Prospects of Determination of Thermal History After Inflation with DECIGO

Space-baced Laser Interferoeter

Jun’ichi Yokoyama (Tokyo)

RESEARCH CENTER FOR THE EARLY UNIVERSE
THE UNIVERSITY OF TOKYO
RESCEU

東京大学 大学院
理学系研究科·理学部
SCHOOL OF SCIENCE, THE UNIVERSITY OF TOKYO

with S. Kuroyanagi and K. Nakayama

Detection of Primordial Tensor Perturbation from Inflation by BICEP2...

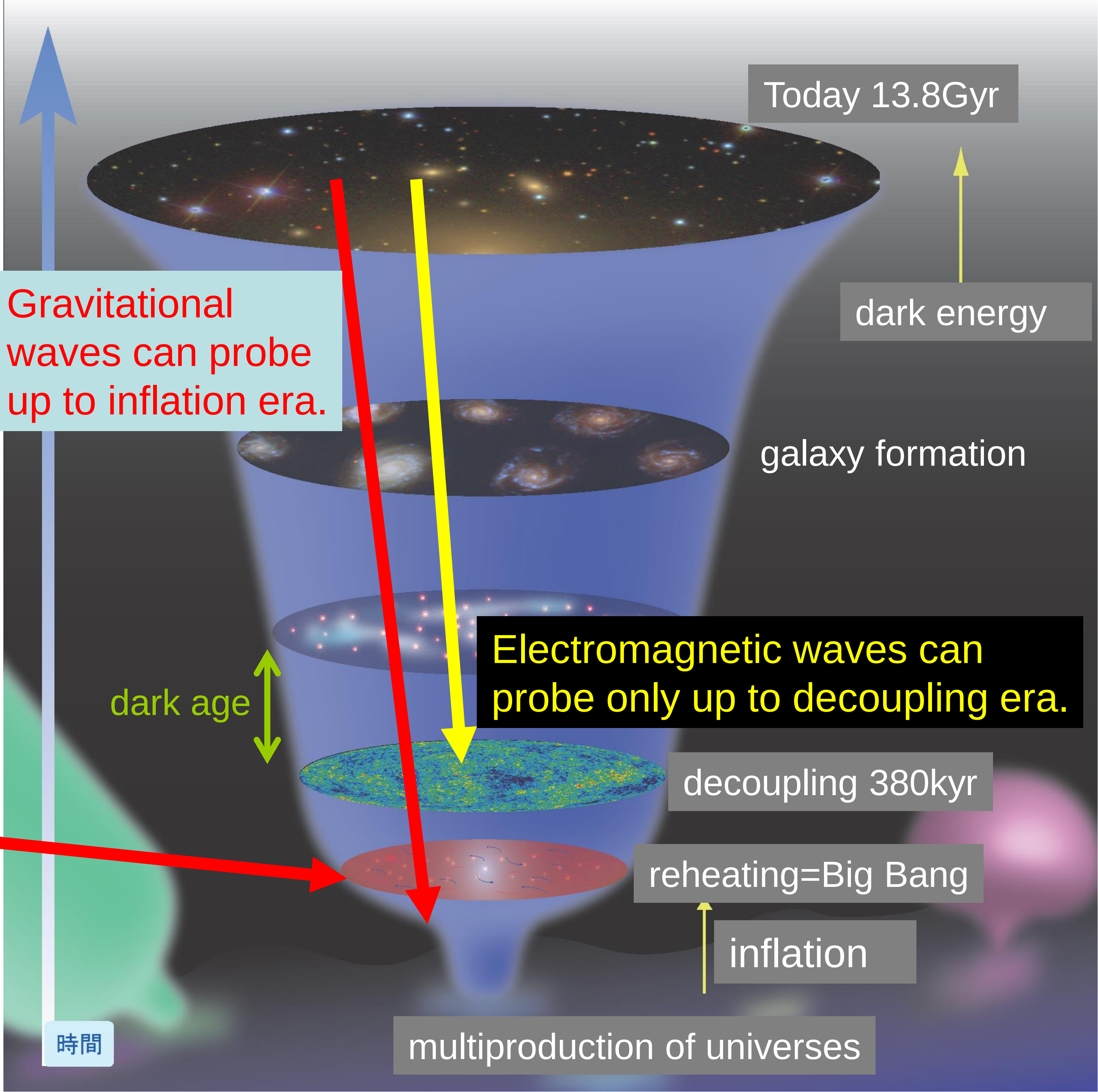
$$r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2}$$

Tensor-to-scalar ratio measures the scale of inflation

$$V[\phi] = (3.2\times 10^{16}\,\mathrm{GeV})^4 r = (7.5\times 10^{15}\,\mathrm{GeV})^4 \left(\frac{r}{0.003}\right)$$

Higher Frequency Tensor Perturbation Carries Information on the Thermal History After Inflation

Why Gravitational Waves ?



Today 13.8Gyr

dark energy

Gravitational waves can probe up to inflation era.

galaxy formation

Electromagnetic waves can probe only up to decoupling era.

dark age

decoupling 380kyr

reheating=Big Bang

inflation

multiproduction of universes

We can probe another tiny dark age between inflation and Big Bang Nucleosynthesis

Shedding new “lignt” on this epoch

Tensor Perturbations (Quantum Gravitational Waves)

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$$h_{ij} = h_+ \varepsilon_{ij}^+ + h_\times \varepsilon_{ij}^\times$$

transverse
-traceless

They are equivalent with two massless scalar fields.

$$h_{ij}(t, \boldsymbol{x}) = \sum_{\lambda=+,\times} \int \frac{d^3k}{(2\pi)^{3/2}} \epsilon_{ij}^\lambda(\boldsymbol{k}) h_{\boldsymbol{k}}^\lambda(t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

satisfies massless
Klein-Gordon eqn

Quantization in De Sitter background yields nearly scale-invariant long-wave perturbations during inflation.

