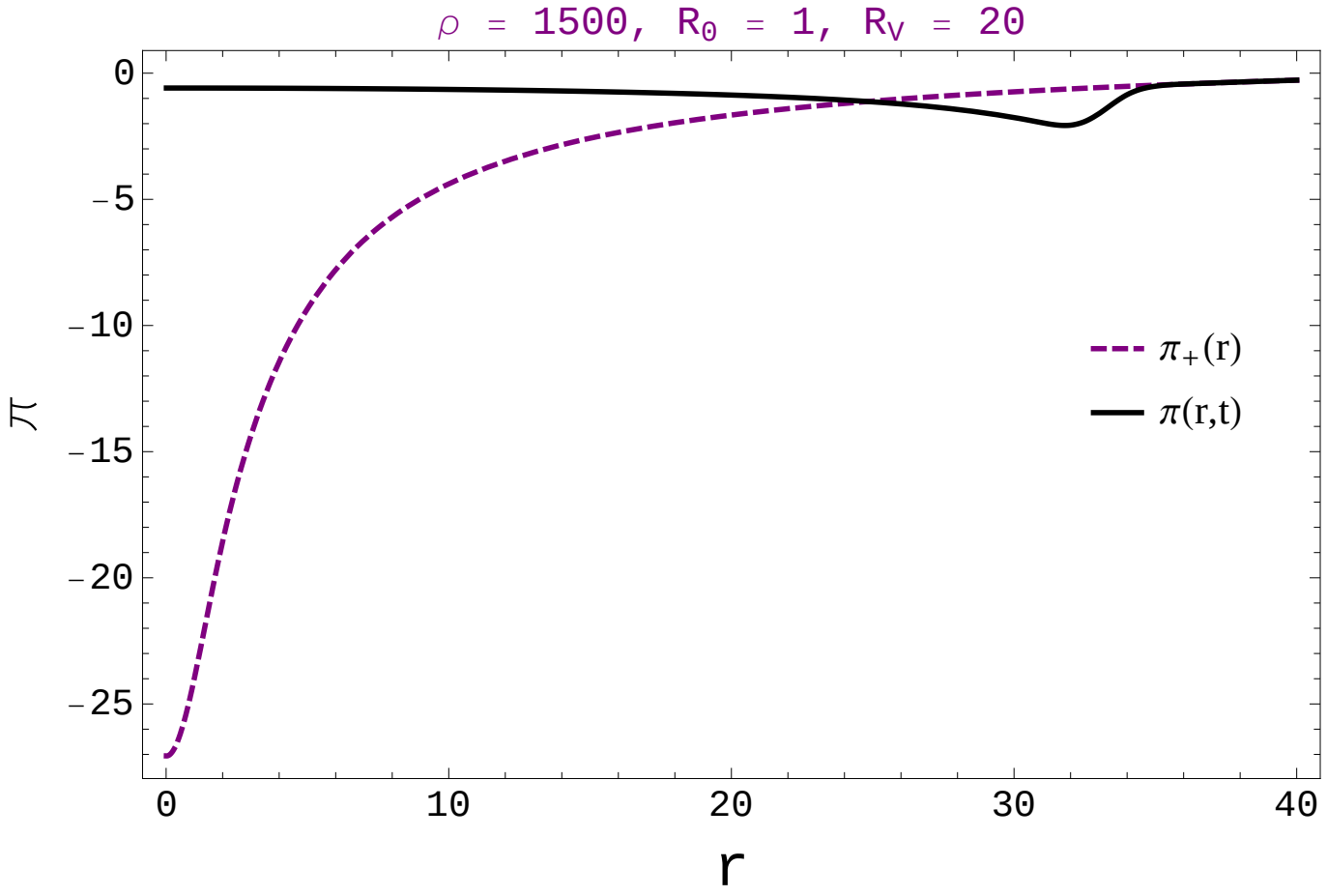

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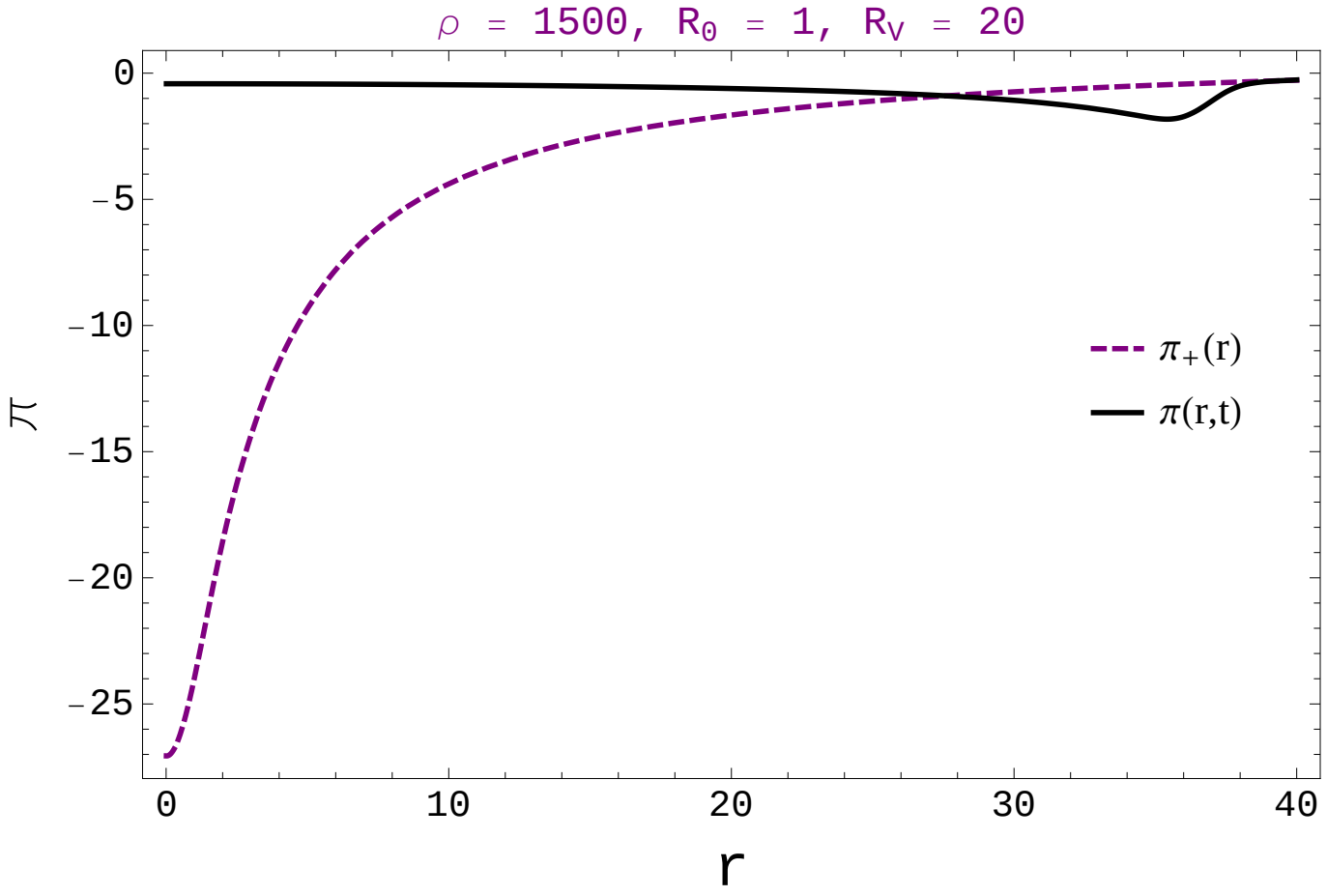

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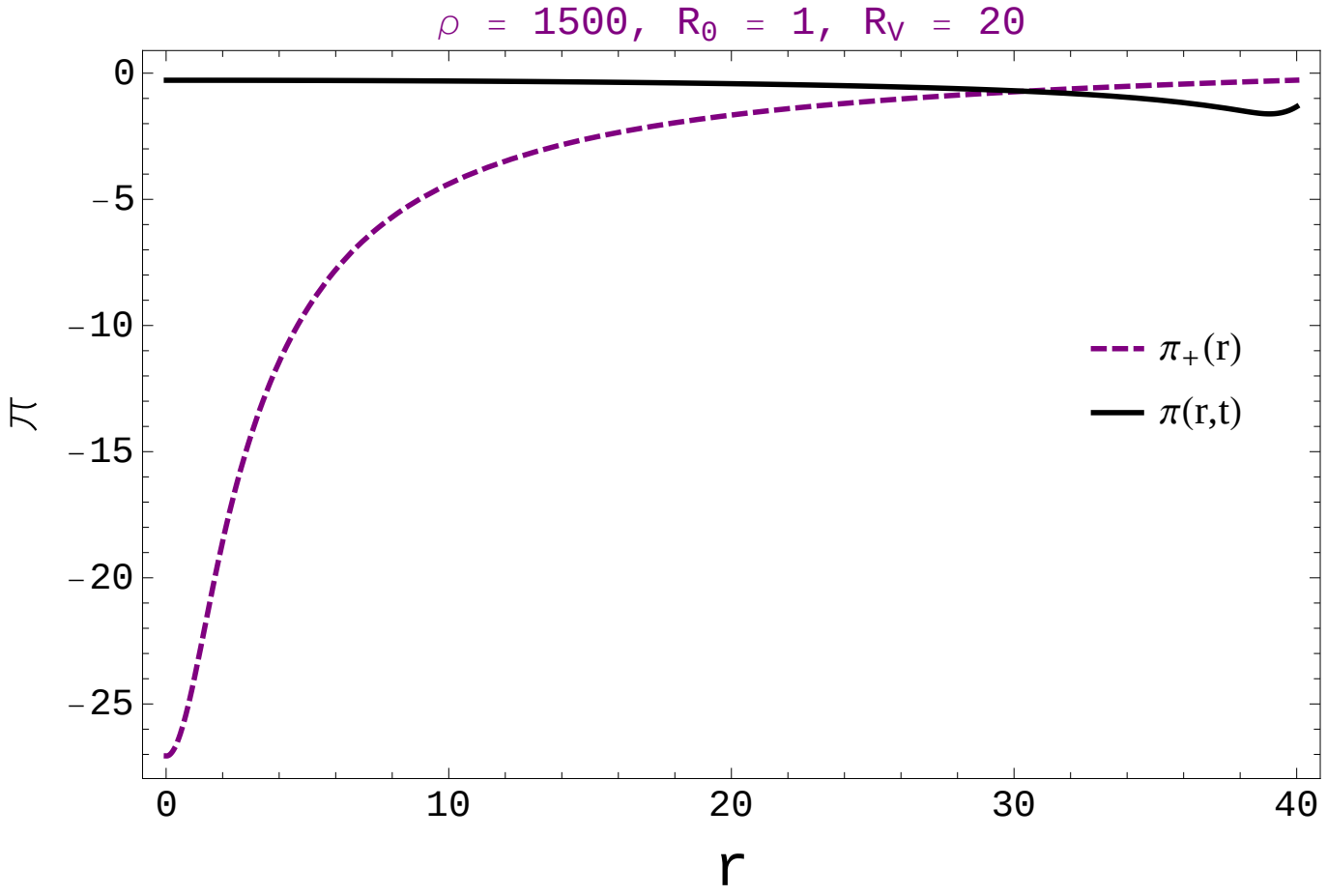

