Using B-modes to Constrain the Lack of Correlation in the CMB

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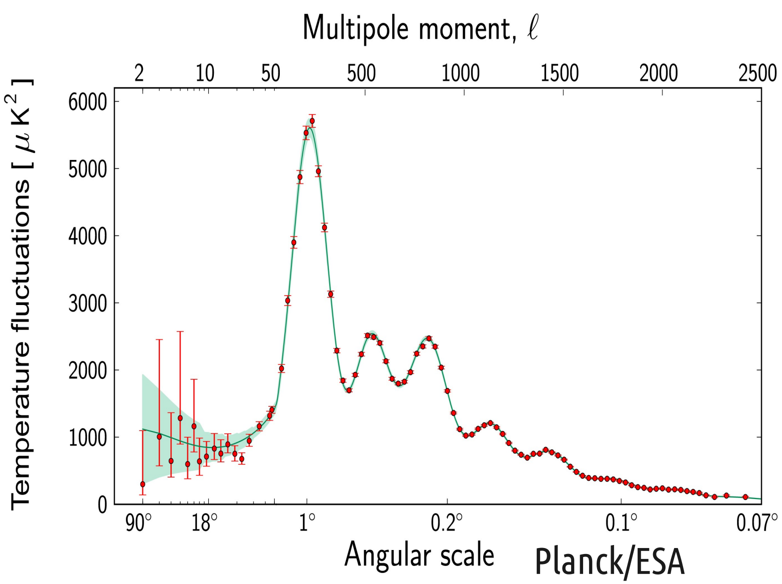
WMAP and Planck have given us excellent measurements of the temperature power spectrum which (mostly) support LCDM cosmology…



(very) brief background

$$\Delta T(\mathbf{n}) = \sum_{l>0} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\mathbf{n})$$

$$\langle a_{lm} a^*_{l'm'} \rangle = \delta_{ll'} \, \delta_{mm'} \, C_l$$

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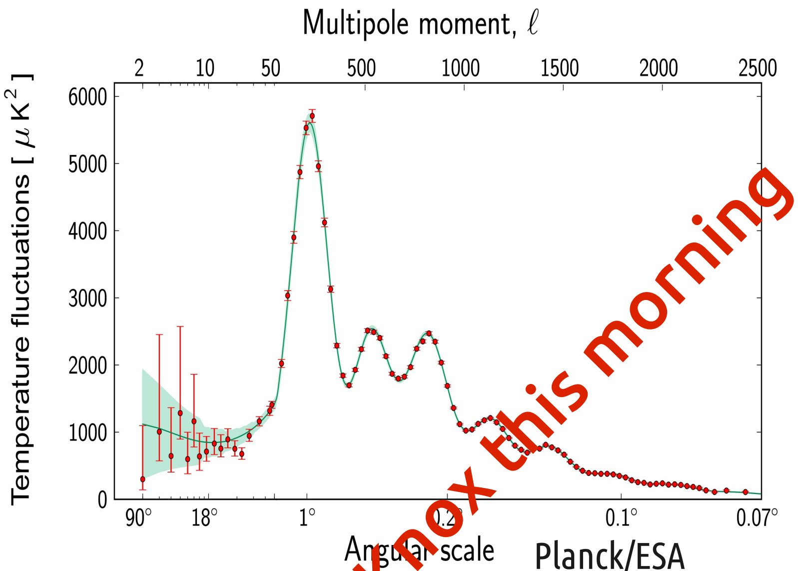

Talk by Lloyd Knox this morning

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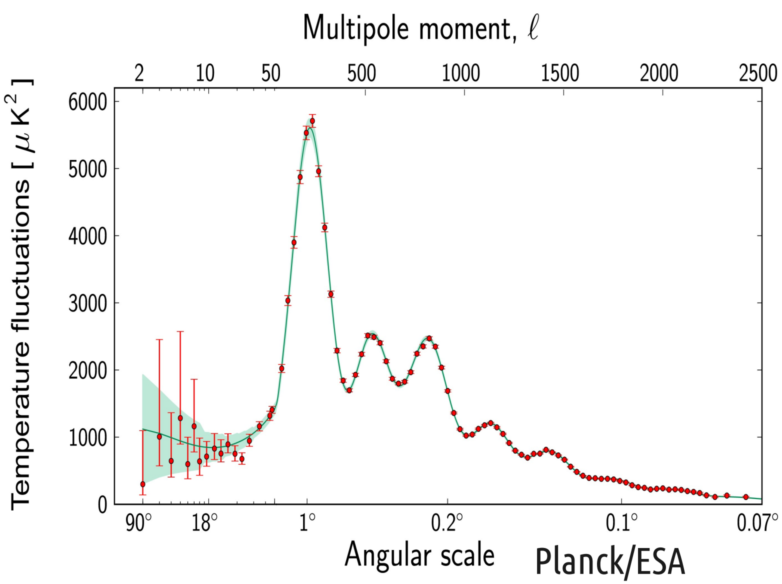