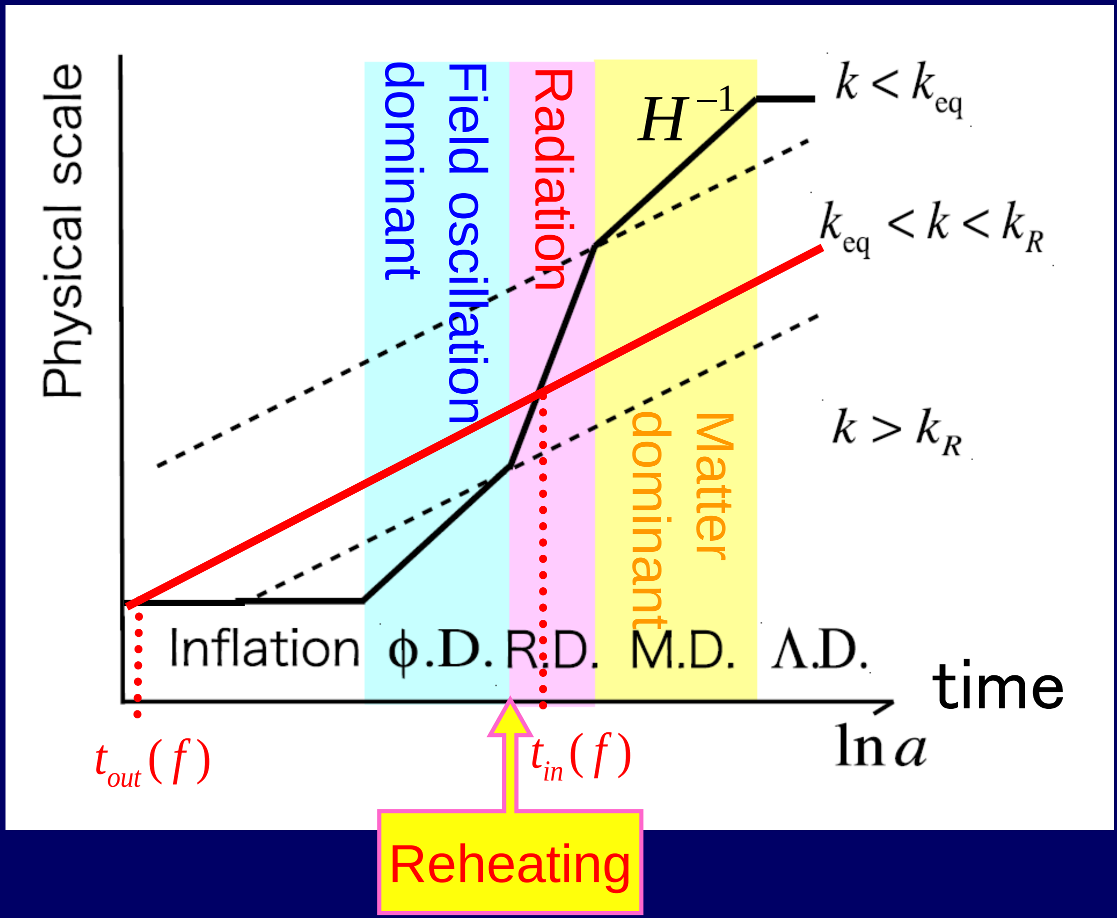
$$\Delta_h^2(k) = \langle h_{ij} h^{ij}(k) \rangle = 64\pi G \left(\frac{H(t_k)}{2\pi} \right)^2$$

Starobinsky (1979)

Evolution of gravitational waves in the inflationary Universe

- Amplitude of GW is constant when its wavelength is longer than the Hubble radius between $t_{out}(f)$ and $t_{in}(f)$.

- After entering the Hubble radius, the amplitude decreases as $\propto a^{-1}(t)$ and the energy density as $\propto a^{-4}(t)$.



When $a(t) \propto t^p \quad (p<1)$, the tensor perturbation evolves as

$$h(f,a) \propto a(t)^{\frac{1-3p}{2p}} J_{\frac{3p-1}{2(1-p)}}\left(\frac{p}{1-p}\frac{k}{a(t)H(t)}\right), \quad k = 2\pi f a(t_0)$$

Density parameter in GW per logarithmic frequency interval

$$\Omega_{GW}(f,t) = \frac{1}{\rho_{cr}(t)} \frac{d\rho_{GW}(f,t)}{d\ln f}$$

When the mode reentered the Hubble horizon at $t \equiv t_{in}(f)$,
the angular frequency is equal to $\omega = H\left(t_{in}(f)\right)$, so we find

$$\frac{d\rho_{GW}(f,t_{in}(f))}{d\ln f} = \frac{\omega^2}{32\pi G} h_{\inf}^2(f) = \frac{H^2\left(t_{in}(f)\right)}{32\pi G} h_{\inf}^2(f) = \frac{1}{24}\rho_{cr}(t_{in}(f))\Delta_h^2(f)$$

$$\boxed{\Omega_{GW}(f,t_{in}(f)) = \frac{1}{24}\Delta_h^2(f)}$$

After entering the Hubble horizon,

$$\Omega_{GW}(f,t) = \frac{\rho_{GW,\ln f}(f,t)}{\rho_{tot}(t)} \quad \begin{array}{l} \propto a^{-4}(t) \\ \propto a^{-3(1+w)}(t) \end{array}$$

$$w \equiv p/\rho_{tot}$$

: equation of state
in the early Universe

$$\Omega_{GW}(f,t) \approx \frac{1}{24}\Delta_h^2(f)\left(\frac{a(t_{in}(f))}{a(t)}\right)^{1-3w}$$

Radiation dominated era: constant

Field oscillation dominated era: decreases $\propto a^{-1}(t)$

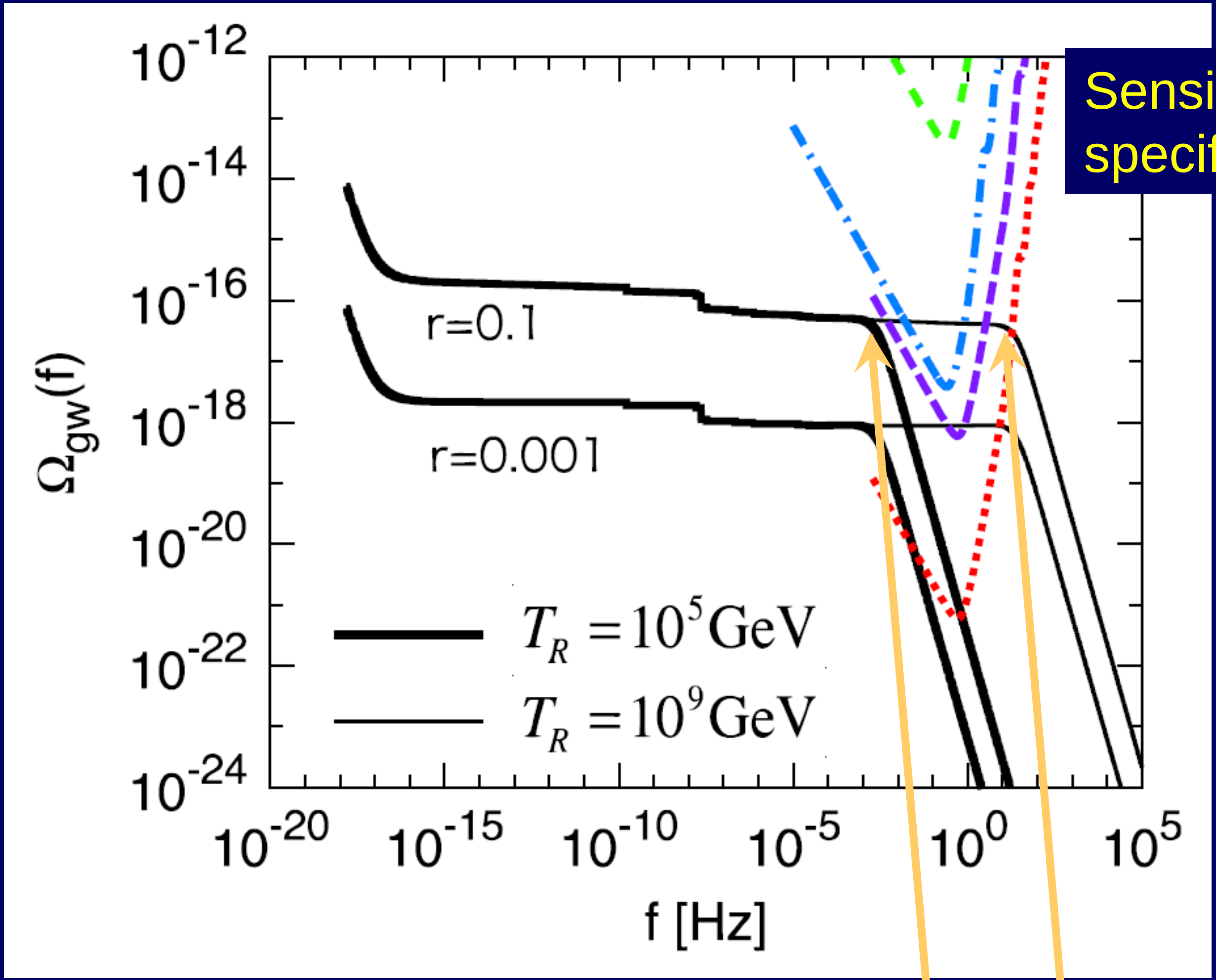
High frequency modes which entered the Hubble radius in the field oscillation regime acquires a suppression $\propto f^{-2}$.