In all of the above cases, for a large variety of initial conditions, the evolution was stable and well-defined, and the field evolves as expected.

However, for some cases, we found that large perturbations drive the field towards a problematic point that we term **Cauchy Breakdown** [3]

[3] C. D. Leonard, J. Ziprick, G. Kunstatter, and R. B. Mann, JHEP 1110, 028 (2011), arXiv:1106.2054 [gr-qc].

Cauchy Breakdown

Recall that small perturbations $\delta\pi$ propagate in an **effective metric** $Z^{\mu\nu}$:

$$S_{\delta\pi} = \int d^4x \left[\frac{1}{2} Z^{\mu\nu} \partial_\mu \delta\pi \partial_\nu \delta\pi + \frac{1}{2M_4} \delta\pi T \right]$$

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Well-posed initial value problem: $Z^{tt} < 0$ and $Z^{rr}, Z^{\theta\theta}, Z^{\phi\phi} > 0$ (or both inequalities reversed)

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$$\begin{aligned} \Pi_+ &: \; Z^{tt} < 0, \; Z^{rr}, Z^{\theta\theta}, Z^{\phi\phi} > 0 \\ \Pi_- &: \; Z^{tt} > 0, \; Z^{rr}, Z^{\theta\theta}, Z^{\phi\phi} < 0 \end{aligned}$$

Cauchy Breakdown

A breakdown of the Cauchy problem occurs when the surfaces of constant time become null with respect to the effective metric

Cauchy Breakdown:

$$Z^{tt} = -\left(6 + 8\frac{\Pi}{r} + 4\Pi'\right) \to 0 \qquad c_s = -\sqrt{-\frac{Z^{rr}}{Z^{tt}}} \to \infty$$

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- Not an issue for the static screening solutions $\Pi_\pm$, but occurs for an evolving field

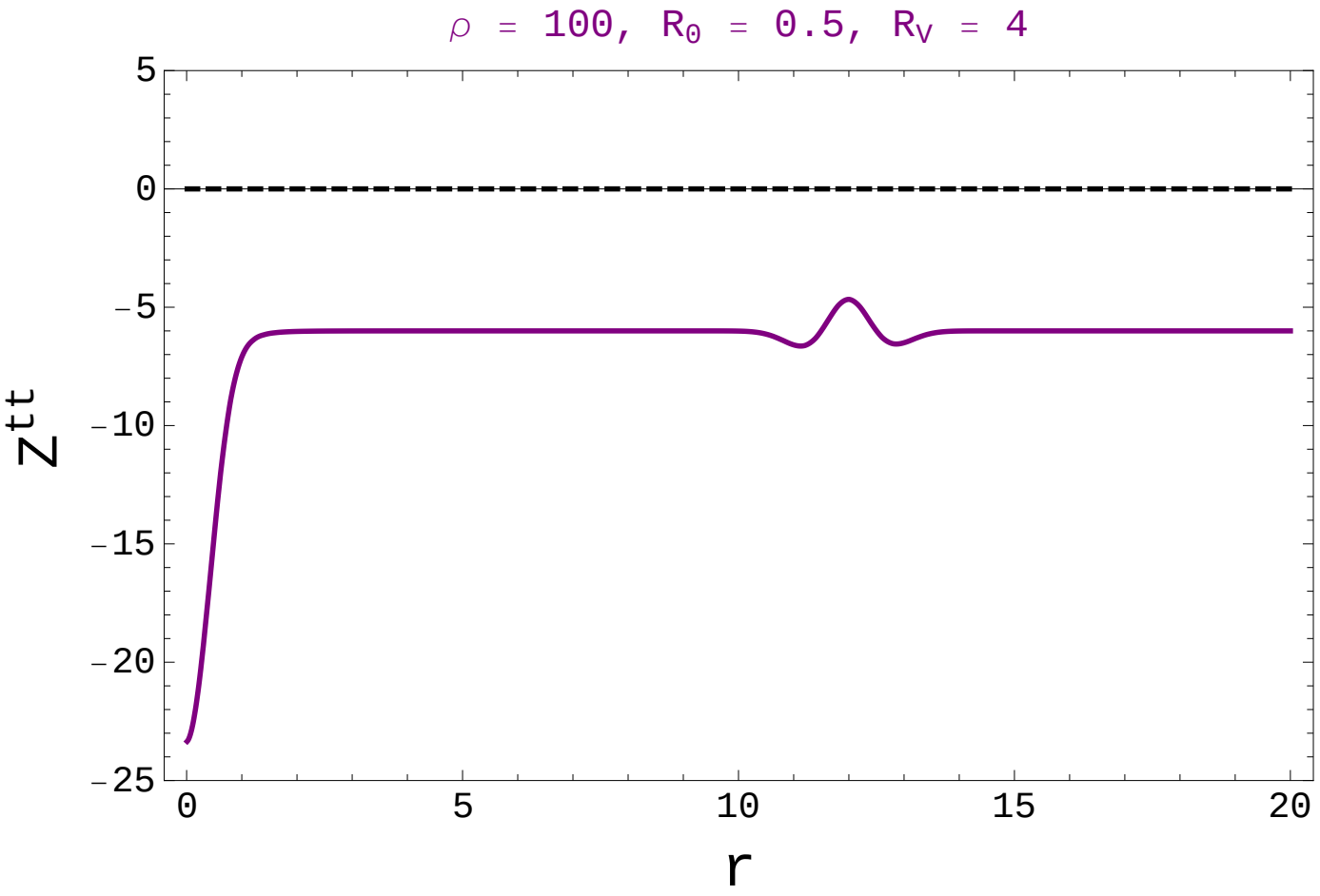
Cauchy Breakdown

Qualitative results:

Cauchy breakdown will occur for...

1. **Large perturbations** on top of the screening solution

Cauchy Breakdown



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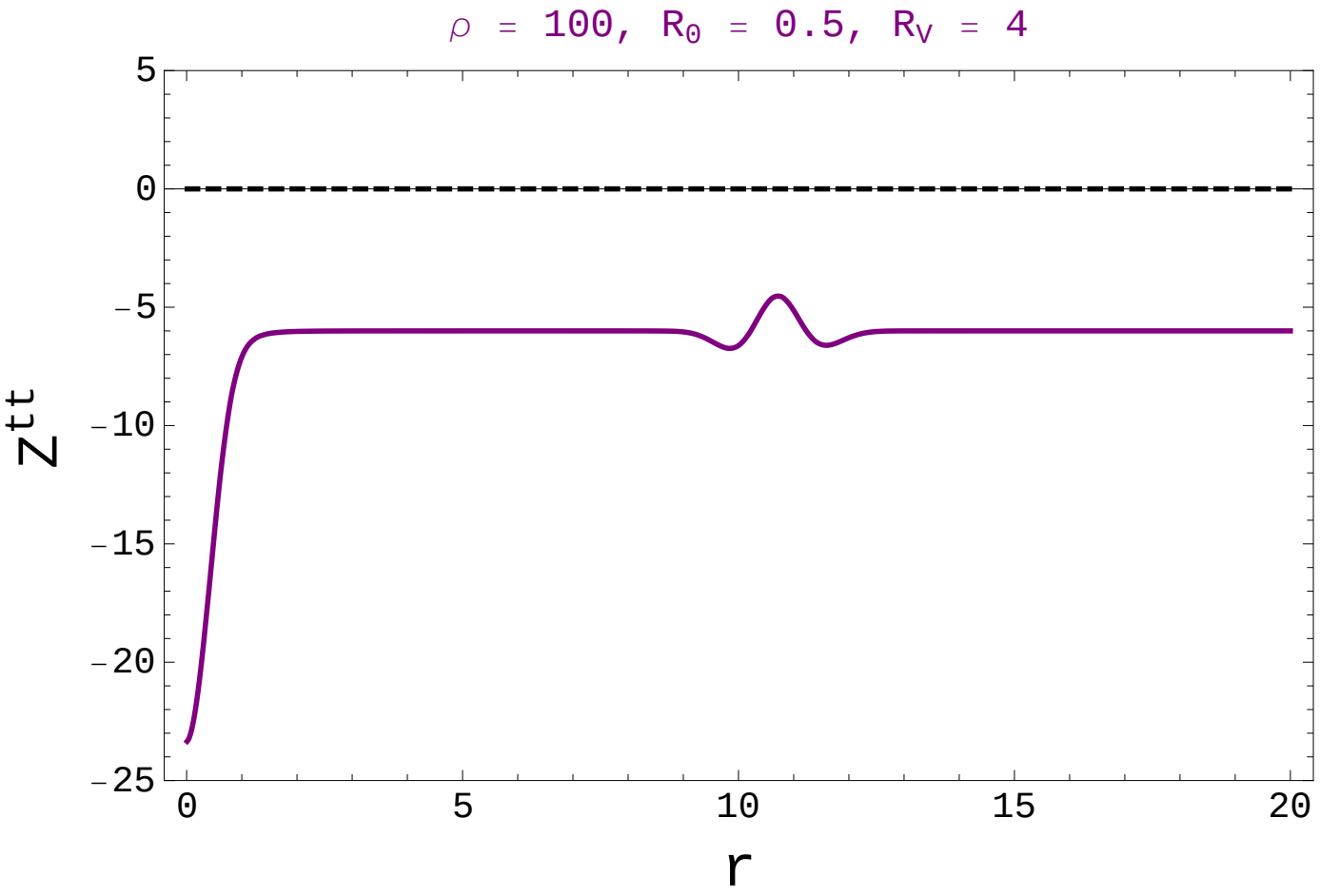