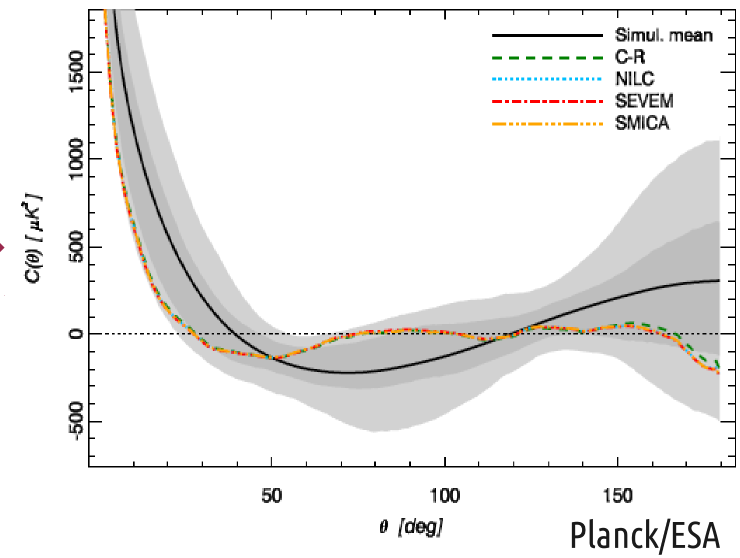

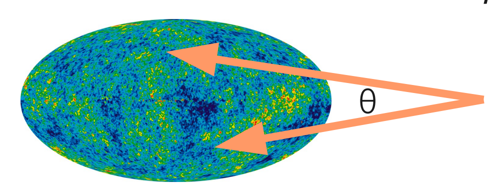

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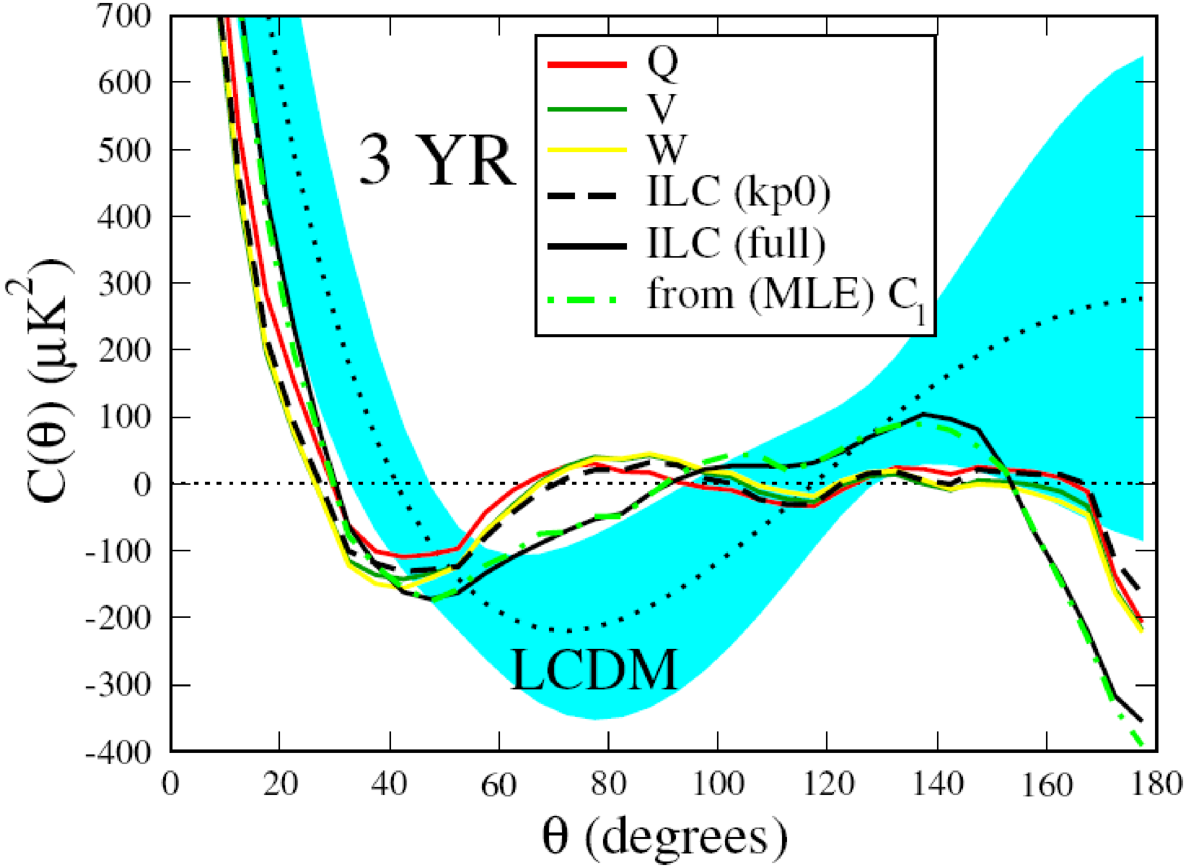
$$C^{TT}(\theta) \equiv \langle T(\hat{\mathbf{n}}_1) T(\hat{\mathbf{n}}_2) \rangle$$

$$C(\theta) = \sum_{\ell} \frac{\ell(\ell+1)}{4\pi} C_\ell P_\ell(\cos\theta)$$

ACF measurements from WMAP



Defined a statistic:

$$S_{1/2} = \int_{1/2}^{-1} d(\cos\theta)\, C(\theta)^2$$

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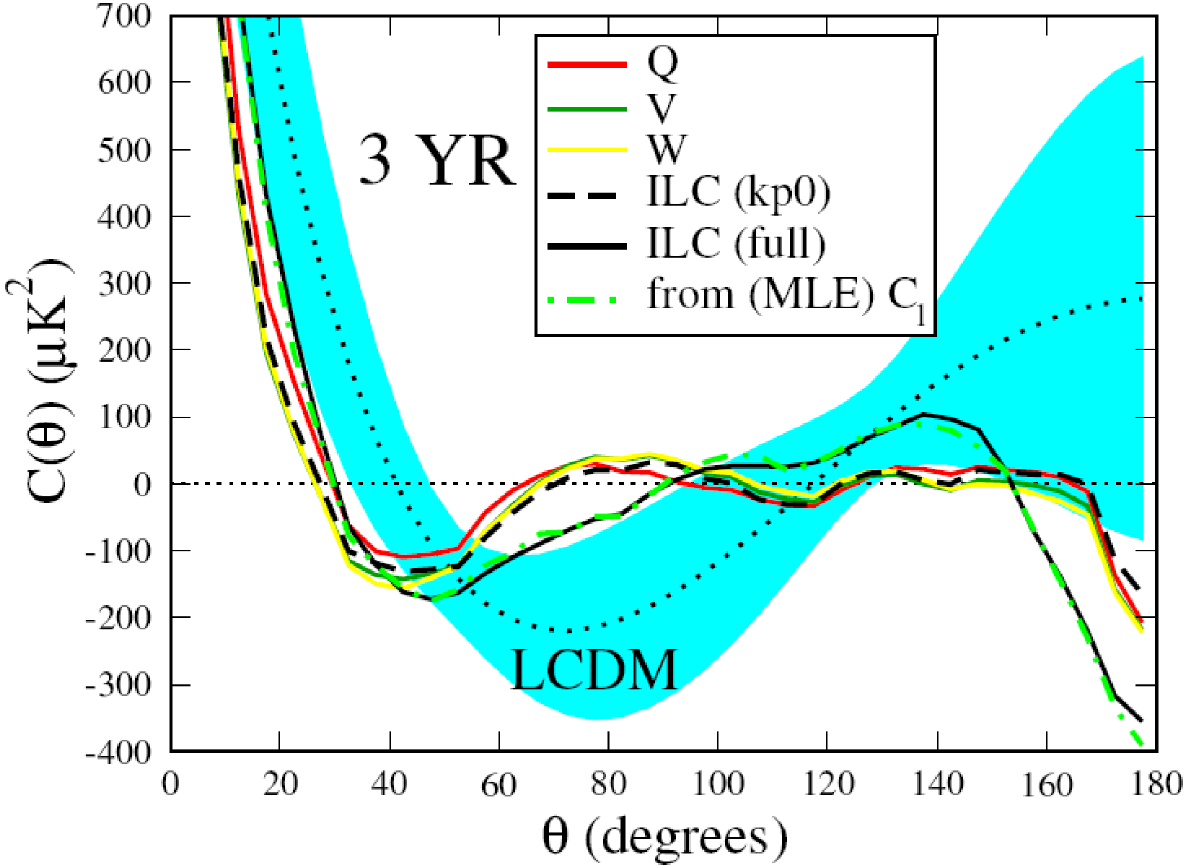

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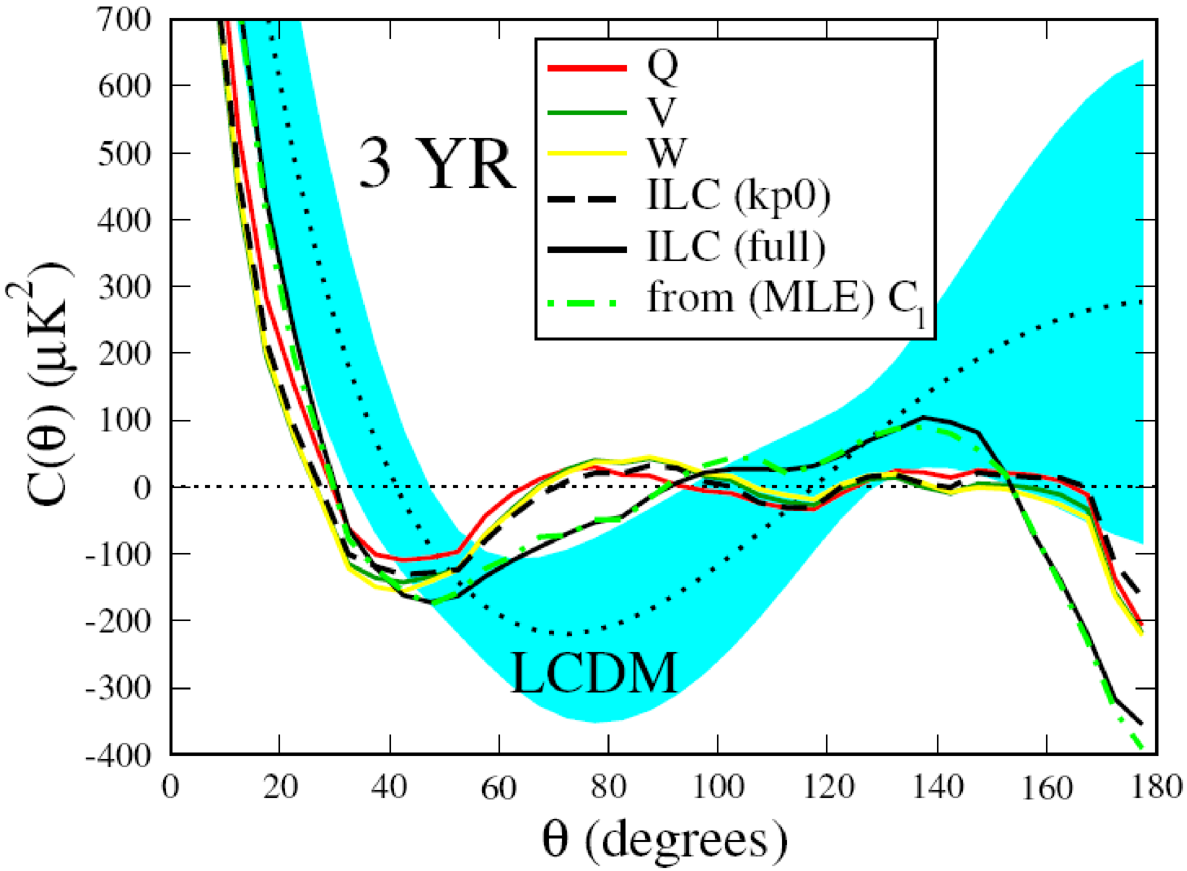

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depending on the choice of data set