We may determine the equation of state in the early Universe.

We may determine thermal history of the early Universe.

N. Seto & JY (03), Boyle & Steinhardt (08), Nakayama, Saito, Suwa, JY (08), Kuroyanagi et al (11)..

Thermal History is inprinted on the spectrum of GWs.



Sensitivity curves of various specifications of DECIGO

$$f_{\rm R} = \frac{k_{\rm R}}{2\pi a_0} \simeq 0.26{\rm Hz}\left(\frac{g_{*s}(T_{\rm R})}{106.75}\right)^{1/6}\left(\frac{T_{\rm R}}{10^7{\rm GeV}}\right)$$

Conceptual design of DECIGO

DECihertz Interferometer Gravitational-wave Observatory

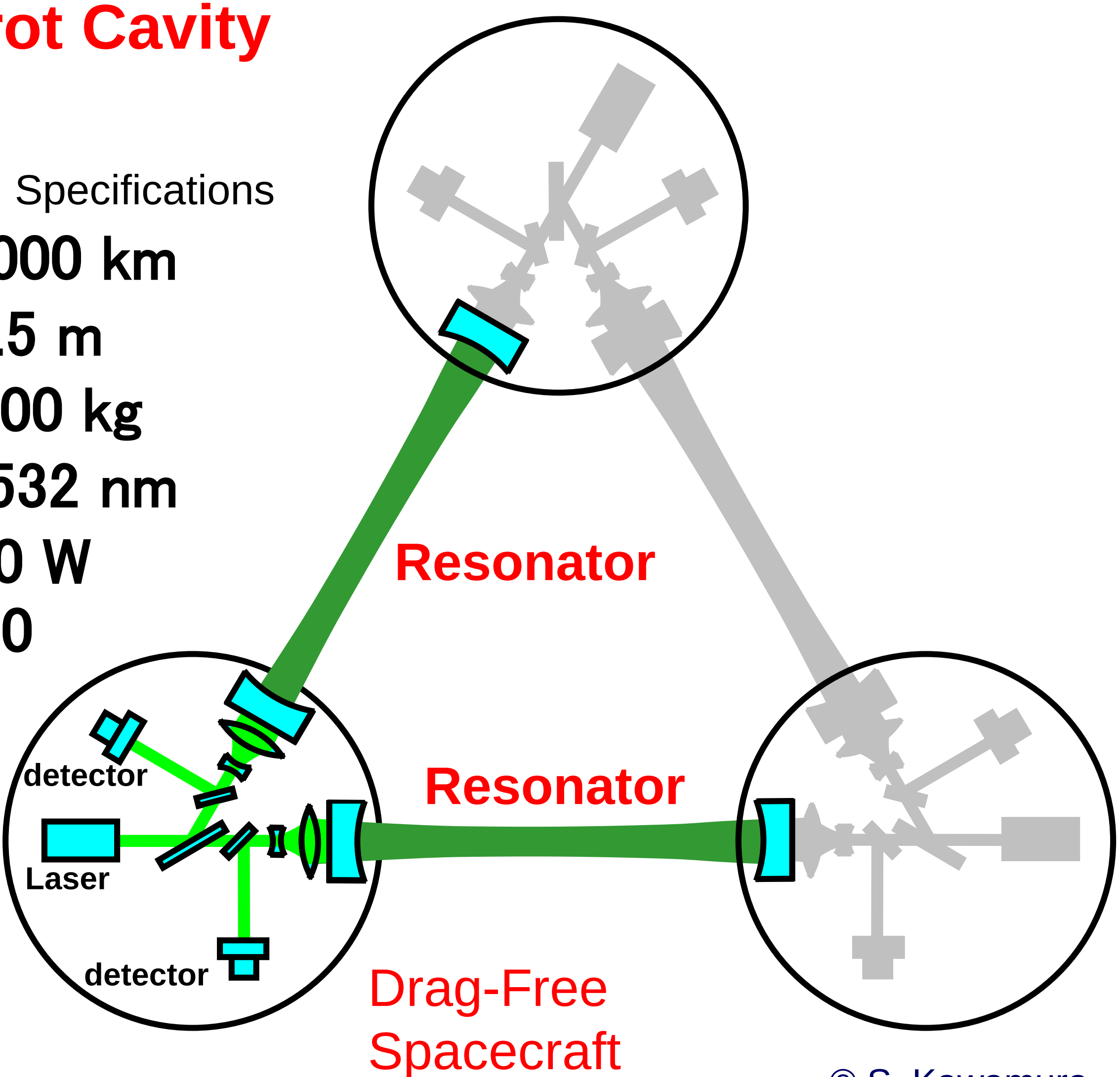
N. Seto, S. Kawamura, & T. Nakamura, PRL 87(2001)221103

Now include Fabry-Perot Cavity =Light Resonator

	Original Specifications
Arm length :	L=1000 km
Mirror Diameter :	R=0.5 m
Mirror Mass :	M=100 kg
Laser Wavelength :	λ =532 nm
Laser Power :	P=10 W
Finesse :	$\mathcal{F}$=10