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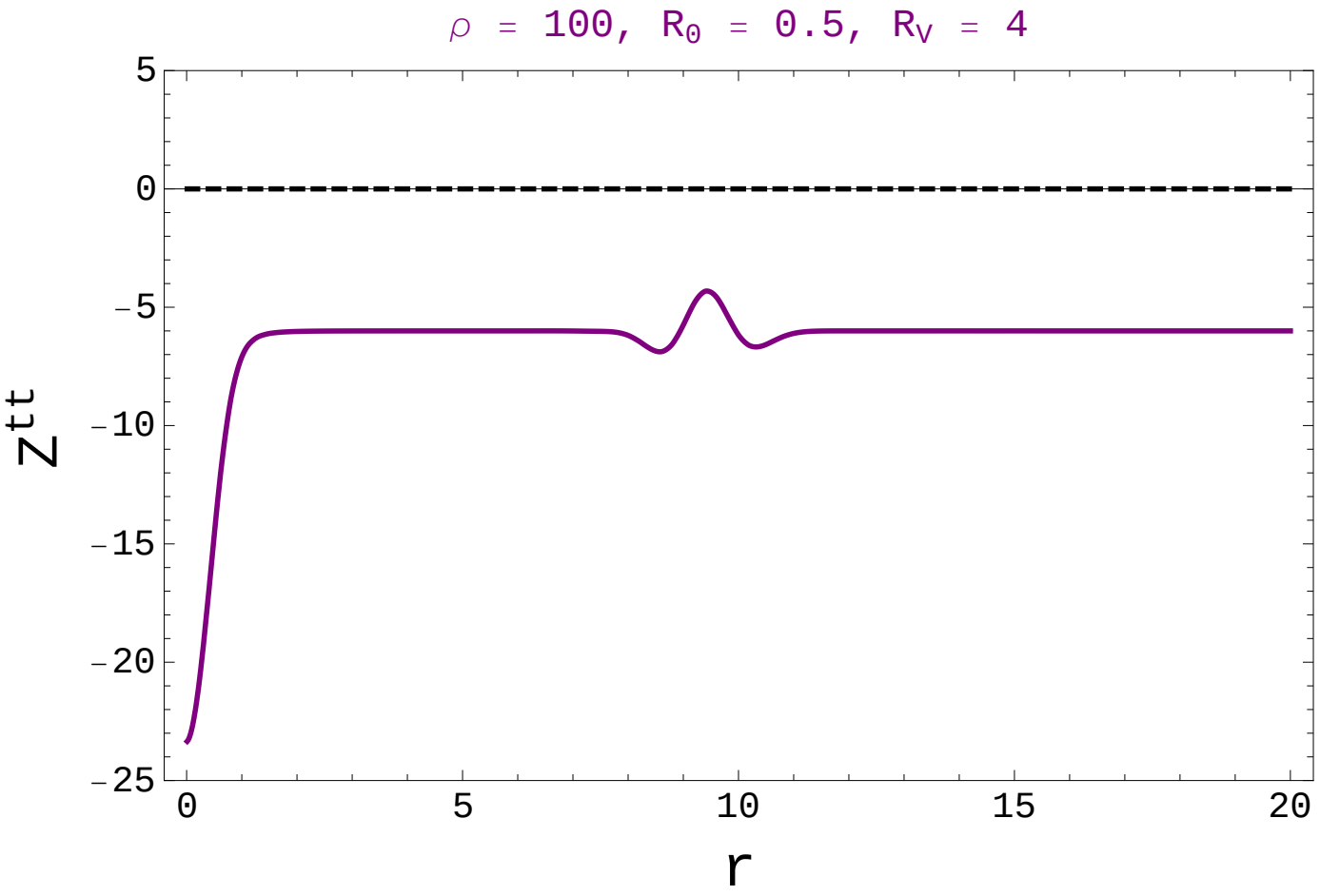

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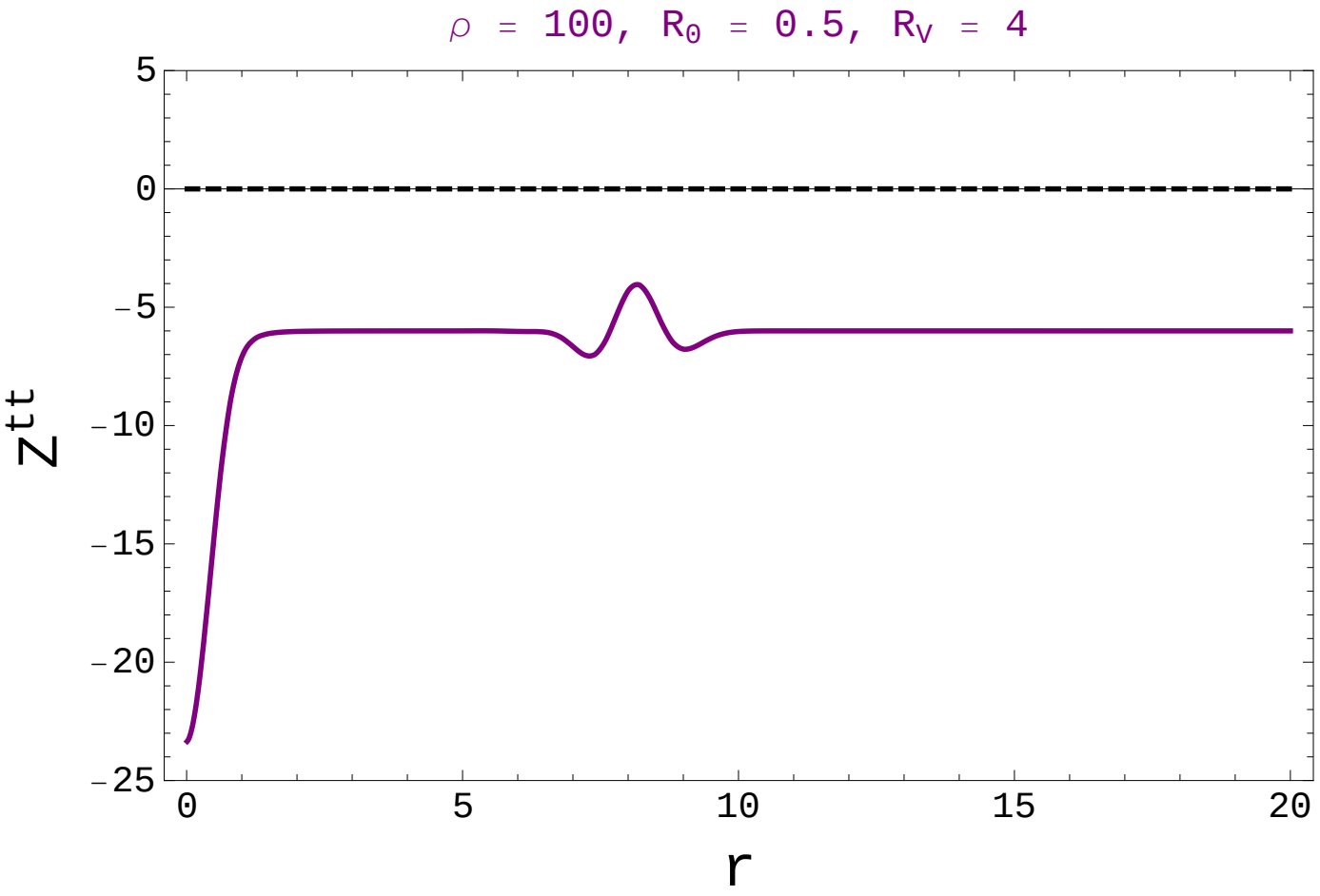