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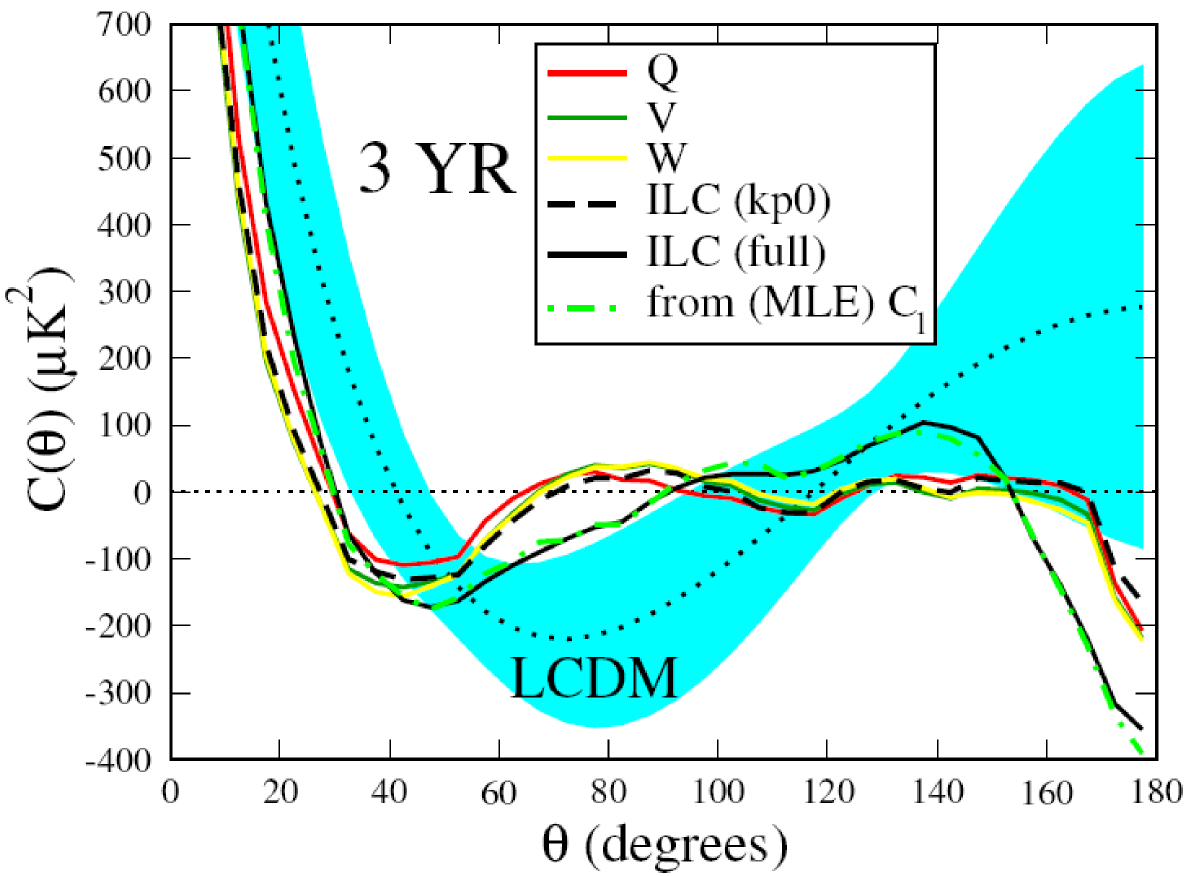

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This is an *a posteriori* statistic

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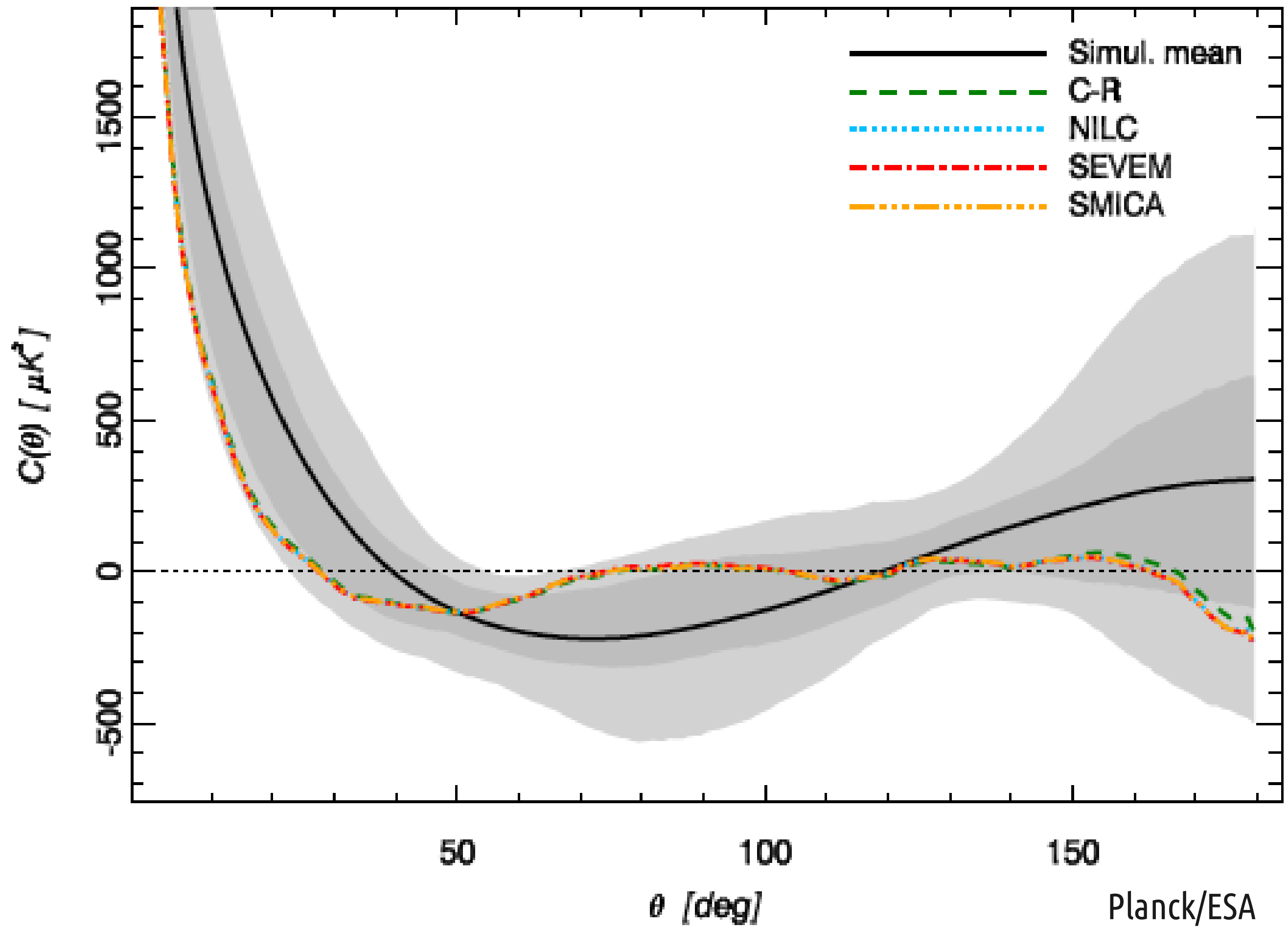
ACF measurements from Planck



