Prospects of Determination of Thermal History After Inflation with DECIGO

Space-baced Laser Interferoeter

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Detection of Primordial Tensor Perturbation from Inflation by BICEP2...

\[ r \equiv \frac{\Delta^2_h}{\Delta^2_R} \]

Tensor-to-scalar ratio measures the scale of inflation

\[ V[\phi] = (3.2 \times 10^{16} \text{ GeV})^4 r = (7.5 \times 10^{15} \text{ GeV})^4 \left( \frac{r}{0.003} \right) \]

Higher Frequency Tensor Perturbation Carries Information on the Thermal History After Inflation
Why Gravitational Waves?

We can probe another tiny dark age between inflation and Big Bang Nucleosynthesis. Shedding new “light” on this epoch.

Gravitational waves can probe up to inflation era.

Electromagnetic waves can probe only up to decoupling era.

Today 13.8Gyr

dark energy

galaxy formation

dark age

multiproduction of universes

reheating = Big Bang

inflation

decoupling 380 kyr
They are equivalent with two massless scalar fields.

Quantization in De Sitter background yields nearly scale-invariant long-wave perturbations during inflation.

Starobinsky (1979)
Amplitude of GW is constant when its wavelength is longer than the Hubble radius between $t_{out}(f)$ and $t_{in}(f)$.

After entering the Hubble radius, the amplitude decreases as $\propto a^{-1}(t)$ and the energy density as $\propto a^{-4}(t)$.

When $a(t) \propto t^p$ ($p < 1$), the tensor perturbation evolves as

$$h(f, a) \propto a(t)^{1-3p/2p} J^{3p-1/2(1-p)} \left( \frac{p}{1-p} \frac{k}{a(t)H(t)} \right)^{1/2}, \quad k = 2\pi f a(t_0)$$
Density parameter in GW per logarithmic frequency interval

\[ \Omega_{GW}(f, t) = \frac{1}{\rho_{cr}(t)} \frac{d \rho_{GW}(f, t)}{d \ln f} \]

When the mode reentered the Hubble horizon at \( t \equiv t_{in}(f) \), the angular frequency is equal to \( \omega = H(t_{in}(f)) \), so we find

\[ \frac{d \rho_{GW}(f, t_{in}(f))}{d \ln f} = \frac{\omega^2}{32\pi G} h_{inf}^2(f) = \frac{H^2(t_{in}(f))}{32\pi G} h_{inf}^2(f) = \frac{1}{24} \rho_{cr}(t_{in}(f)) \Delta_h^2(f) \]

\[ \Omega_{GW}(f, t_{in}(f)) = \frac{1}{24} \Delta_h^2(f) \]
After entering the Hubble horizon,

\[ \Omega_{GW}(f, t) = \frac{\rho_{GW, \ln f}(f, t)}{\rho_{tot}(t)} \propto a^{-4}(t) \]

\[ \propto a^{-3(1+w)}(t) \]

\[ w \equiv \frac{p}{\rho_{tot}} \]

: equation of state in the early Universe

\[ \Omega_{GW}(f, t) \approx \frac{1}{24} \Delta_h^2(f) \left( \frac{a(t_{in}(f))}{a(t)} \right)^{1-3w} \]

Radiation dominated era: constant
Field oscillation dominated era: decreases \( \propto a^{-1}(t) \)

High frequency modes which entered the Hubble radius in the field oscillation regime acquires a suppression \( \propto f^{-2} \).

We may determine the equation of state in the early Universe.

We may determine thermal history of the early Universe.

N. Seto & JY (03), Boyle & Steinhardt (08), Nakayama, Saito, Suwa, JY (08), Kuroyanagi et al (11)....
Thermal History is inprinted on the spectrum of GWs.