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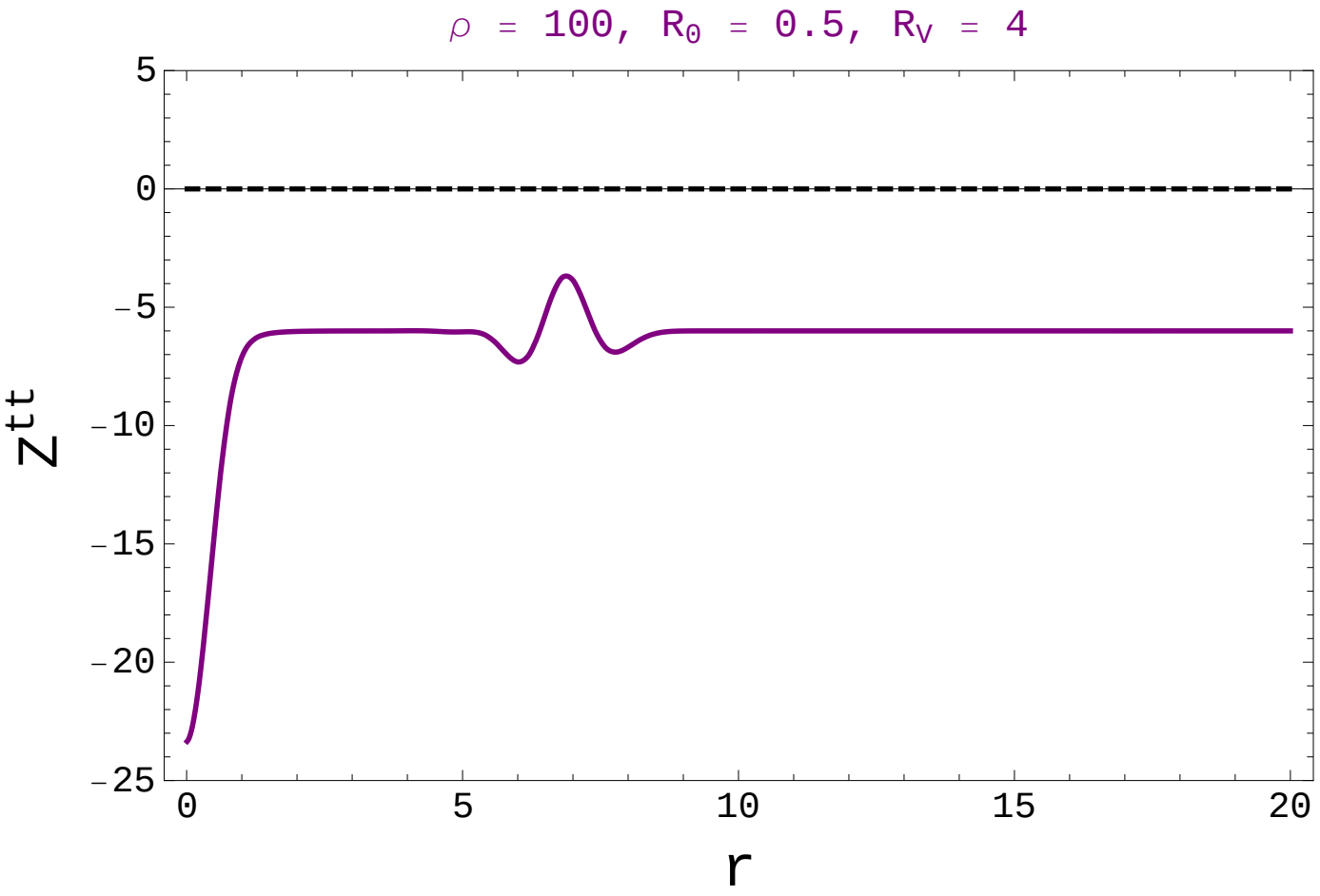

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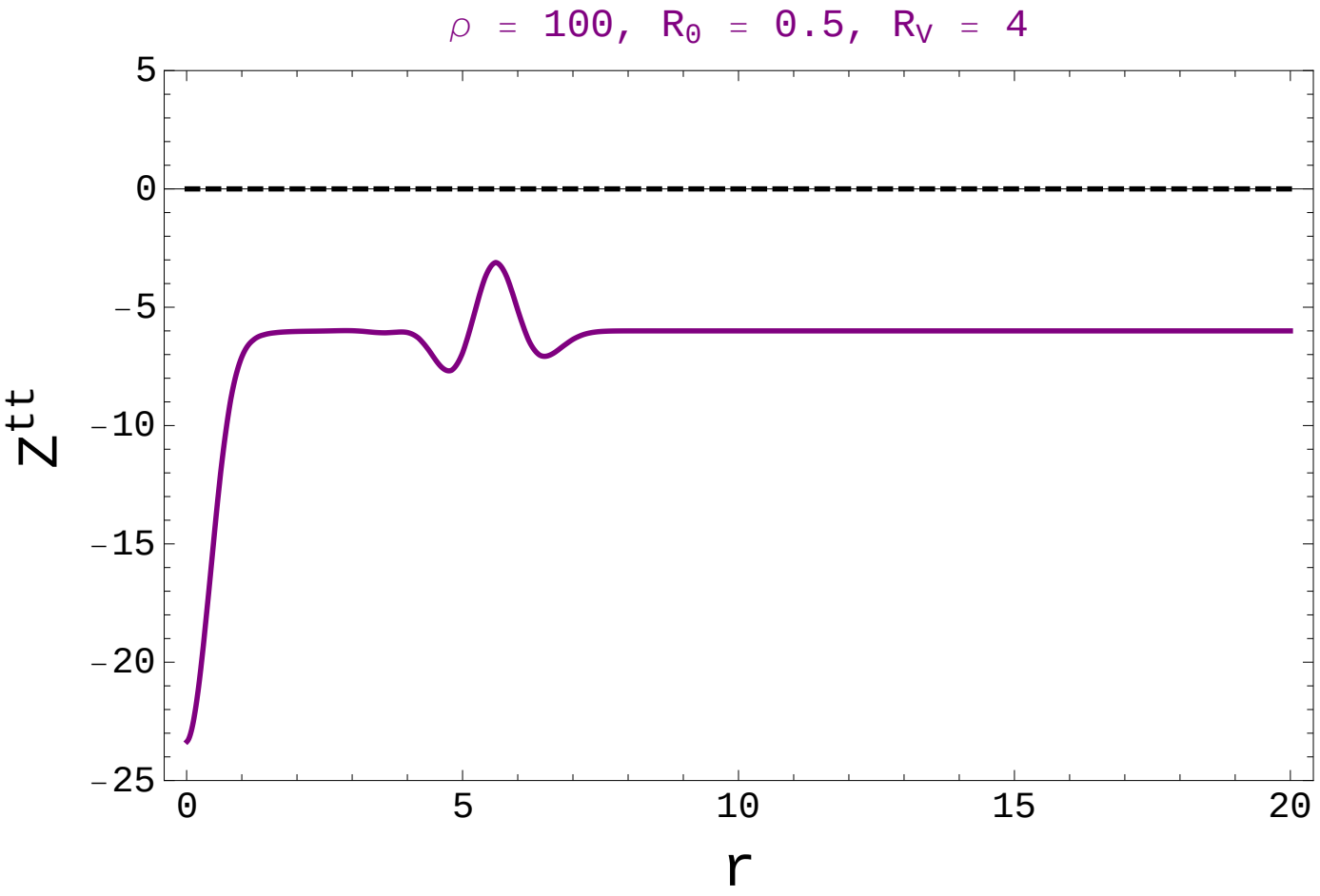

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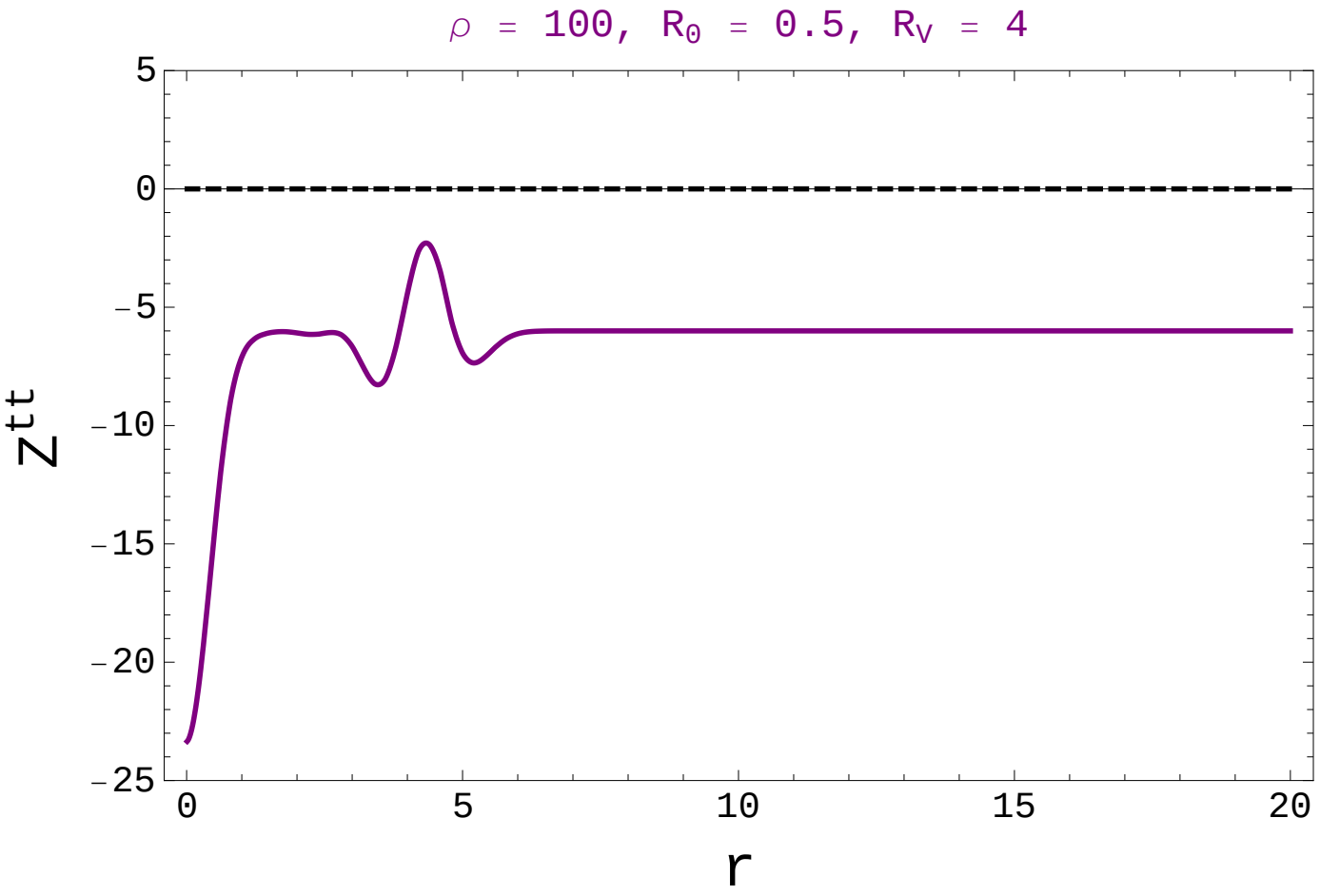