Planck/ESA

Lots of work has gone into characterizing the lack of correlation in temperature data…

Recent analysis of Planck maps by Copi, Huterer, Schwarz, Starkman – arXiv: 1310:3831

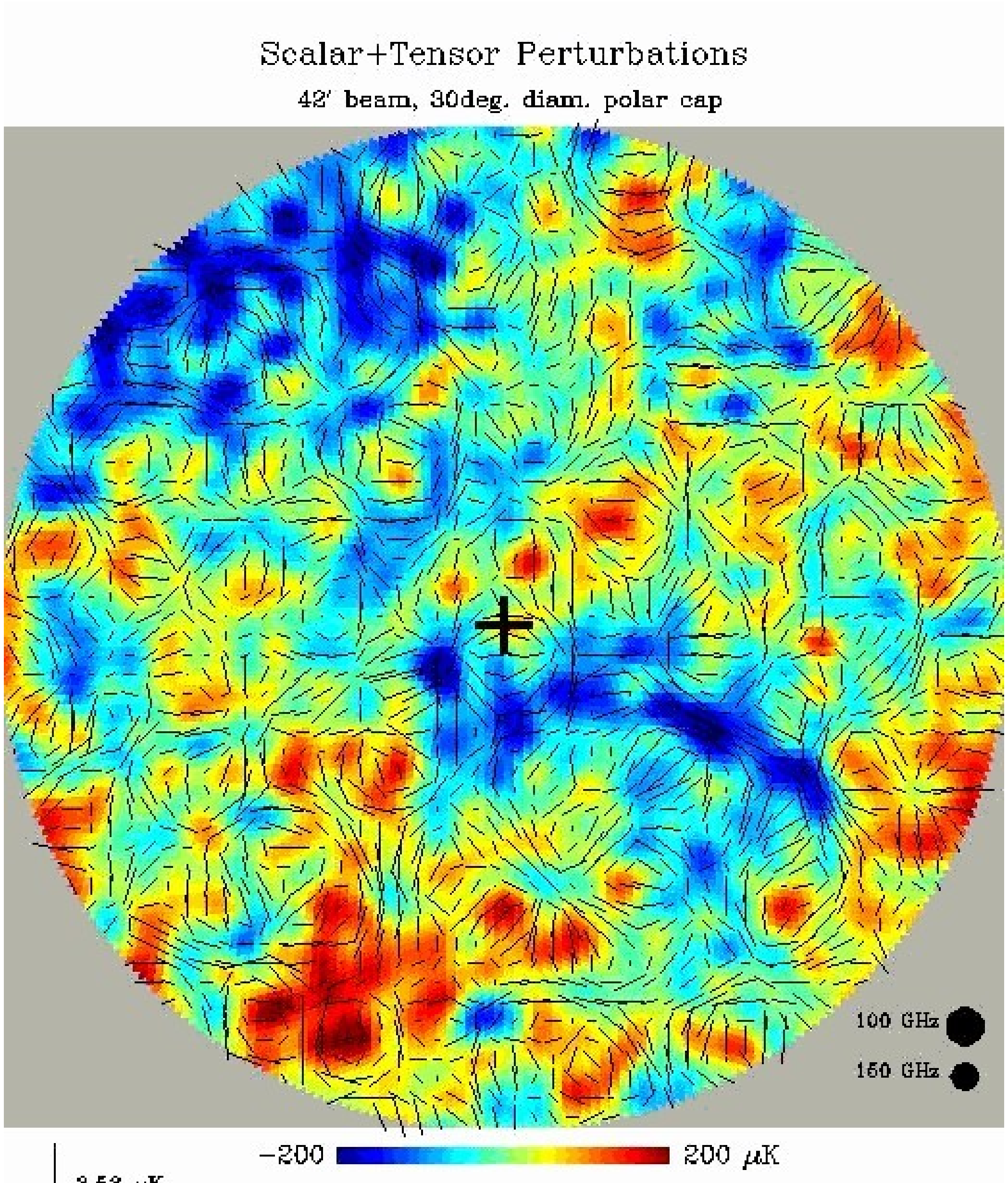
A nice (short) review of the lack of correlation at large angles arXiv: 1201.2459

Detailed comparison of WMAP to Planck large angle anomalies arXiv: 1303.5083

But we need to move beyond temperature to learn something new

This work:

Use the B-mode correlation function as an independent test of the lack of correlation seen in temperature



BICEP collaboration

$$(Q(\hat{n}) \pm \imath U(\hat{n})) = \sum_{\ell m} a_{\ell m}^{\pm 2} \; {}_{\pm 2}Y_{\ell m}$$

$$a_{\ell m}^{E} = \frac{1}{2}\left[a_{\ell m}^{2} + a_{\ell m}^{-2}\right]$$

$$a_{\ell m}^{B} = \frac{\imath}{2}\left[a_{\ell m}^{2} - a_{\ell m}^{-2}\right]$$