Sensitivity curves of various specifications of DECIGO

\[ f_R = \frac{k_R}{2\pi a_0} \approx 0.26\text{Hz} \left( \frac{g_{*s}(T_R)}{106.75} \right)^{1/6} \left( \frac{T_R}{10^7\text{GeV}} \right) \]
Conceptual design of DECIGO
DECihertz Interferometer Gravitational-wave Observatory

Now include Fabry-Perot Cavity
=Light Resonator

Original Specifications

Arm length: \( L = 1000 \text{ km} \)
Mirror Diameter: \( R = 0.5 \text{ m} \)
Mirror Mass: \( M = 100 \text{ kg} \)
Laser Wavelength: \( \lambda = 532 \text{ nm} \)
Laser Power: \( P = 10 \text{ W} \)
Finesse: \( \mathcal{F} = 10 \)

Average time photons spend inside the resonator cavity

\[ \tau_s \approx \frac{L \mathcal{F}}{c \pi} \]
Thermal History is inprinted on the spectrum of GWs.

In order to probe higher reheating temperature we need sufficient sensitivity at higher frequency.

\[ f_R = \frac{k_R}{2\pi a_0} \approx 0.26 \text{Hz} \left( \frac{g_{*s}(T_R)}{106.75} \right)^{1/6} \left( \frac{T_R}{10^7 \text{GeV}} \right) \]
At the present time, the energy density of GW is given by

\[
\Omega_{GW}(f,t_0) = \left(\frac{2\pi f}{12H_0^2}\right)^2 \Delta_h^2(f)T_h^2(f) = \frac{4\pi^2}{3H_0^2} f^3 S_h^2(f)
\]

Amplitude per logarithmic frequency interval \(d\ln f\)

Transfer function depending on thermal history

Strain power spectrum with a measure \(df\)

This \(f^3\) dependence makes it very difficult to detect higher frequency stochastic GWs.

\[
\left\langle h_{ij}h^{ij}\right\rangle = \int_{-\infty}^{\infty} d\ln f \Delta_h^2(f)T_h^2(f) = 2 \int_{-\infty}^{\infty} df S_h^2(f) = 4 \int_{-\infty}^{\infty} d\ln f f S_h^2(f)
\]
In order to probe higher frequency with the same sensitivity to $\Omega_{GW}$,
In order to probe higher frequency with the same sensitivity to $\Omega_{GW}$, Strain sensitivity must be improved in proportion to $f^{-3/2}$.

Lower thicker curves indicate sensitivity achieved by 3yr correlation analysis.
On the basis of BICEP2 result, we reconsider sensitivity curves of DECIGO for direct detection of inflationary GW & determination of the reheating temperature.

\[
S_{\text{rad}}(f) = \frac{16F}{(2\pi f)^2 ML} \sqrt{\frac{\hbar P}{\pi \lambda c}} \left[ 1 + \left( \frac{f}{f_c} \right)^2 \right]^{-1/2}
\]

\[
S_{\text{shot}}(f) = \frac{\sqrt{\hbar \pi c \lambda}}{4FL \sqrt{\tilde{P}}} \left[ 1 + \left( \frac{f}{f_c} \right)^2 \right]^{1/2}
\]

Radiation Pressure Noise
Fluctuations in radiation pressure induces unwanted motion of the mirror

Shot Noise
Poisson noise due to quantum nature of laser
In order to achieve sufficient sensitivity at higher frequency, it is important to suppress shot noise by \( \lambda \downarrow P \uparrow F \uparrow L \uparrow \).

But \( F \uparrow L \uparrow \) would also lower \( f_c \) and the frequency range of our interest would fall above \( f_c \) where we find

\[
S_{\text{shot}}(f) \approx \sqrt{\frac{\hbar \pi \lambda}{c \tilde{P}}} f^2
\]