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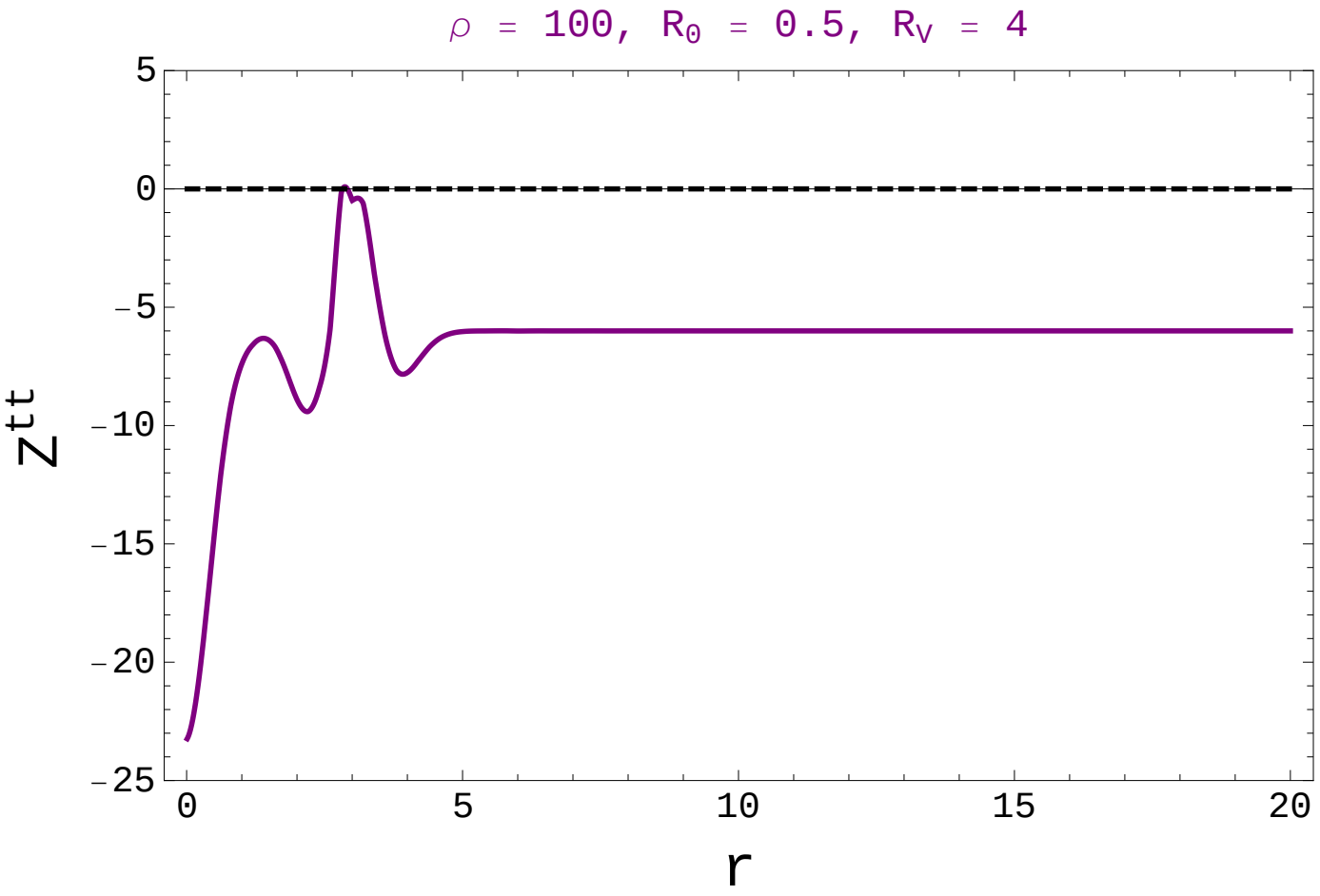

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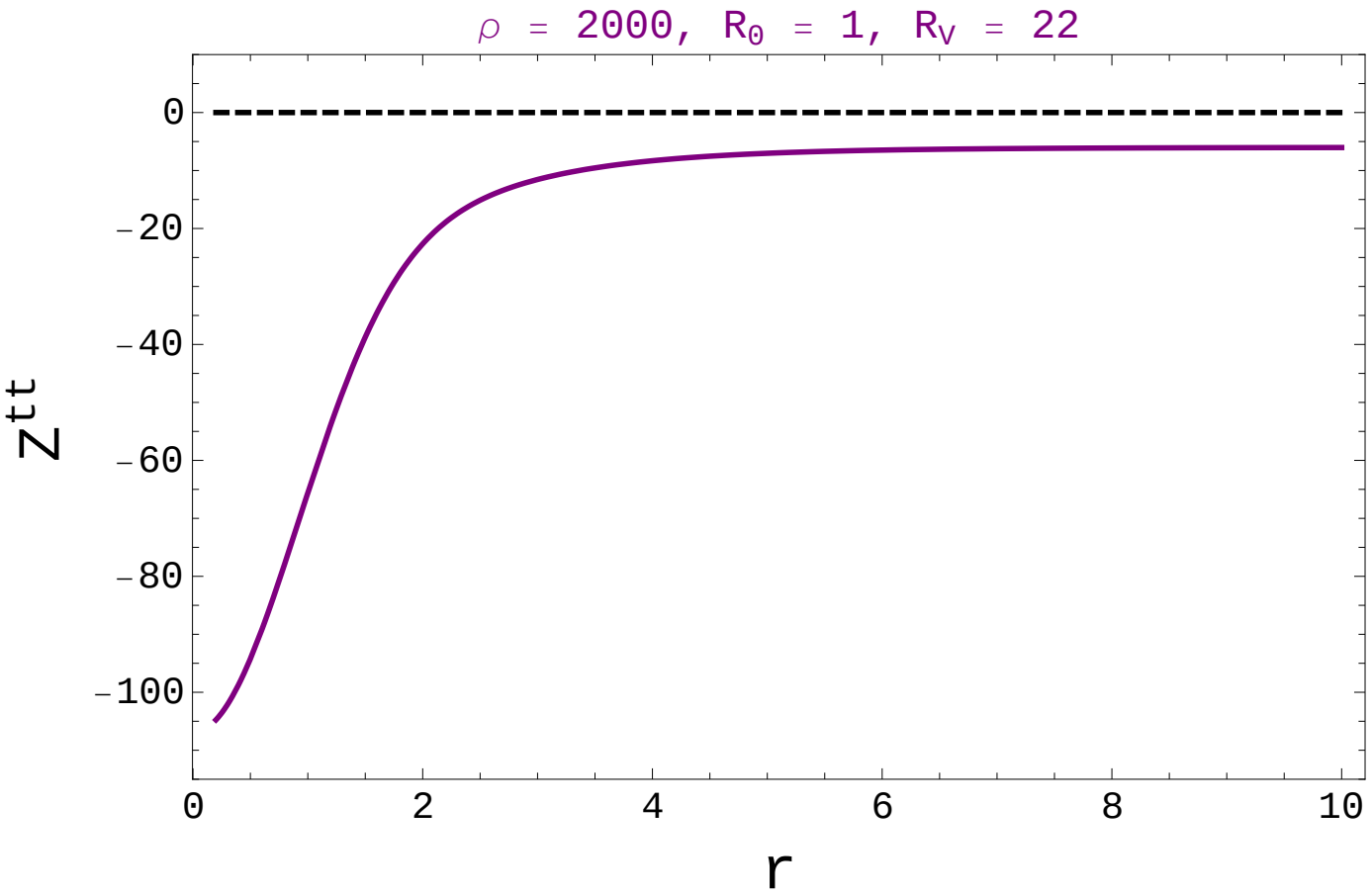

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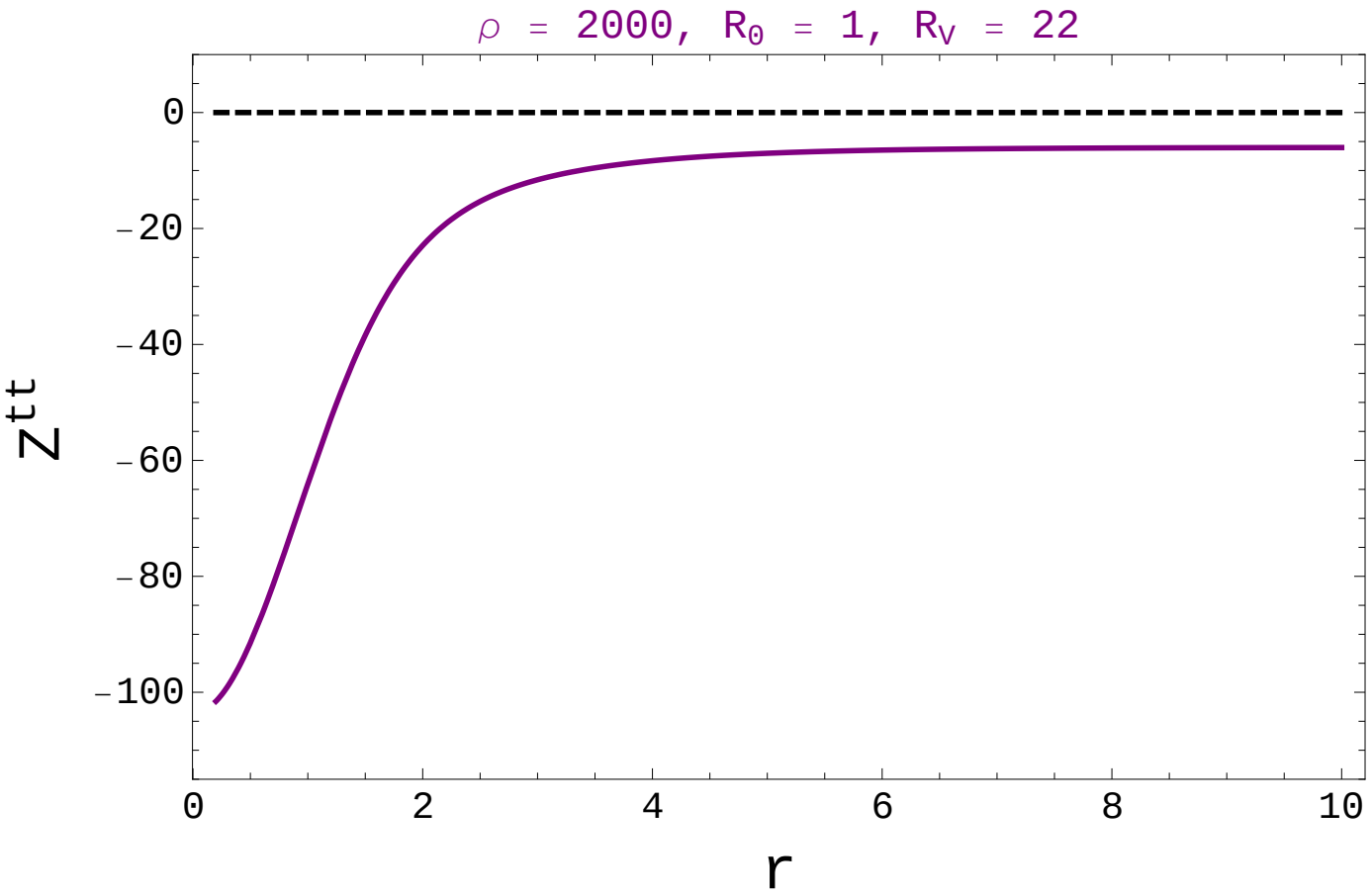