Local BB correlation

$$(Q(\hat{n}) \pm \imath U(\hat{n})) = \sum_{\ell m} a^{\pm 2}_{\ell m}\ {}_{\pm 2}Y_{\ell m}$$

$$a^B_{\ell m} = \frac{\imath}{2}\left[a^2_{\ell m} - a^{-2}_{\ell m}\right]$$

$$\left.\begin{matrix} \eth^2 \\ \bar{\eth}^2 \end{matrix}\right\} \text{Spin raising and lowering operators} \left\{\begin{matrix} -(\sin\theta)^s\left[\dfrac{\partial}{\partial\theta} + \left(\dfrac{\imath}{\sin\theta}\right)\dfrac{\partial}{\partial\phi}\right](\sin\theta)^{-s} \\ \sqrt{(\ell - s)(\ell + s + 1)}\ {}_{s+1}Y_{\ell m} \end{matrix}\right.$$

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Spin raising and lowering operators

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$$\sqrt{(\ell - s)(\ell + s + 1)} \; {}_{s+1}Y_{\ell m}$$

$$B(\hat{\mathbf{n}}) = \frac{-\imath}{2}\left[\bar{\eth}^2(Q(\hat{\mathbf{n}}) + \imath U(\hat{\mathbf{n}})) - \eth^2(Q(\hat{\mathbf{n}}) - \imath U(\hat{\mathbf{n}}))\right]$$

Zaldarriaga & Seljak '97

Local BB correlation

$$B(\hat{n}) = \sum_{\ell m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\, a^{B}_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$$

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$$\downarrow$$

$$C^{BB}(\theta) = \langle B(\hat{\mathbf{n}}_{\mathbf{1}}) B(\hat{\mathbf{n}}_{\mathbf{2}}) \rangle$$

$$= \sum_{\ell} \frac{2\ell+1}{4\pi} \left(\frac{(\ell+2!)}{(\ell-2)!} \right) C_\ell^{BB} P_\ell(\cos\theta)$$

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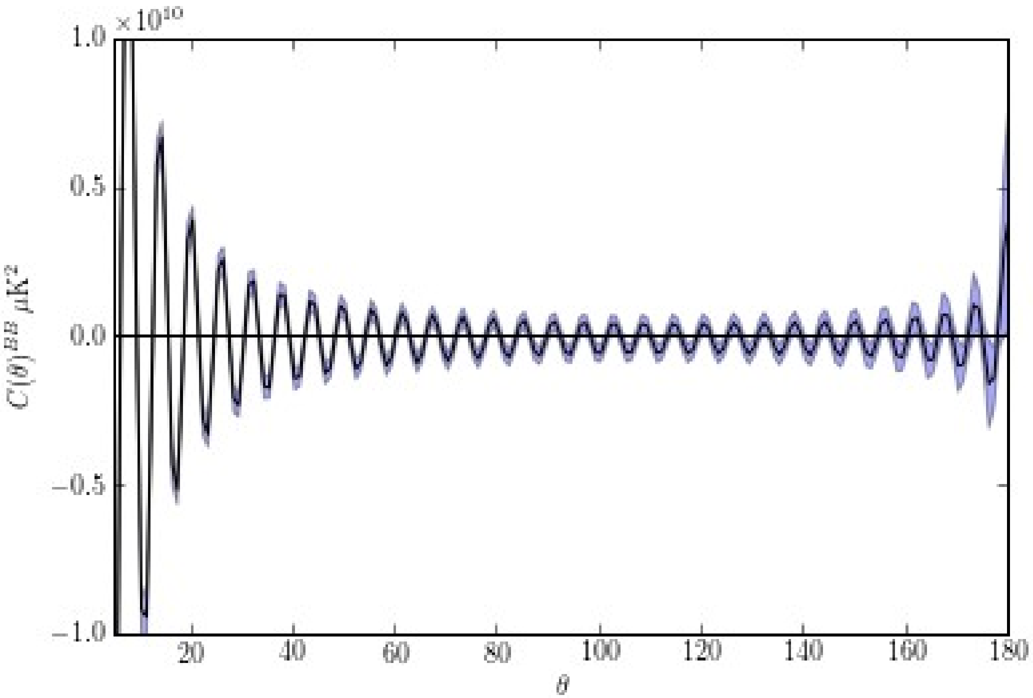
This goes like ℓ^4

This is not a matter of normalization or definition;
it's required for a local quantity

Local BB correlation



Smoothed such that lensing contribution to PS is suppressed

With this definition, large scales no longer correspond to low multipoles.

Local BB correlation

Pros:

Local function on the sky

Can be calculated from map observations directly

Local BB correlation

Pros:

- Local function on the sky
- Can be calculated from map observations directly

Cons:

- Dominated by large multipoles
- Requires smoothing/limited by resolution

QQ and UU correlation

$$C^{QQ} \equiv \langle Q(\hat{\mathbf{n}}_\mathbf{1}) Q(\hat{\mathbf{n}}_\mathbf{2}) \rangle \qquad C^{UU} \equiv \langle U(\hat{\mathbf{n}}_\mathbf{1}) U(\hat{\mathbf{n}}_\mathbf{2}) \rangle$$

$$C^{QQ}(\theta) = \sum_\ell \frac{2\ell+1}{2\pi} \left(\frac{2(\ell-2!)}{(\ell+2)!} \right) \left[C_\ell^{BB} G^-_{\ell,2}(\cos\theta) + C_\ell^{EE} G^+_{\ell,2}(\cos\theta) \right]$$

$$C^{UU}(\theta) = \sum_\ell \frac{2\ell+1}{2\pi} \left(\frac{2(\ell-2!)}{(\ell+2)!} \right) \left[C_\ell^{EE} G^-_{\ell,2}(\cos\theta) + C_\ell^{BB} G^+_{\ell,2}(\cos\theta) \right]$$