Average time photons spend inside the resonator cavity

$$\tau_s \approx \frac{L}{c}\frac{\mathcal{F}}{\pi}$$



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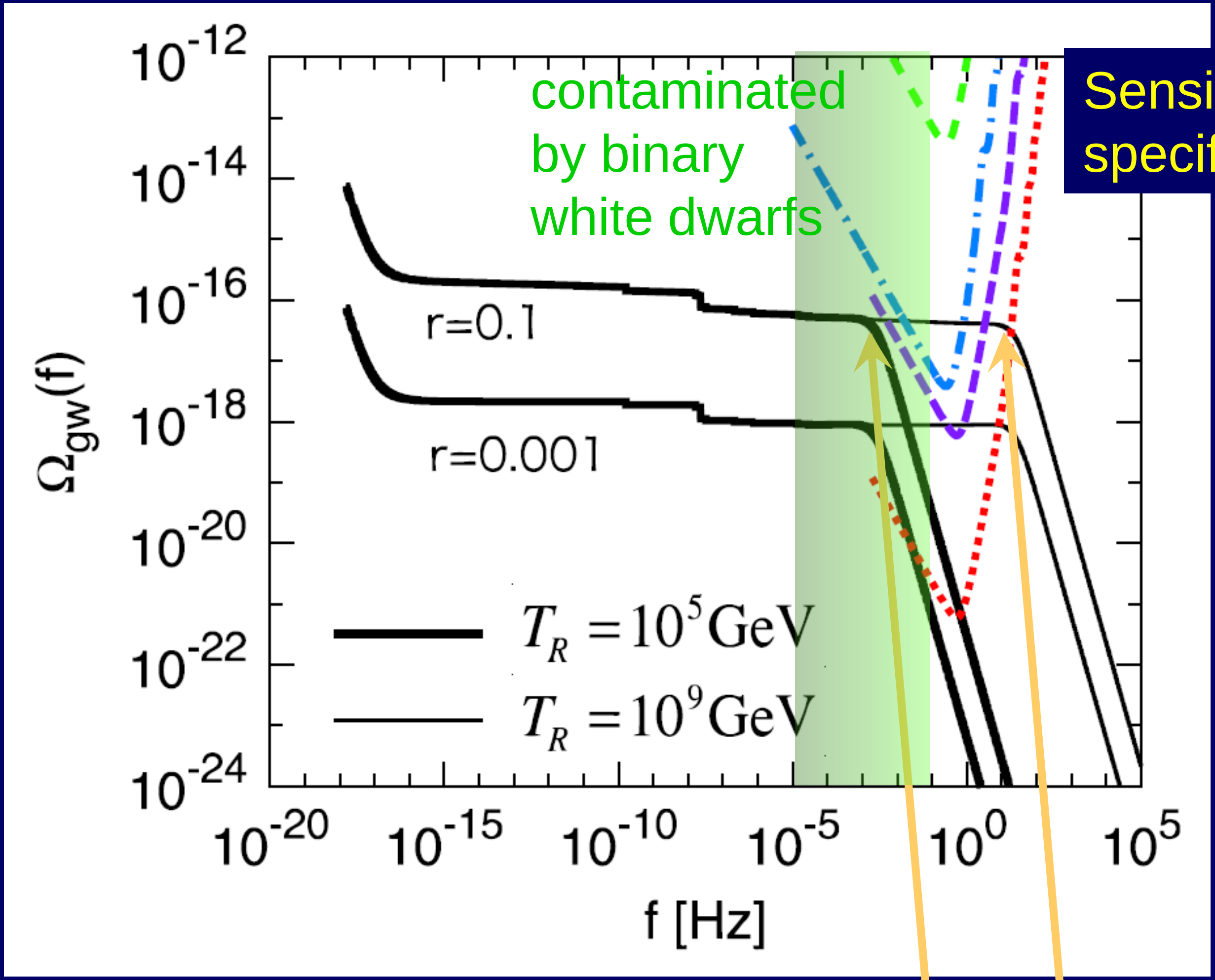
Thermal History is inprinted on the spectrum of GWs.



Sensitivity curves of various specifications

In order to probe higher reheating temperature we need sufficient sensitivity at higher frequency.

$$f_{\mathrm{R}} = \frac{k_{\mathrm{R}}}{2\pi a_0} \simeq 0.26\mathrm{Hz}\left(\frac{g_{*s}(T_{\mathrm{R}})}{106.75}\right)^{1/6}\left(\frac{T_{\mathrm{R}}}{10^7\mathrm{GeV}}\right)$$

At the present time, the energy density of GW is given by

$$\Omega_{GW}(f,t_0)=\frac{(2\pi f)^2}{12H_0^2}\Delta_h^2(f)T_h^2(f)=\frac{4\pi^2}{3H_0^2}f^3S_h^2(f)$$

Amplitude per logarithmic frequency interval $d\ln f$

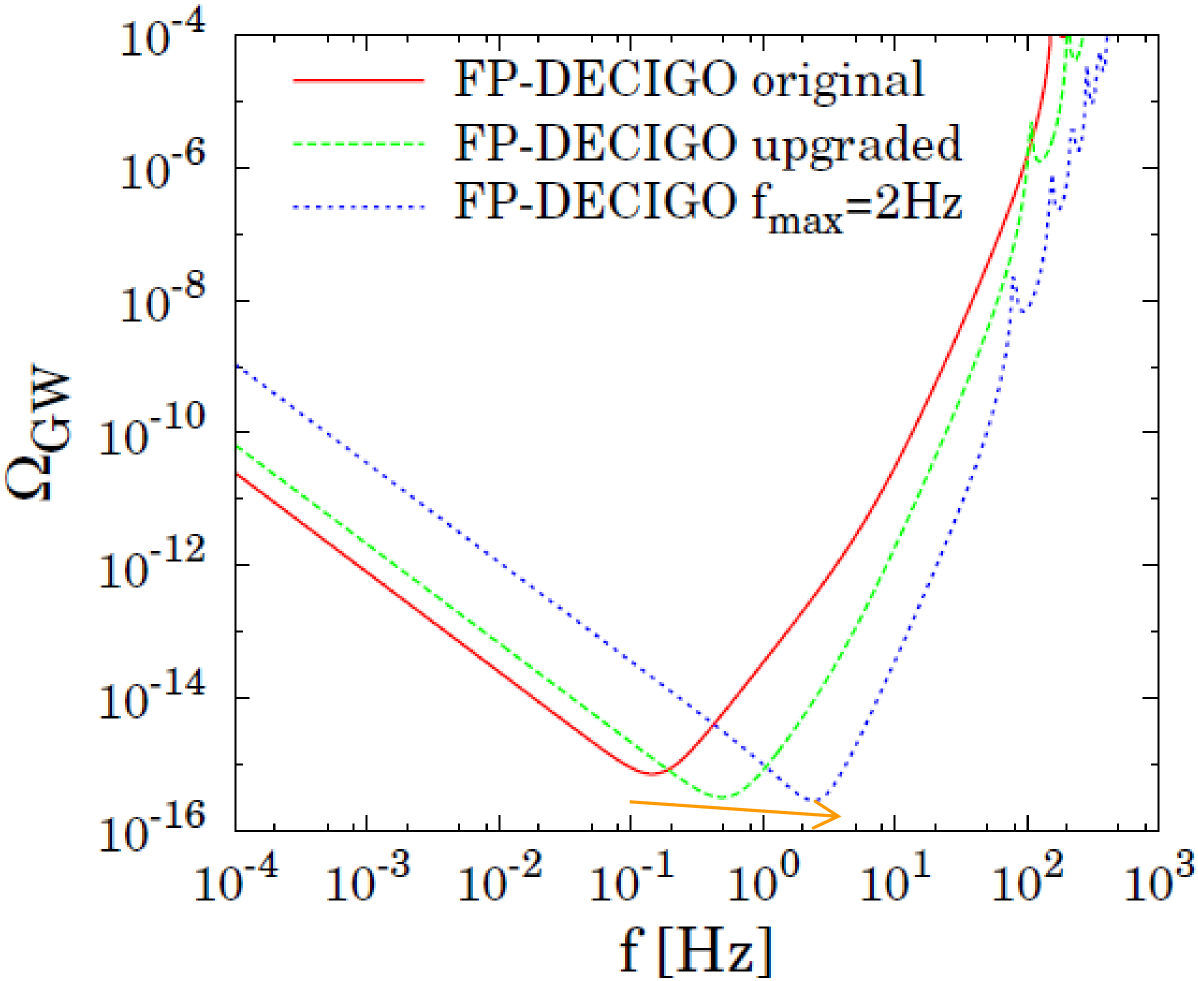
Transfer function depending on thermal history

Strain power spectrum with a measure df

This f^3 dependence makes it very difficult to detect higher frequency stochastic GWs.

$$\left\langle h_{ij}h^{ij}\right\rangle=\int_{-\infty}^{\infty}d\ln f\,\Delta_h^2(f)T_h^2(f)=2\int_{-\infty}^{\infty}df\,S_h^2(f)=4\int_{-\infty}^{\infty}d\ln f\,fS_h^2(f)$$