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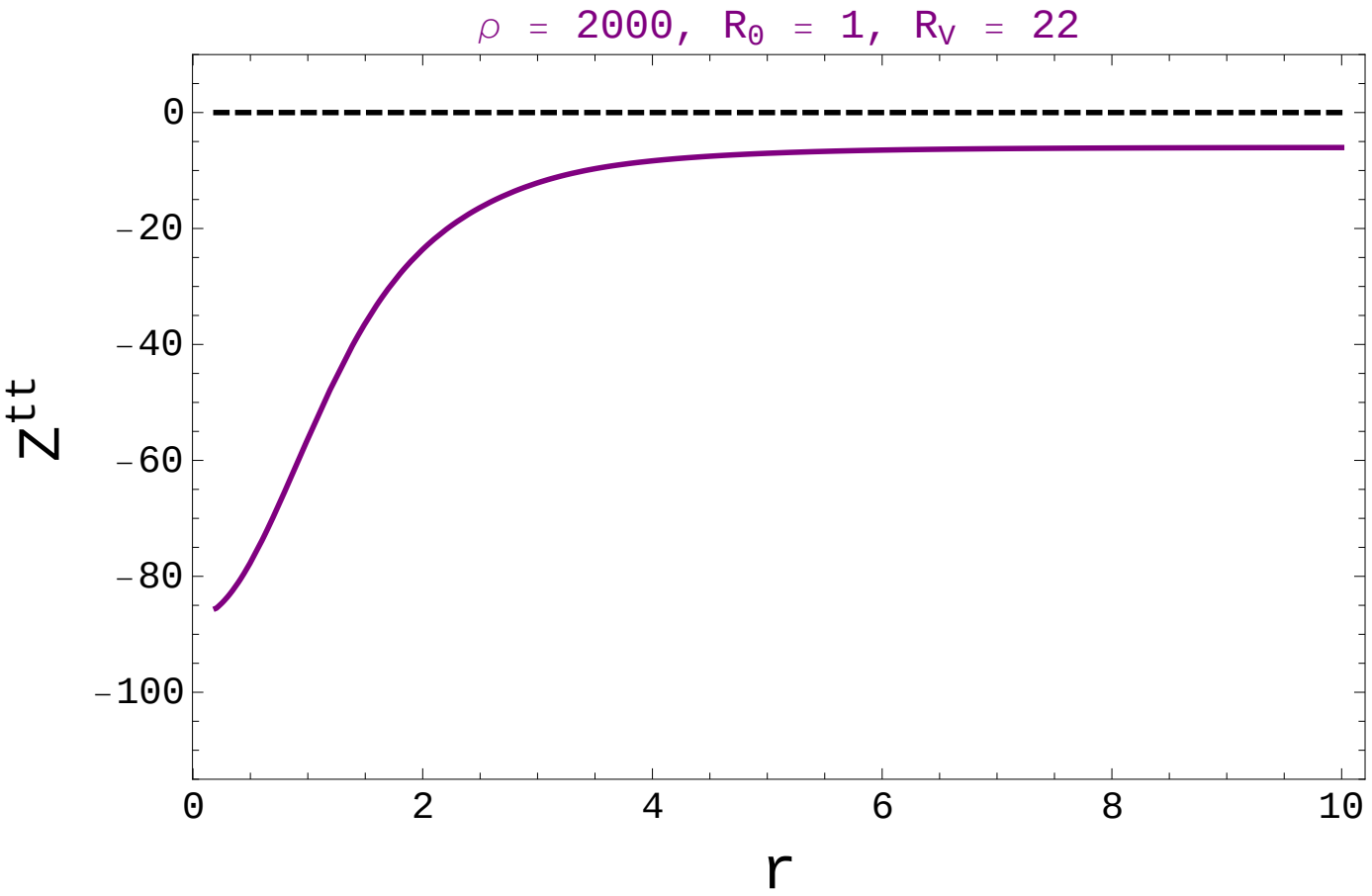

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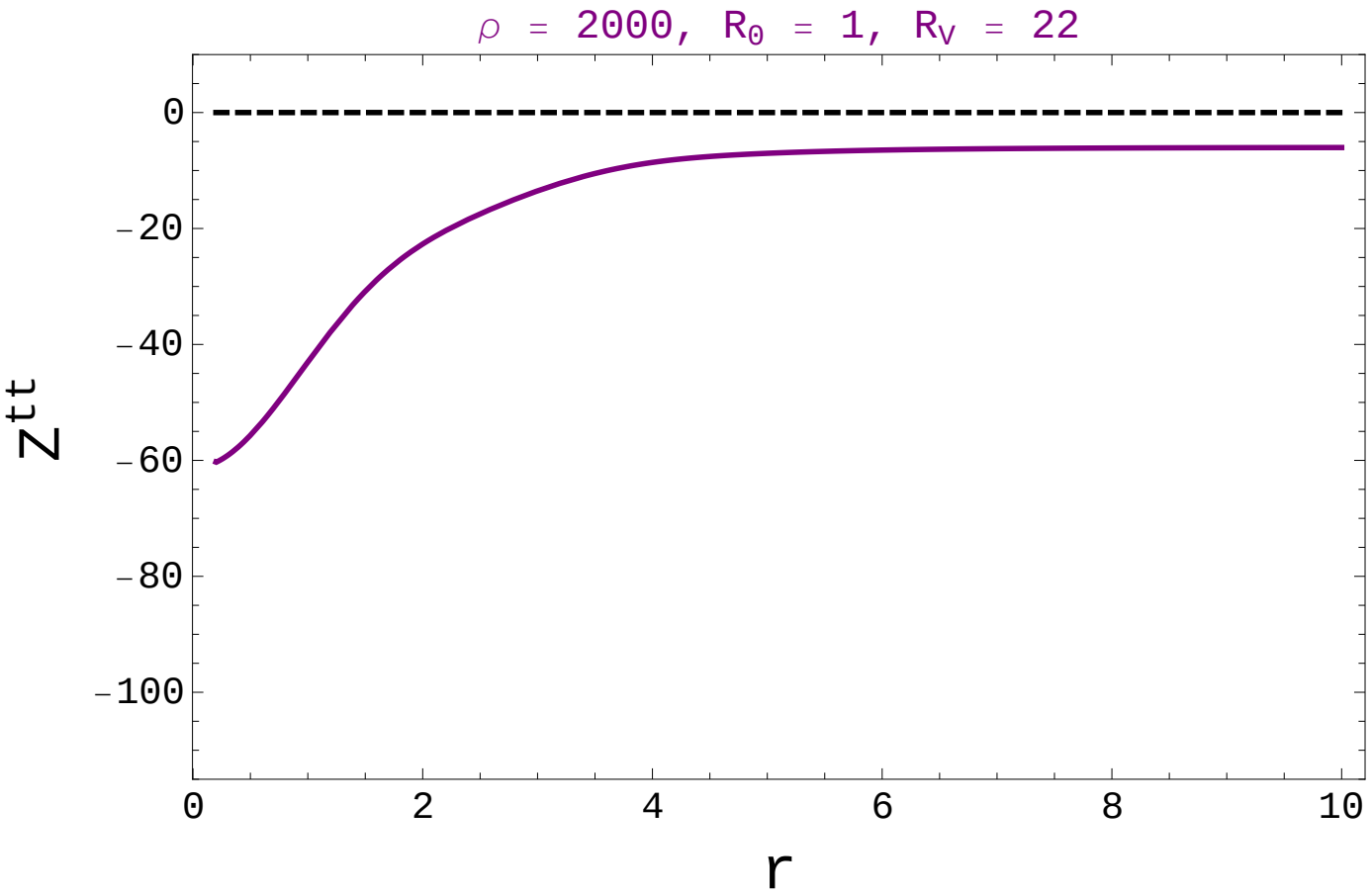