Kamionkowski, Kosowsky, Stebbins '97

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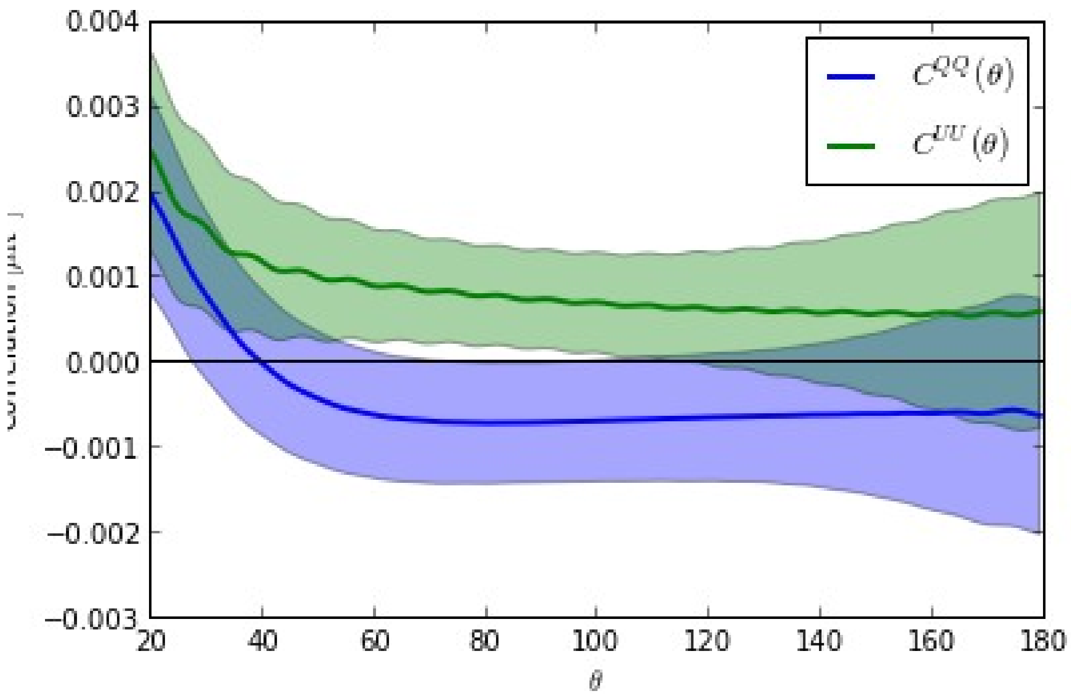
$$\boxed{P_\ell(\cos\theta)}$$

QQ and UU correlation

$$C^{\tilde{Q}\tilde{Q}} = \sum_{\ell} \frac{2\ell+1}{4\pi} \left(\frac{2(\ell-2)!}{(\ell+2)!} \right) C_\ell^{BB} G^{-}_{\ell,2}(\cos\theta)$$

$$C^{\tilde{U}\tilde{U}} = \sum_{\ell} \frac{2\ell+1}{4\pi} \left(\frac{2(\ell-2)!}{(\ell+2)!} \right) C_\ell^{BB} G^{+}_{\ell,2}(\cos\theta)$$



Not dominated by large multipoles, could provide better test for suppression

QQ and UU correlation

Pros:

- Dominated by low multipoles
- Distinguishable features which help with defining statistics

QQ and UU correlation

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Distinguishable features which help with defining statistics

Cons:

Non-local function:

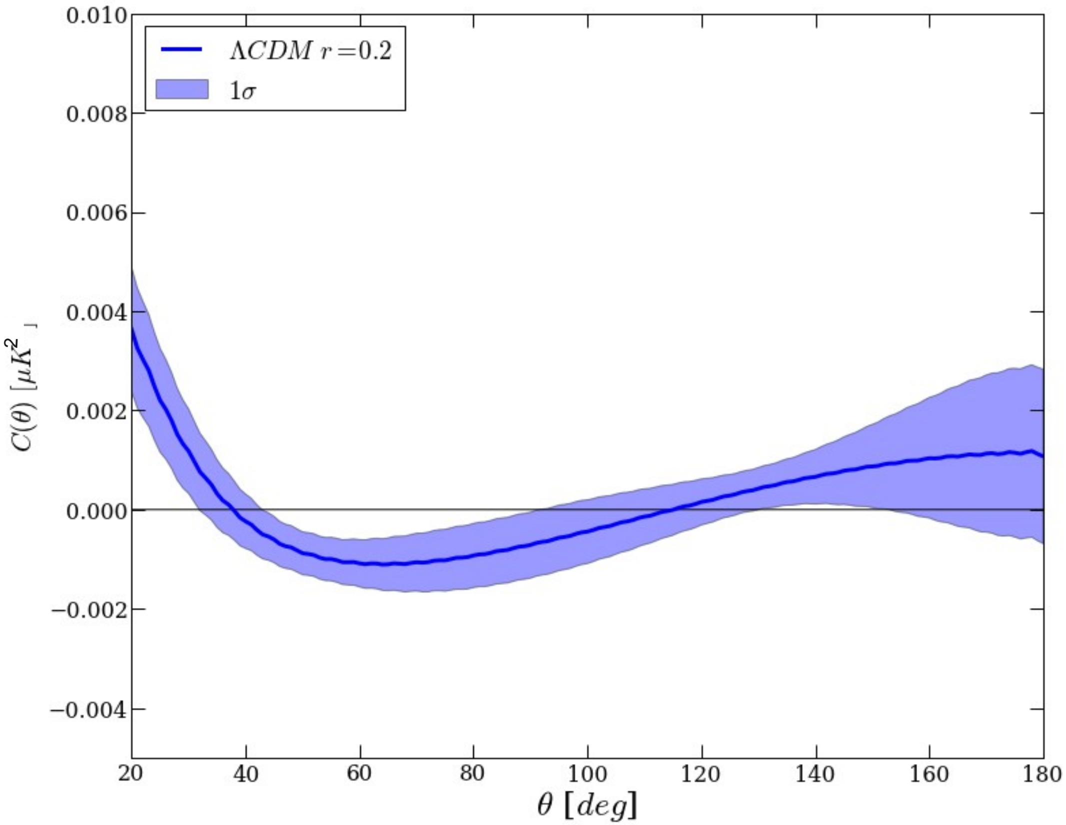
$$_{\pm 2}a_{\ell m} = \int \mathrm{d}\Omega \left(Q(\hat{n}) \pm \imath U(\hat{n})\right) \; _{\pm 2}Y_{\ell m}(\hat{n})$$