In order to probe higher frequency with the same sensitivity to Ω_{GW},



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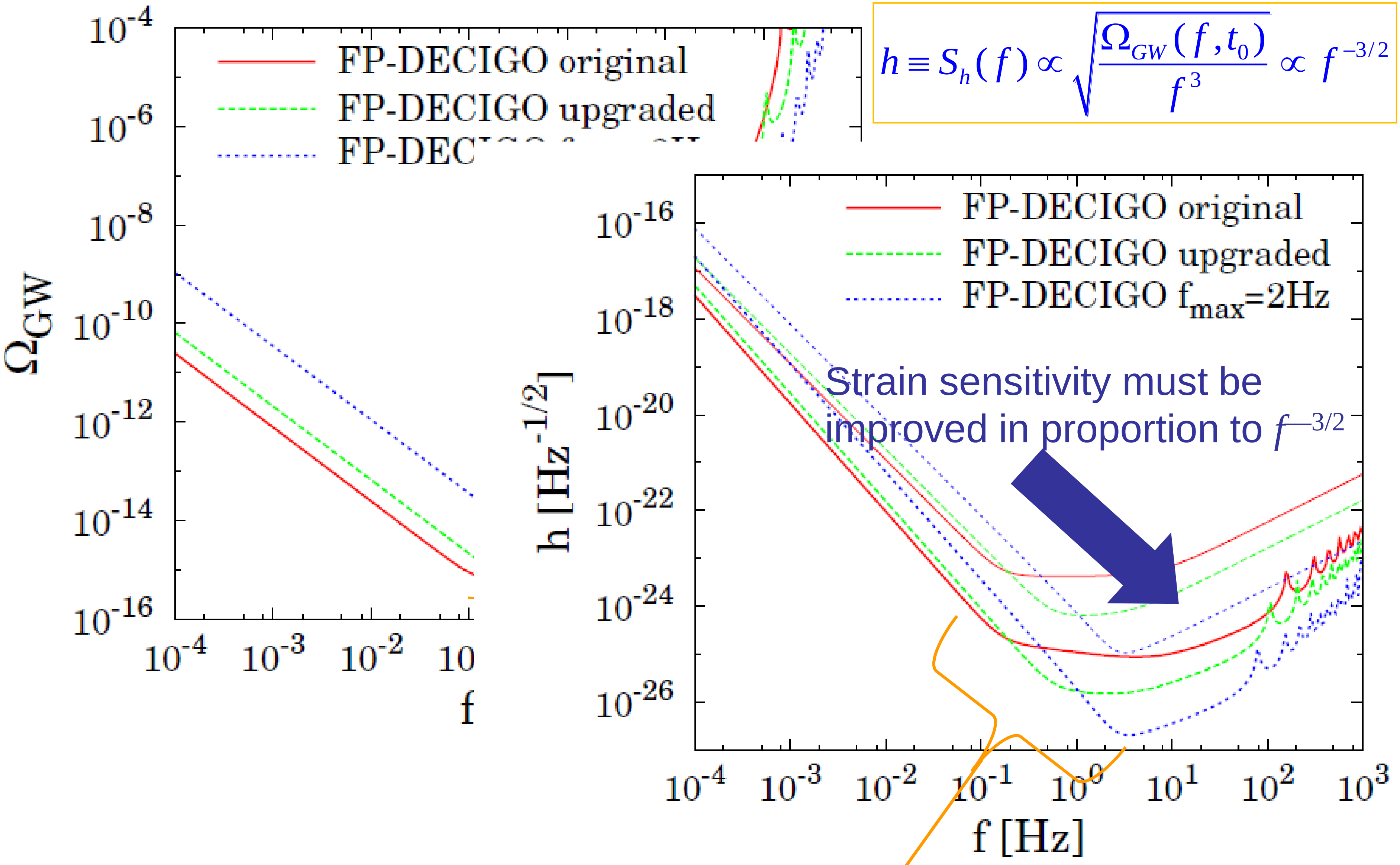
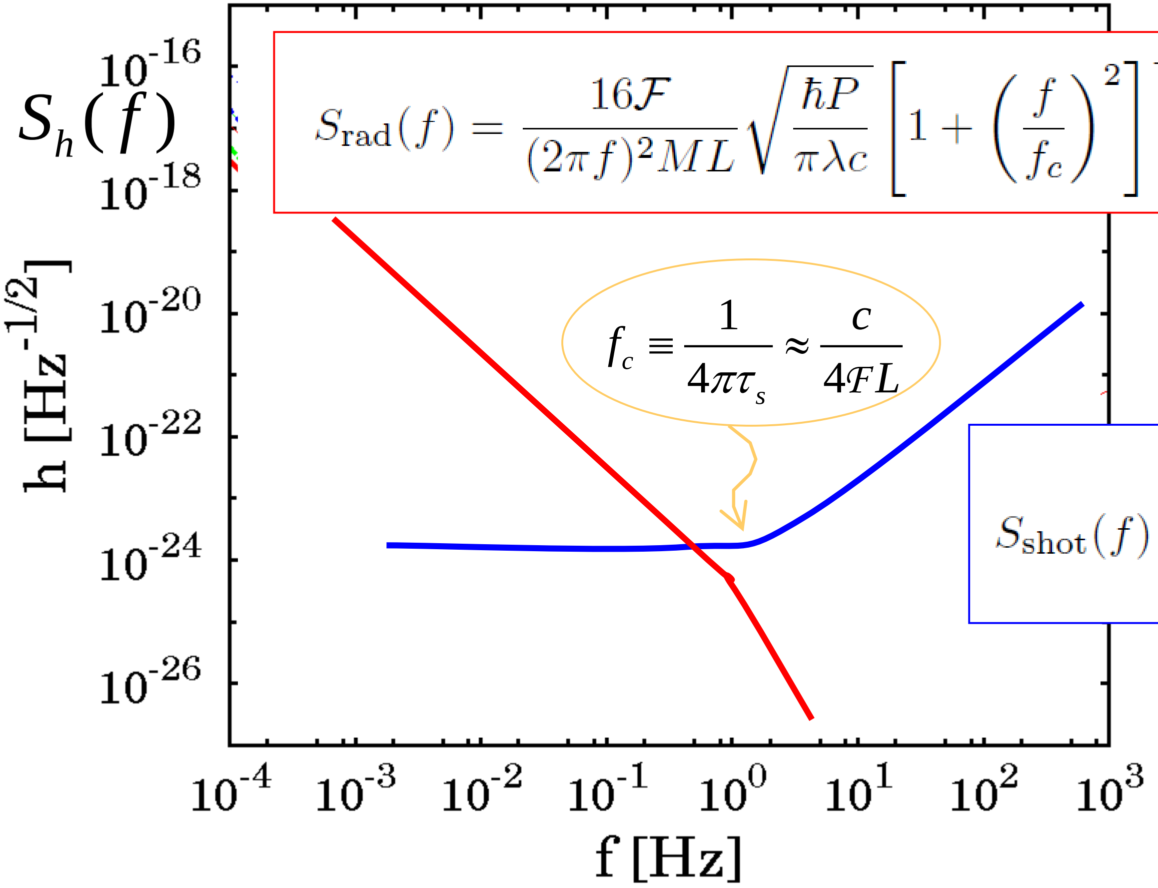


$$h \equiv S_h(f) \propto \sqrt{\frac{\Omega_{GW}(f,t_0)}{f^3}} \propto f^{-3/2}$$