Hence we can control the shot noise only by \( \lambda \downarrow P \uparrow \).
On the basis of BICEP2 result, we reconsider sensitivity curves of DECIGO for direct detection of inflationary GW & determination of the reheating temperature.
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<table>
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<tr>
<th>Specifications</th>
<th>Original</th>
<th>Upgraded</th>
<th>$f_{\text{max}} = 2\text{Hz}$</th>
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<tr>
<td>Arm length:</td>
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<td>300W</td>
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The figure shows the sensitivity curves for FP-DECIGO original, upgraded, and with $f_{\text{max}}=2\text{Hz}$. The x-axis is labeled as [FZ], likely a typo, and should be [Hz].
We consider quadratic chaotic inflation
\[ V[\phi] = \frac{1}{2} m^2 \phi^2 \]
and natural inflation
\[ V[\phi] = \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right] \]
\[ f = 7 M_{Pl} \]
yielding
\[ r \approx 0.07 \]
as fiducial models.

The original DECIGO does not have sufficient sensitivity to
detect the stochastic GW background predicted by these models.

We determine maximum possible reheat temperature DECIGO
can measure by Fisher matrix analysis for upgraded, \( f_{max} = 2 \text{Hz} \)
and ultimate versions.

noises are assumed to be quantum limited.
Marginalized $1\sigma$ uncertainty in $T_R$ as a fraction of $T_R$ for quadratic chaotic inflation

$T_R$ can be determined within $1\sigma$ if

$T_R < 5.2 \times 10^7 \text{ GeV}$

$T_R < 2.1 \times 10^8 \text{ GeV}$

$T_R < 7.0 \times 10^8 \text{ GeV}$
Marginalized $1\sigma$ uncertainty in $T_R$ as a fraction of $T_R$ for natural inflation with $f = 7M_{Pl}$

$T_R$ can be determined within $1\sigma$ if

- $T_R < 4.5 \times 10^7 \text{ GeV}$
- $T_R < 1.8 \times 10^8 \text{ GeV}$
- $T_R < 4.6 \times 10^8 \text{ GeV}$
DECIGO can measure the reheat temperature $T_R$ if it lies in the range $5 \times 10^6 \text{GeV} < T_R < 2 \times 10^8 \text{GeV}$

The ultimate DECIGO can measure the reheat temperature $T_R$ if it lies in the range $6 \times 10^4 \text{GeV} < T_R < 7 \times 10^8 \text{GeV}$
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One may naively think that high-scale inflation predicts high reheat temperature, and the upper bound we obtained is too low.

However, in order to realize high-scale inflation with a large $r$ and a large field excursion $\phi_i - \phi_e \gg M_{Pl}$ (Lyth-Turner Bound) we often introduce symmetries in model building

- Chaotic inflation: Shift symmetry
- Natural inflation: Nambu-Goldstone

which also constrain coupling of the inflaton and delay reheating.
An example of Chaotic inflation in Supergravity

\[
K = \frac{1}{2} (\phi + \phi^\dagger)^2 + |X|^2 + |H_u|^2 + |H_d|^2,
\]

\[
W = mX\phi + yXH_uH_d,
\]

\[
V[\phi] = \frac{1}{2} m^2 (\text{Im} \phi)^2
\]

\(\text{Im} \phi\) has a shift symmetry and act as the inflaton. The Universe is reheated through Higgs bosons & Higgsinos.

\[
T_R \approx 4 \times 10^8 \left( \frac{y}{10^{-6}} \right) \text{GeV}
\]

\(y < 10^{-6}\) is required for the stability of the inflaton’s trajectory.

The natural inflation model

\[
V[\phi] = \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right]
\]

\[
\Gamma_\phi \approx g^2 \frac{M^3}{f^2} \approx g^2 \frac{\Lambda^6}{f^5}
\]

\[
M = \frac{\Lambda^2}{f}
\]

\[
T_R \approx 5 \times 10^7 \left( \frac{g}{0.1} \right) \text{GeV} \quad \text{for} \quad f = 7M_{\text{Pl}}
\]

(Nakayama, Takahashi, Yanagida 13)

(Freese, Frieman, Olinto 90)
Conclusion

BICEP2 may have determined when inflation took place.

DECIGO/BBO may be able to determine when Big Bang happened.