Can't be calculated directly from map

$$C^{\tilde{Q}\tilde{Q}}(\theta) = C_L^{\tilde{Q}\tilde{Q}}(\theta) + C_{NL}^{\tilde{Q}\tilde{Q}}(\theta)$$

Non-local BB correlation

$$B(\hat{n}) \equiv \sum_{\ell m} a^B_{\ell m} Y_{\ell m}(\hat{n}) \qquad C(\theta) = \sum_{\ell} \frac{2\ell+1}{4\pi} C^{BB}_{\ell} P_{\ell}(\cos\theta)$$



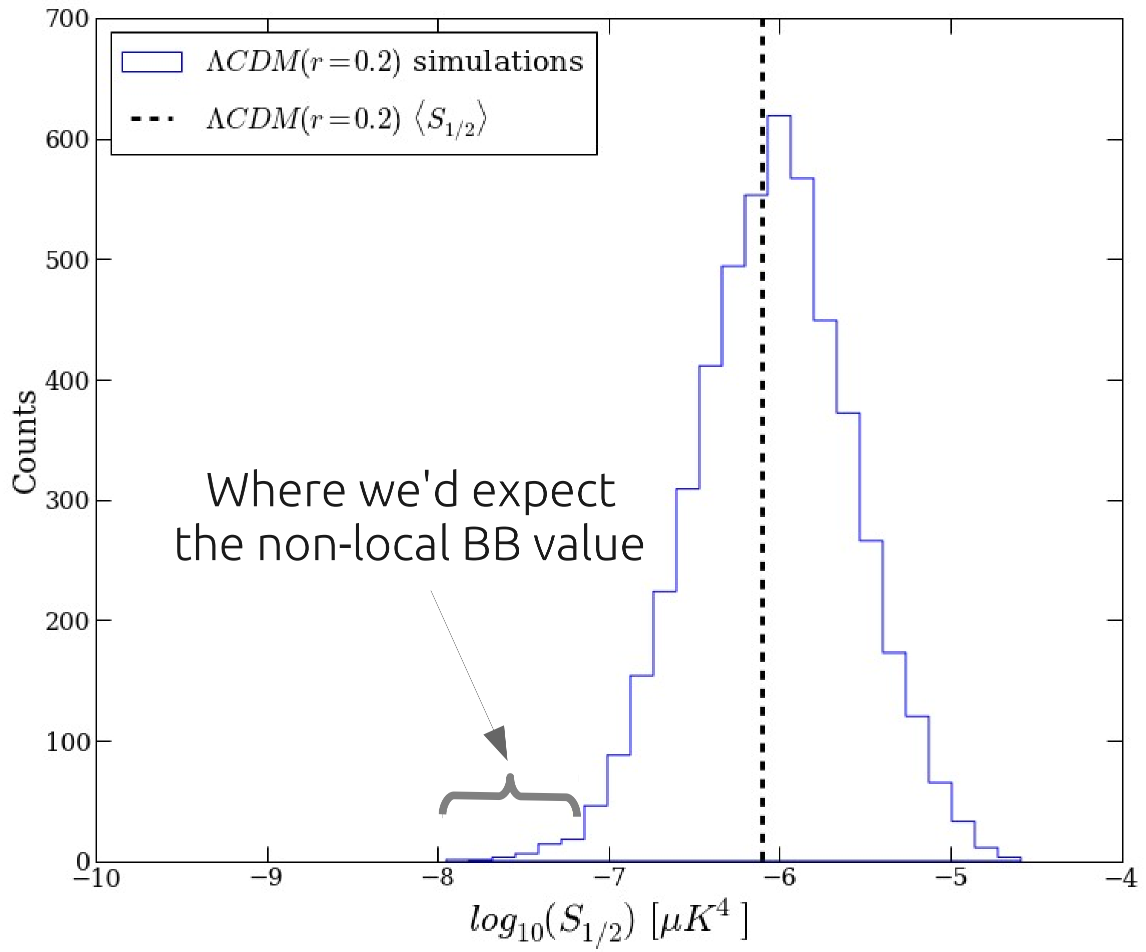
Dominated by low multipoles,
Behavior analogous
to temperature

But, physical interpretation
Is ambiguous

Non-local BB correlation

Reminder:

$$S_{1/2} = \int_{1/2}^{-1} d(\cos\theta)\, C(\theta)^2$$



Non-local BB correlation

Pros:

Dominated by low multipoles

Direct analog to temperature analysis

Non-local BB correlation

Pros:

Dominated by low multipoles

Direct analog to temperature analysis

Cons:

Non-local function, information From the full sky required

Not a quantity that can be Calculated from direct map observations

Conclusions

Local BB

- Local definition of the B-mode map, constructed from observables
- Dominated by large multipoles

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- Dominated by low multipoles, easier to make predictions for statistics

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QQ and UU (B-mode only)

- Extracts B-mode only piece of Q and U maps in harmonic space (therefore non-local)
- Dominated by low multipoles, easier to make predictions for statistics

Non-local BB

- Closest analog to temperature expressions
- Purely non-local; interpretation as a real-space correlation more ambiguous

Overall usefulness of each function predominantly dictated by which model you compare to LCDM