Strain sensitivity must be improved in proportion to $f^{-3/2}$

Lower thicker curves indicate sensitivity achieved by 3yr correlation analysis

On the basis of BICEP2 result, we reconsider sensitivity curves of DECIGO for direct detection of inflationary GW & determination of the reheating temperature.



$$S_{\rm rad}(f) = \frac{16\mathcal{F}}{(2\pi f)^2 ML}\sqrt{\frac{\hbar P}{\pi\lambda c}}\left[1+\left(\frac{f}{f_c}\right)^2\right]^{-1/2}$$

Radiation Pressure Noise

Fluctuations in radiation pressure induces unwanted motion of the mirror

$$f_c \equiv \frac{1}{4\pi\tau_s} \approx \frac{c}{4\mathcal{F}L}$$

$$S_{\rm shot}(f) = \frac{\sqrt{\hbar\pi c\lambda}}{4\mathcal{F}L\sqrt{\tilde{P}}}\left[1+\left(\frac{f}{f_c}\right)^2\right]^{1/2}$$

Shot Noise

Poisson noise due to quantum nature of laser

In order to achieve sufficient sensitivity at higher frequency, it is important to suppress shot noise

$$S_{\text{shot}}(f) = \frac{\sqrt{\hbar\pi c\lambda}}{4\mathcal{F}L\sqrt{\tilde{P}}}\left[1+\left(\frac{f}{f_c}\right)^2\right]^{1/2}$$

by $\lambda \searrow$ $P \nearrow$ $\mathcal{F} \nearrow$ $L \nearrow$.

$$f_c \equiv \frac{1}{4\pi\tau_s} \approx \frac{c}{4\mathcal{F}L}$$

But $\mathcal{F} \nearrow$ $L \nearrow$ would also lowers $f_c \searrow$ and the frequency range of our interest would fall above f_c where we find

$$S_{shot}(f) \cong \sqrt{\frac{\hbar\pi\lambda}{c\tilde{P}}}\, f^{\,2}$$

Hence we can control the shot noise only by $\lambda \searrow$ $P \nearrow$.

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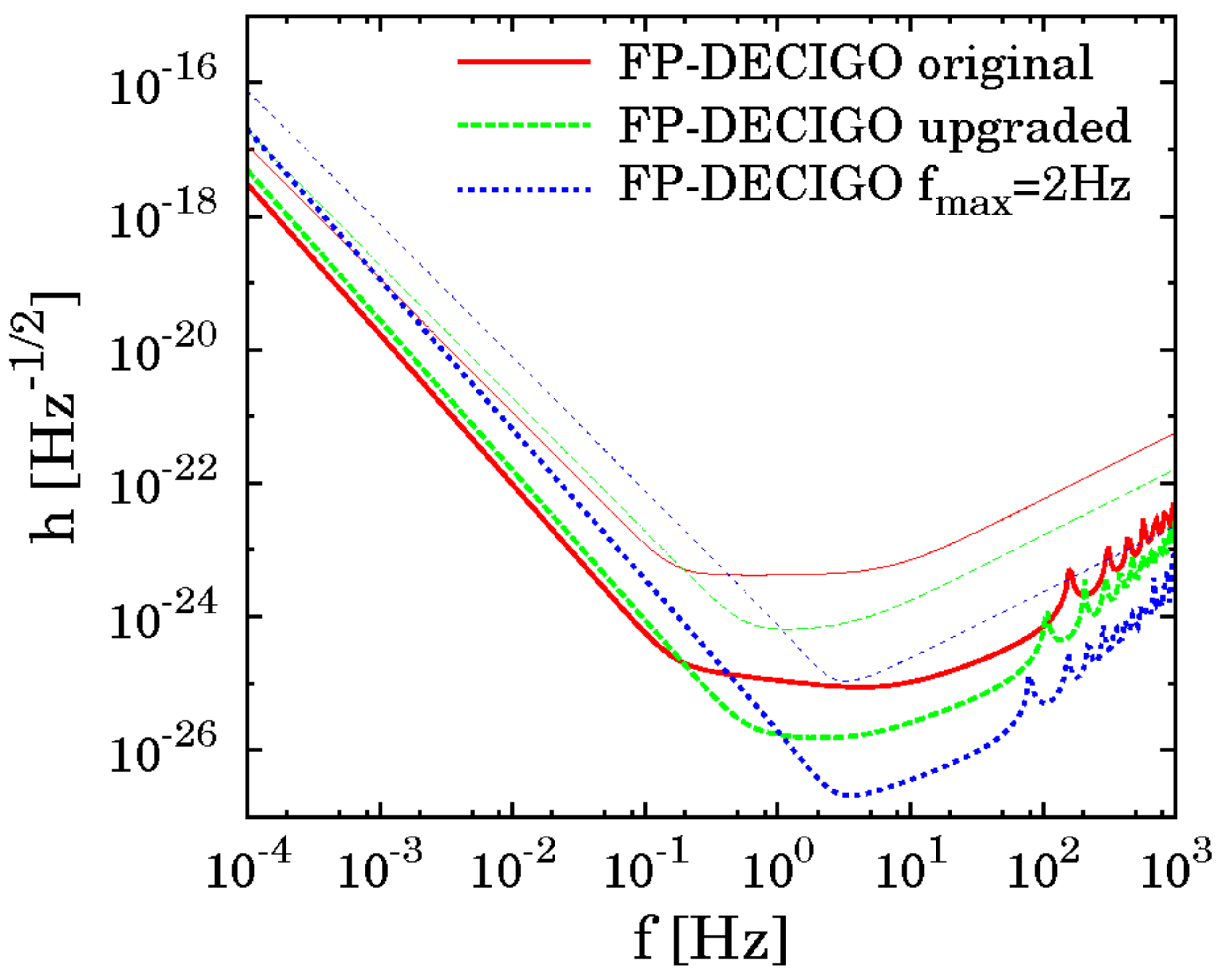