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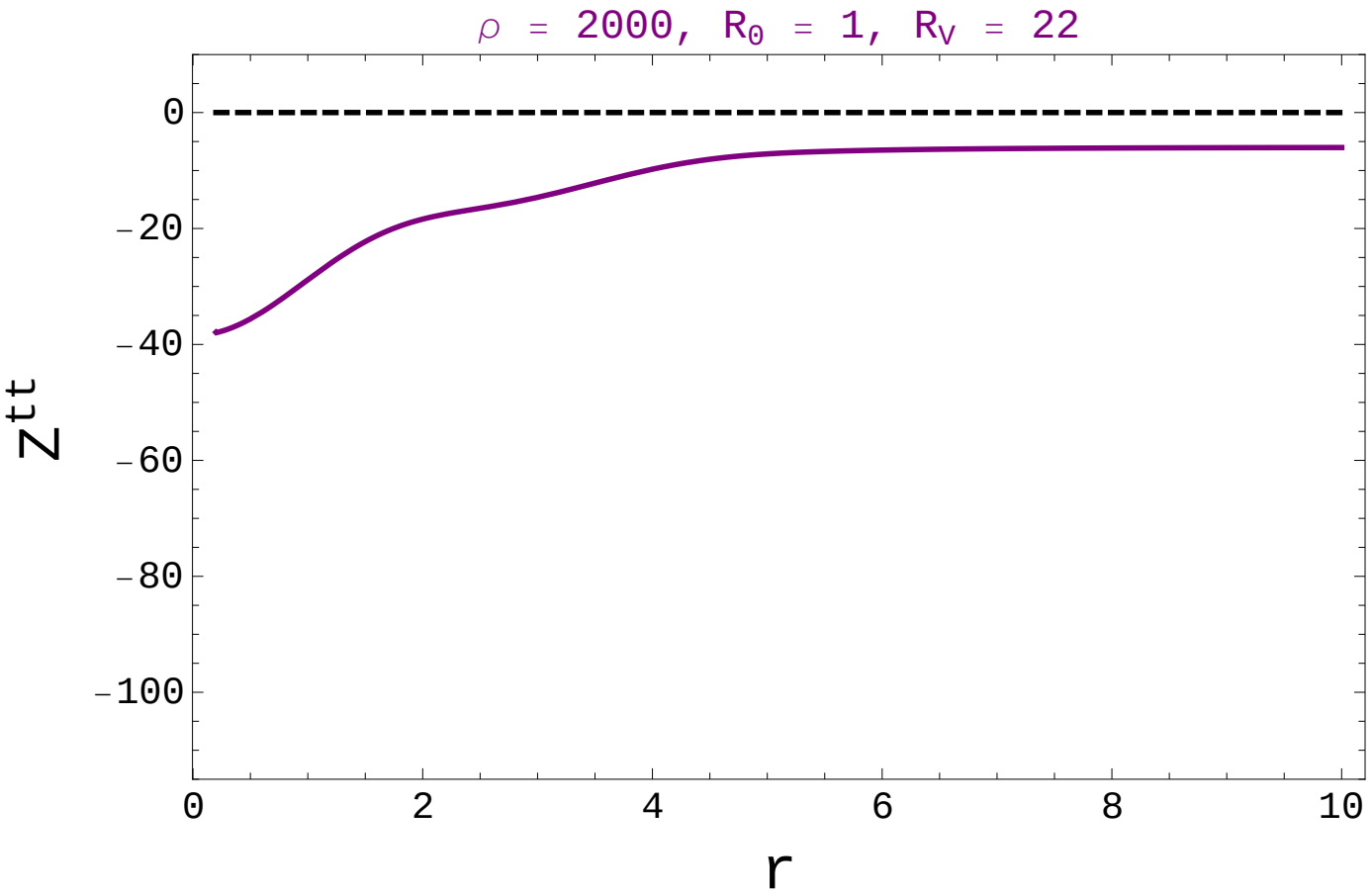

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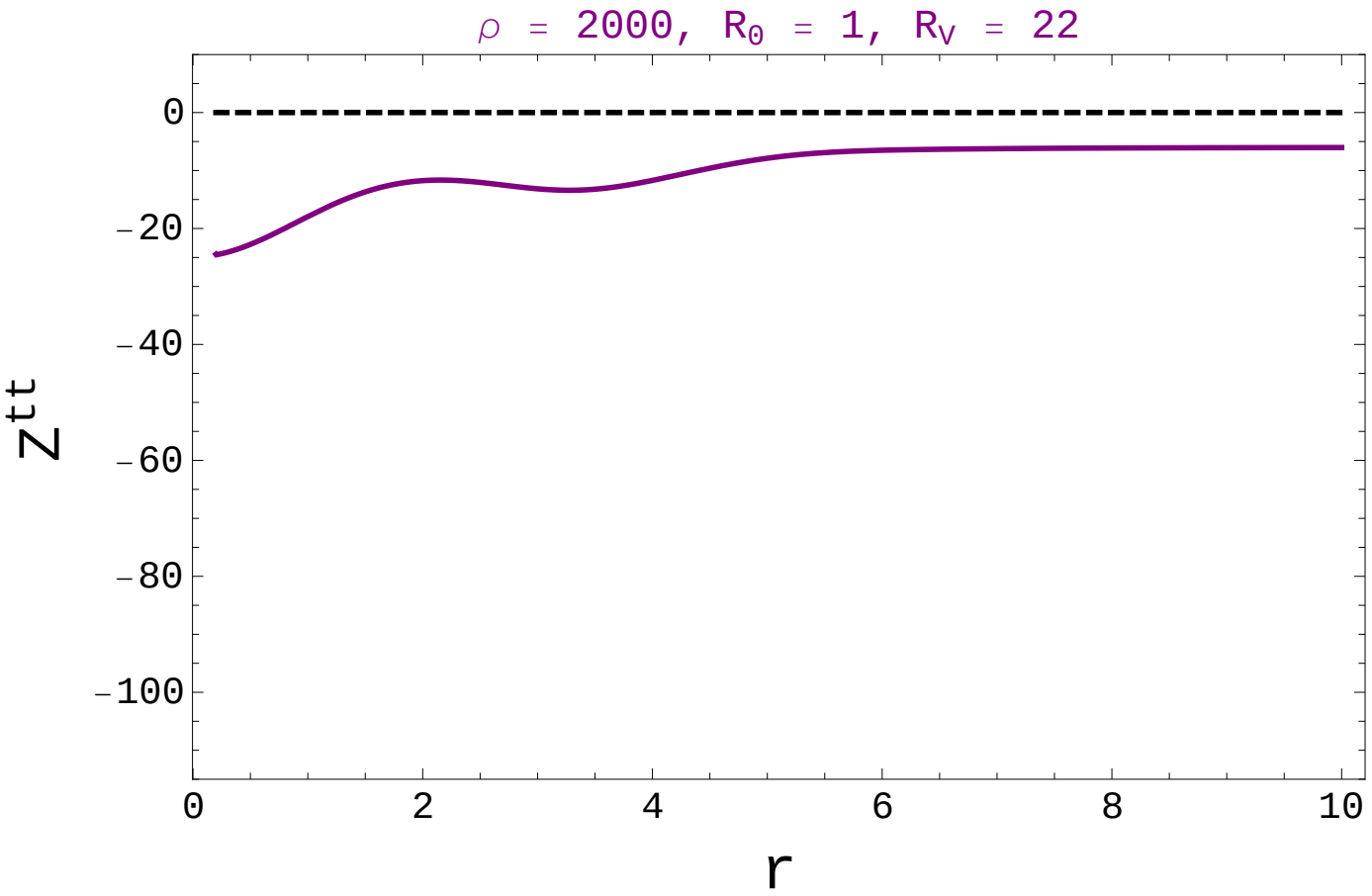