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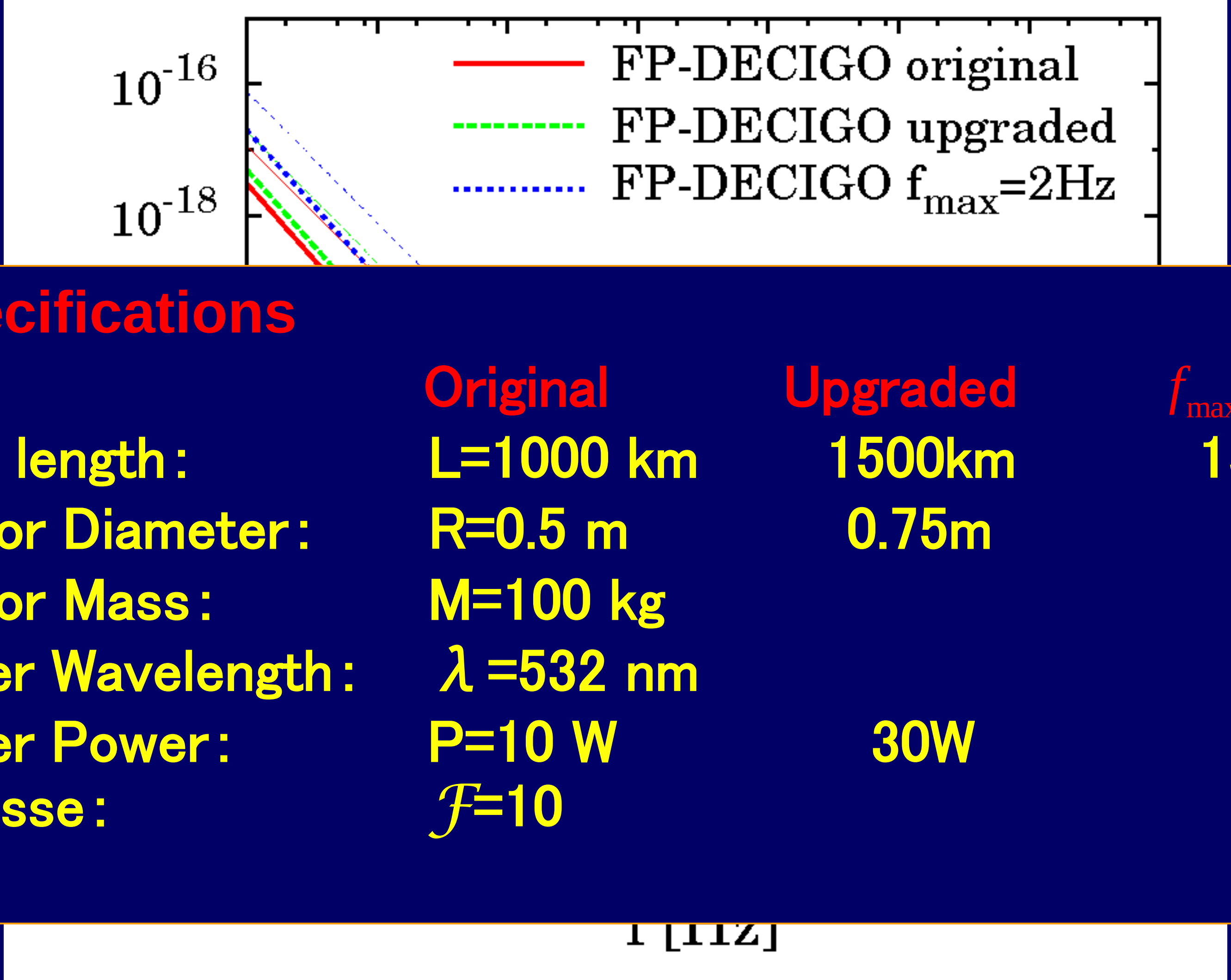


Specifications

	Original	Upgraded	$f_{max} = 2\mathrm{Hz}$
Arm length :	L=1000 km	1500km	1500km
Mirror Diameter :	R=0.5 m	0.75m	
Mirror Mass :	M=100 kg		
Laser Wavelength :	λ =532 nm		157nm
Laser Power :	P=10 W	30W	300W
Finesse :	$\mathcal{F}$=10		

We consider quadratic chaotic inflation

(Linde 83)

$$V[\phi] = \frac{1}{2} m^2 \phi^2$$

$$r \approx 0.14$$

and natural inflation

(Freese, Frieman, Olinto 90)

$$V[\phi] = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f} \right) \right]$$

with $f = 7M_{Pl}$ yielding $r \approx 0.07$

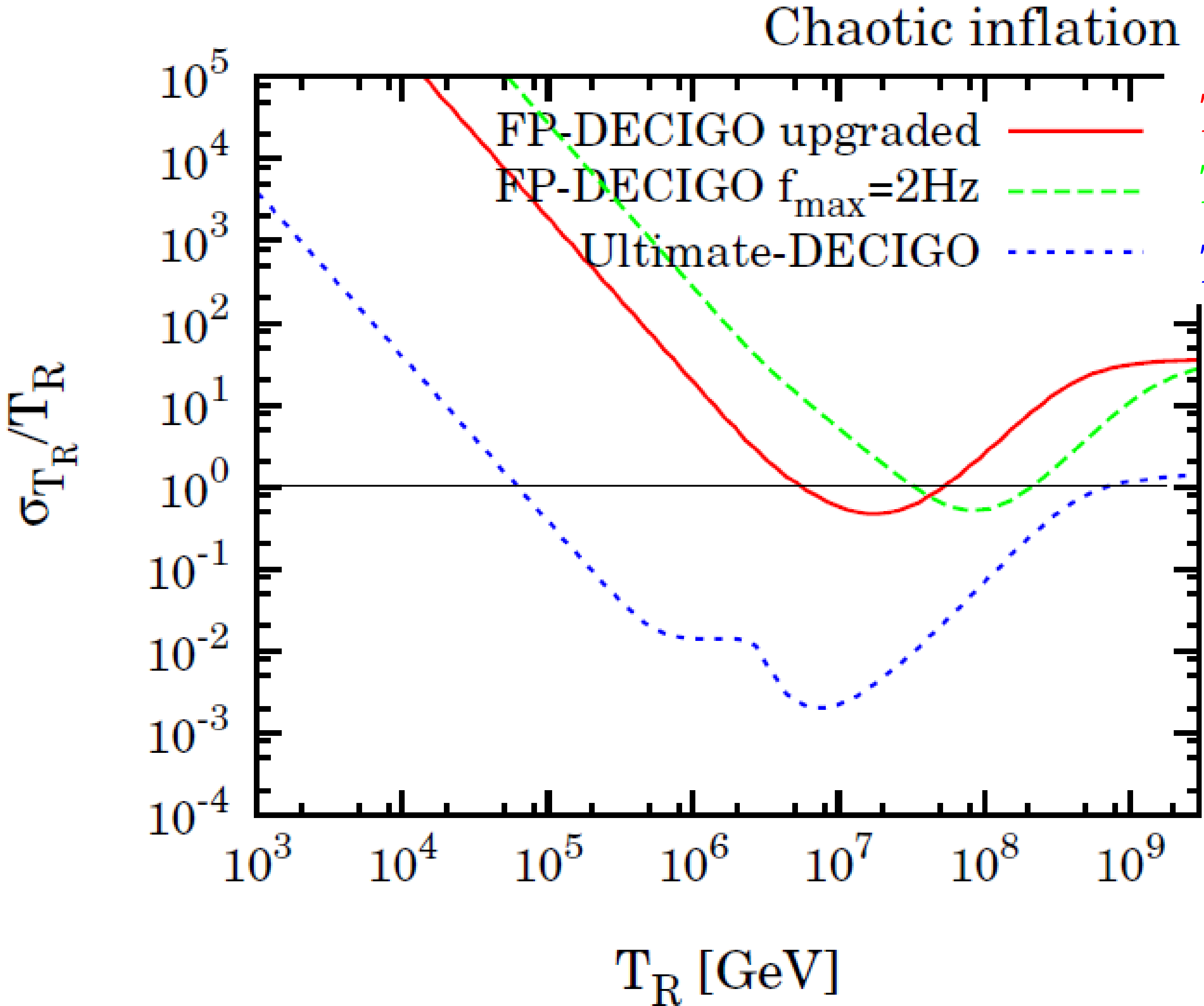
as fiducial models.

The original DECIGO does not have sufficient sensitivity to detect the stochastic GW background predicted by these models.

We determine maximum possible reheat temperature DECIGO can measure by Fisher matrix analysis for upgraded, $f_{\max} = 2\mathrm{Hz}$ and ultimate versions.

noises are assumed to be quantum limited.

Marginalized 1σ uncertainty in T_R as a fraction of T_R for quadratic chaotic inflation



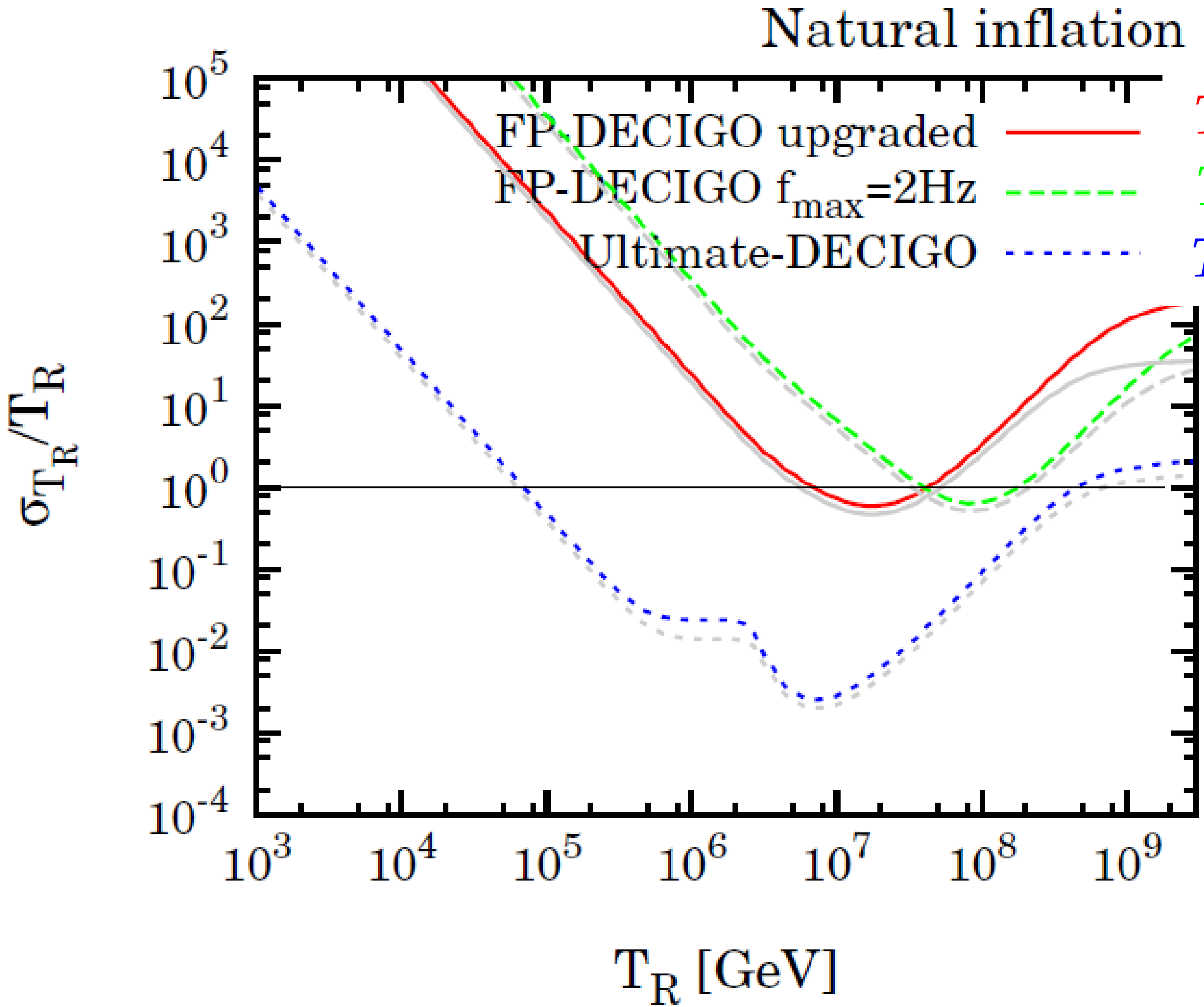
T_R can be determined within 1σ if

$T_R < 5.2\times10^{7}$ GeV

$T_R < 2.1\times10^{8}$ GeV

$T_R < 7.0\times10^{8}$ GeV

Marginalized 1σ uncertainty in T_R as a fraction of T_R for natural inflation with $f = 7M_{Pl}$



T_R can be determined within 1σ if

$T_R < 4.5\times10^7\,\mathrm{GeV}$

$T_R < 1.8\times10^8\,\mathrm{GeV}$

$T_R < 4.6\times10^8\,\mathrm{GeV}$

DECIGO can measure the reheat temperature T_R if it lies in the range $5\times10^{6}\,\mathrm{GeV}<T_R<2\times10^{8}\,\mathrm{GeV}$

The ultimate DECIGO can measure the reheat temperature T_R if it lies in the range $6\times10^{4}\,\mathrm{GeV}<T_R<7\times10^{8}\,\mathrm{GeV}$

DECIGO can measure the reheat temperature T_R if it lies in the range $5\times10^6\,\mathrm{GeV} < T_R < 2\times10^8\,\mathrm{GeV}$

The ultimate DECIGO can measure the reheat temperature T_R if it lies in the range $6\times10^4\,\mathrm{GeV} < T_R < 7\times10^8\,\mathrm{GeV}$

One may naïvely think that high-scale inflation predicts high reheat temperature, and the upper bound we obtained is too low.

However, in order to realize high-scale inflation with a large r and a large field excursion $\phi_{\mathrm{i}} - \phi_e \gg M_{Pl}$ (Lyth-Turner Bound) we often introduce symmetries in model building

- Chaotic inflation: Shift symmetry (Kawasaki, Yamaguchi, Yanagida 00)
- Natural inflation: Nambu-Goldstone (Freese, Frieman, Olinto 90)

which also constrain coupling of the inflaton and delay reheating.

★ An example of Chaotic inflation in Supergravity

$$K = \frac{1}{2}(\phi + \phi^\dagger)^2 + |X|^2 + |H_u|^2 + |H_d|^2,$$
$$W = mX\phi + yXH_uH_d,$$

$$\Rightarrow \quad V[\phi] = \frac{1}{2}m^2\left(\mathrm{Im}\,\phi\right)^2$$

$\mathrm{Im}\,\phi$ has a shift symmetry and act as the inflaton.
The Universe is reheated through Higgs bosons & Higgsinos.

$$T_R \simeq 4\times10^8\left(\frac{y}{10^{-6}}\right)\ \mathrm{GeV}$$

(Nakayama, Takahashi, Yanagida 13)

$y < 10^{-6}$ is required for the stability of the inflaton's trajectory.

★ The natural inflation model

$$V[\phi] = \Lambda^4\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$$\Gamma_\phi \approx g^2\frac{M^3}{f^2} \approx g^2\frac{\Lambda^6}{f^5} \quad \Rightarrow \quad T_R \approx 5\times10^7\left(\frac{g}{0.1}\right)\ \mathrm{GeV} \quad \text{for } f = 7M_{Pl}$$

$$M \equiv \frac{\Lambda^2}{f}$$

(Freese, Frieman, Olinto 90)

Conclusion

BICEP2 may have determined when inflation took place.

DECIGO/BBO may be able to determine when Big Bang happened.

上州無智亦無大

剛毅木訥易被欺

唯以正直接萬人

至誠依神期勝利

鑑三