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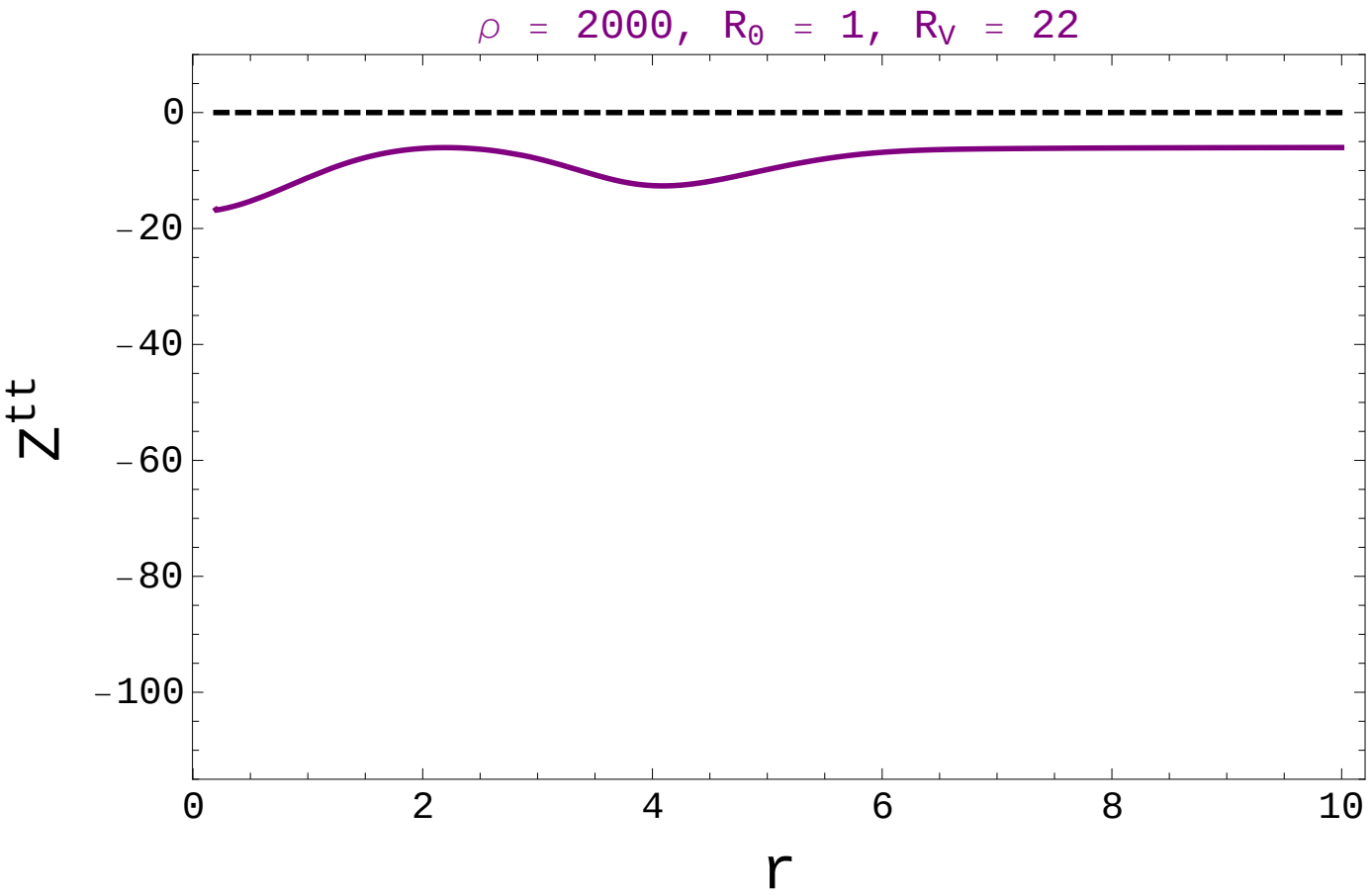
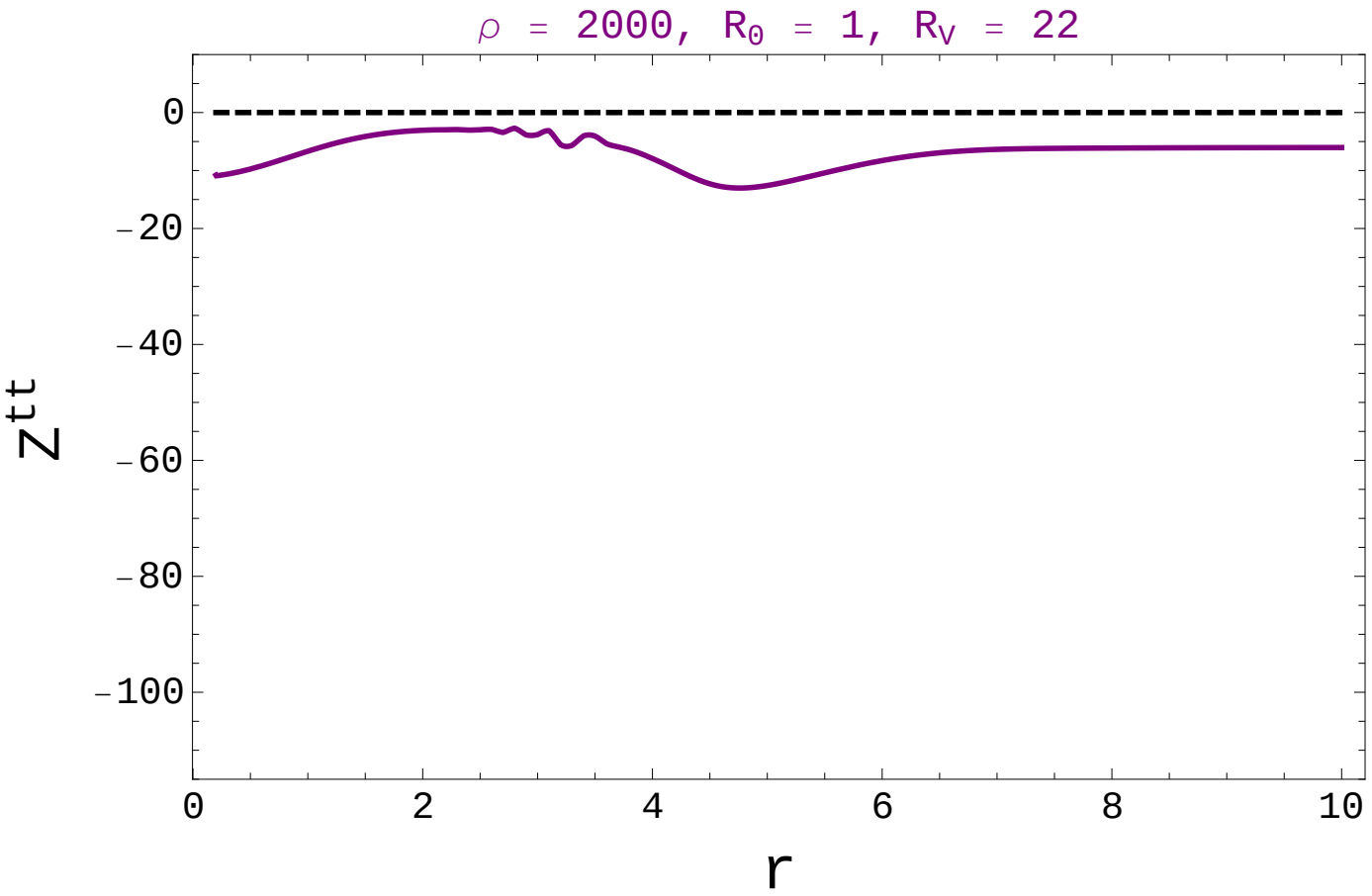


Cauchy Breakdown



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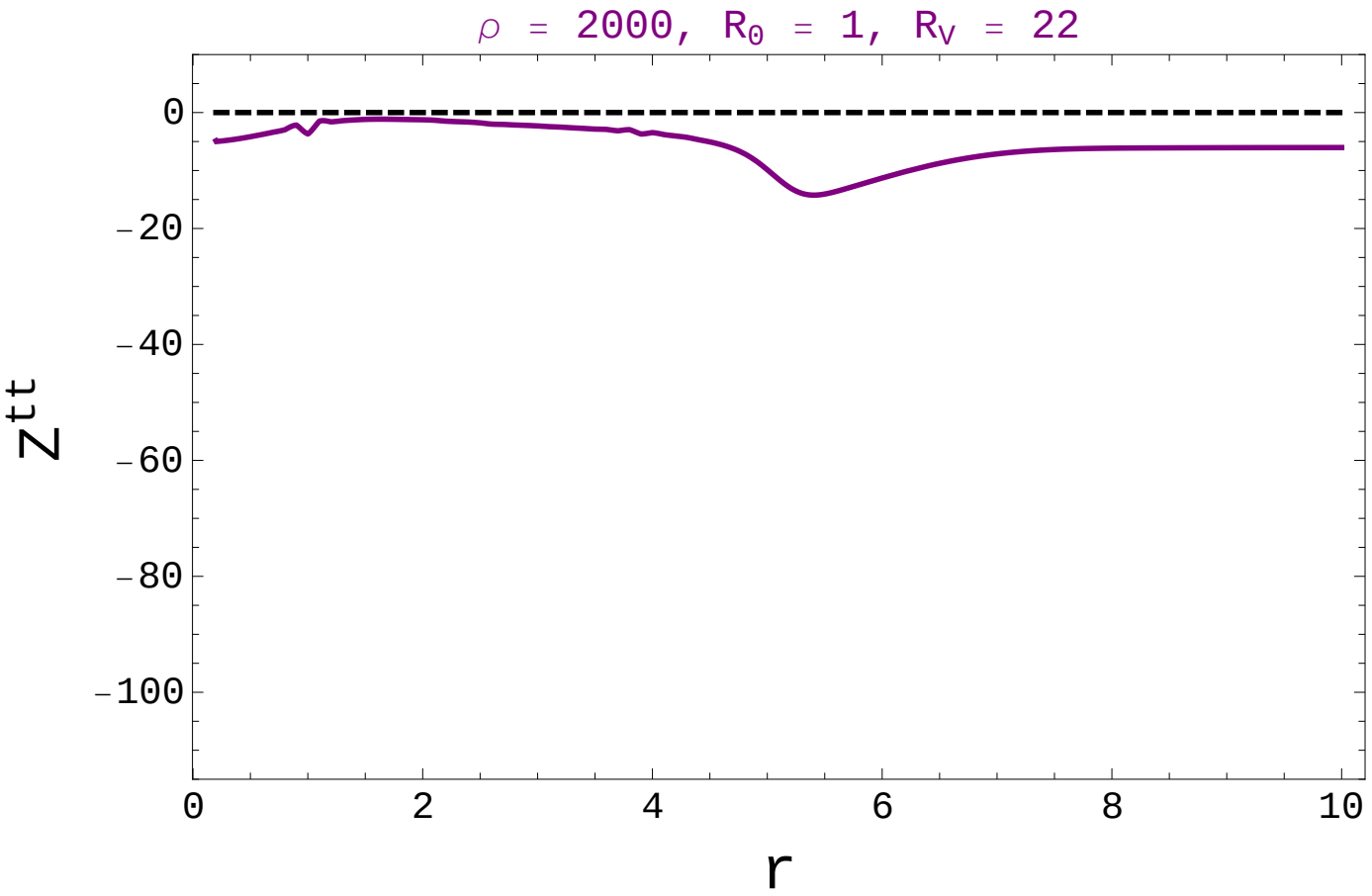

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For astrophysical objects, Cauchy breakdown will be an issue

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- Does it depend on choice of coordinate system? Can we find a transformation to well-defined Cauchy surface on which to evolve? Probably not...
- Is this just a phenomena in the decoupling limit that will resolve itself in the full theory?

Cauchy Breakdown

- This is an issue that must be resolved if we want to use these theories to model astrophysical phenomena.
- Not possible to evolve past the point of Cauchy breakdown without a physical prescription.
- Does it depend on choice of coordinate system? Can we find a transformation to well-defined Cauchy surface on which to evolve? Probably not...
- Is this just a phenomena in the decoupling limit that will resolve itself in the full theory?
- It is likely that Cauchy breakdown coincides with the breakdown of the classical effective field theory, and occurs when quantum corrections become important
  $\Rightarrow$ Need an understanding of the UV completion to evolve past this breakdown.

Conclusion

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4. We have also shown that Cauchy breakdown occurs when there is a large hierarchy between R_0 and R_V which is the case for realistic objects.

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3. Investigate further into Cauchy breakdown to determine its origin and possible prescription for resolution